Introduction to Physics

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PREFACE

About This Book

Introduction to Physics is an adaptation of *Concepts of Physics*, itself an adaptation based on modules from OpenStax *College Physics*. Some of the modules removed from *College Physics* have been put back in, most notably modules on momentum, rotation and special relativity. Sections on linear momentum has been added from James Rittenbach's derived copy of *Concepts of Physics*. Additional minor edits have been made as necessary.

This textbook is intended for a one-semester introduction to physics course, requiring as little mathematics as possible. Students are expected to have taken some algebra and geometry at high-school level, but no higher level of mathematics is required (and use of algebra and geometry is minimized as much as possible, in order to emphasize the concepts of physics). The courses this textbook is intended for often go by the name of "conceptual physics," "descriptive physics," or "introduction to physics."

This edition is put together for use by College of Alameda's Physics 10 course, starting in Summer 2018 (updated and revised as needed thereafter). Much gratitude is due to Prof. Bobby Bailey for putting together *Concepts of Physics* and to OpenStax for making this Open Educational Resource available.

Andrew Park, Physics Instructor at College of Alameda, June 2018

P.S. Original "Preface" to *Concepts of Physics* follows below.

Preface to Concepts of Physics

Concepts of Physics is an adaptation based on modules from OpenStax College's textbook *College Physics*.

This text is intended for use in a one-semester physical science course that requires algebra but no trigonometry.

Not all physics topics are covered in this work. Of the nearly 300 original *College Physics* modules, only 100 were retained. For these selected modules, content was modified (as needed) to remove/replace any trigonometry-related topics, examples, homework, or figures. Occasionally, original content was edited (or replaced) to reflect the level, and coverage, of the course. Notation was modified in a few cases for consistency between modules. Explicit links between modules (and discussions related to non-retained modules) were removed. Finally, all direct links to PHeT plugin-based (Java or Flash) physics simulations were removed.

The authors of *College Physics* are commended for providing such a comprehensive body of high-quality material. Special thanks to them and to OpenStax. Additional thanks to University of Maryland University College (UMUC) and Debra F. McLaughlin.

Bobby Bailey, January 2015

About OpenStax College

OpenStax College is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of modern college courses. Unlike traditional textbooks, OpenStax College resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax College is working to improve access to higher education for all. OpenStax College is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

1 INTRODUCTION

Figure 1.1 Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature's apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics)

Chapter Outline

1.1. Physics: An Introduction
- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

1.2. Physical Quantities and Units
- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

Science and the Realm of Physics, Physical Quantities, and Units

What is your first reaction when you hear the word "physics"? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people's regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, "invisibility cloaks" that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

1.1 Physics: An Introduction

Figure 1.2 The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone (**Figure 1.3**). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

Figure 1.3 The Apple “iPhone” is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See **Figure 1.4** and **Figure 1.5**.) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car’s ignition system as well as the transmission of electrical signals through our body’s nervous system are much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes (**Figure 1.6** and **Figure 1.7**). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

Figure 1.4 The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

Figure 1.5 These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)

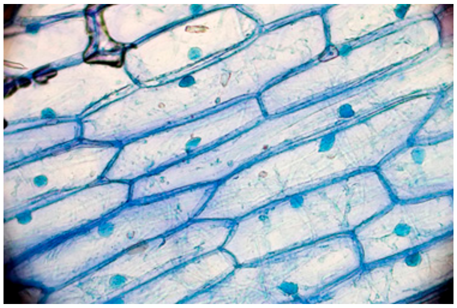


Figure 1.6 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

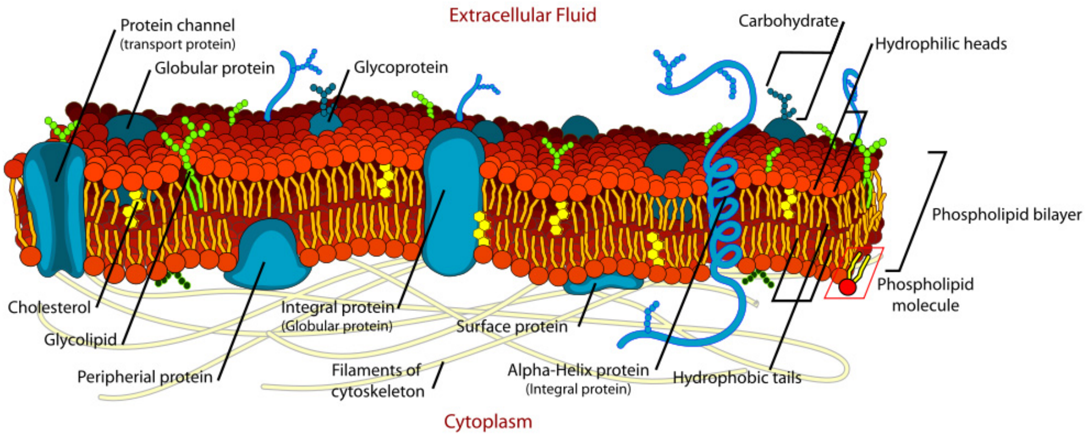


Figure 1.7 An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See **Figure 1.8** and **Figure 1.9**.) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.

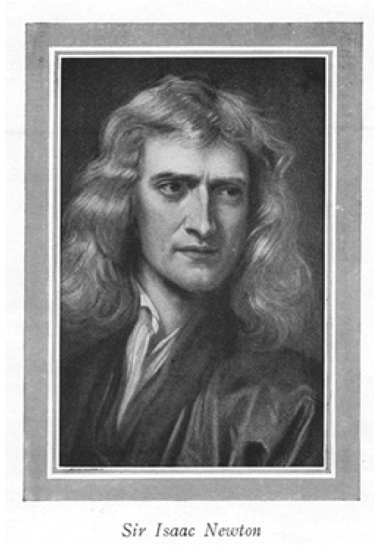


Figure 1.8 **Isaac Newton** (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Figure 1.9 **Marie Curie** (1867–1934) sacrificed monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the nucleus, analogous to the way planets orbit the Sun. (See **Figure 1.10**.) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton's theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton's second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal's principle, which is applicable only in fluids), but the distinction between laws and principles often is not carefully made.

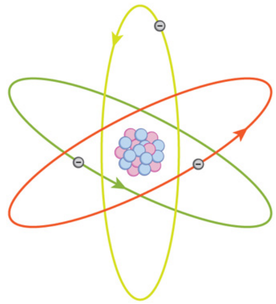


Figure 1.10 What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Models, Theories, and Laws

Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

The Scientific Method

As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called "natural philosophy." From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See **Figure 1.11**, **Figure 1.12**, and **Figure 1.13**.) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

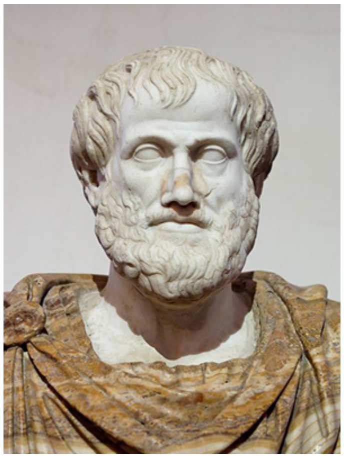


Figure 1.11 Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow (2006)/Ludovisi Collection)

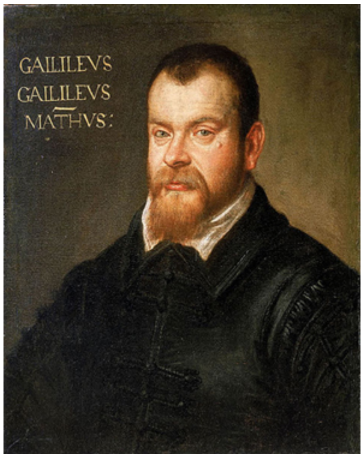


Figure 1.12 **Galileo Galilei** (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)

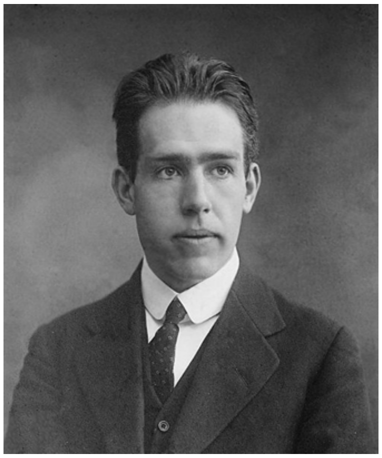


Figure 1.13 **Niels Bohr** (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.

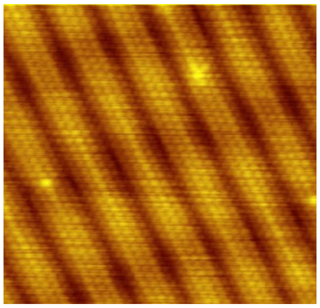


Figure 1.14 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics,* and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results.

Check Your Understanding

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution
Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

1.2 Physical Quantities and Units

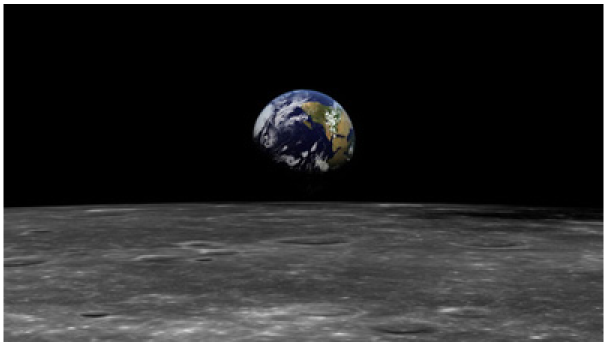


Figure 1.15 The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See **Figure 1.16**.)

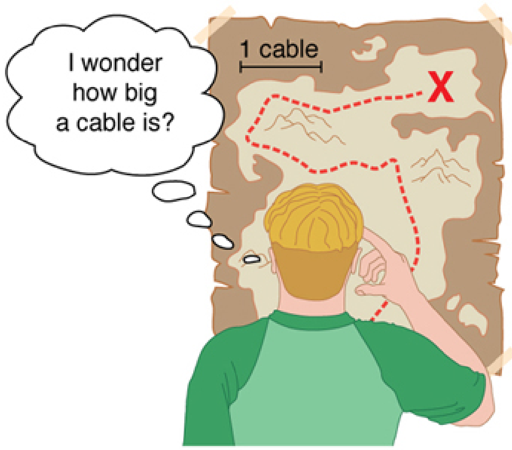


Figure 1.16 Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym "SI" is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

Table 1.1 gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Table 1.1 Fundamental SI Units

Length	Mass	Time	Electric Current
meter (m)	kilogram (kg)	second (s)	ampere (A)

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth's rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See **Figure 1.17**.) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.

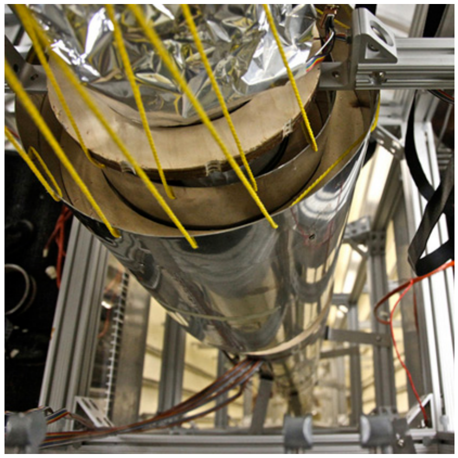


Figure 1.17 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See **Figure 1.18**.) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.

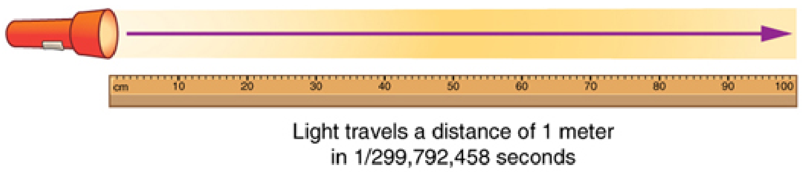


Figure 1.18 The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is currently defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass. In a very near future, the kilogram is expected to be re-defined in a similar as the meter, by precisely fixing the numerical value of a fundamental physical constant.

Electric current and its accompanying unit, the ampere, will be introduced when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. **Table 1.2** gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the Sun is on the order of 10^9 m.

Table 1.2 Metric Prefixes for Powers of 10 and their Symbols

Prefix	Symbol	Value[1]	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^9	gigahertz	GHz	10^9 Hz	a microwave frequency
mega	M	10^6	megacurie	MCi	10^6 Ci	high radioactivity
kilo	k	10^3	kilometer	km	10^3 m	about 6/10 mile
hecto	h	10^2	hectoliter	hL	10^2 L	26 gallons
deka	da	10^1	dekagram	dag	10^1 g	teaspoon of butter
—	—	10^0 (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:

$$80\cancel{\text{m}}\times\frac{1\text{ km}}{1000\cancel{\text{m}}}=0.080\text{ km}. \tag{1.1}$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

1. See Appendix A for a discussion of powers of 10.

Example 1.1 Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}. \tag{1.2}$$

(2) Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}. \tag{1.3}$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr . Thus,

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}. \tag{1.4}$$

Discussion for (a)

To check your answer, consider the following:

(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2}, \tag{1.5}$$

which are obviously not the desired units of km/h.

(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.

(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.

(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.

(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.

(2) Multiplying by these yields

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}}, \tag{1.6}$$

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}. \tag{1.7}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Check Your Understanding

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution
The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Check Your Understanding

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution
The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Glossary

classical physics: physics that was developed from the Renaissance to the end of the 19th century

conversion factor: a ratio expressing how many of one unit are equal to another unit

derived units: units that can be calculated using algebraic combinations of the fundamental units

English units: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units: units that can only be expressed relative to the procedure used to measure them

kilogram: the SI unit for mass, abbreviated (kg)

law: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

meter: the SI unit for length, abbreviated (m)

metric system: a system in which values can be calculated in factors of 10

model: representation of something that is often too difficult (or impossible) to display directly

modern physics: the study of relativity, quantum mechanics, or both

order of magnitude: refers to the size of a quantity as it relates to a power of 10

physical quantity : a characteristic or property of an object that can be measured or calculated from other measurements

physics: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

quantum mechanics: the study of objects smaller than can be seen with a microscope

relativity: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

scientific method: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

second: the SI unit for time, abbreviated (s)

SI units : the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

theory: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

units : a standard used for expressing and comparing measurements

Section Summary

1.1 Physics: An Introduction

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

1.2 Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

1.1 Physics: An Introduction

1. Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

2. How does a model differ from a theory?

3. If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

4. What determines the validity of a theory?

5. Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

6. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

7. Classical physics is a good approximation to modern physics under certain circumstances. What are they?

8. When is it *necessary* to use relativistic quantum mechanics?

9. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

1.2 Physical Quantities and Units

10. Identify some advantages of metric units.

Problems & Exercises

1.2 Physical Quantities and Units

1. The speed limit on some interstate highways is roughly 100 km/h. (a) What is this in meters per second? (b) How many miles per hour is this?

2. A car is traveling at a speed of 33 m/s. (a) What is its speed in kilometers per hour? (b) Is it exceeding the 90 km/h speed limit?

3. Show that $1.0\text{ m/s} = 3.6\text{ km/h}$. Hint: Show the explicit steps involved in converting $1.0\text{ m/s} = 3.6\text{ km/h.}$

4. American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)

5. Soccer fields vary in size. A large soccer field is 115 m long and 85 m wide. What are its dimensions in feet and inches? (Assume that 1 meter equals 3.281 feet.)

6. What is the height in meters of a person who is 6 ft 1.0 in. tall? (Assume that 1 meter equals 39.37 in.)

7. Mount Everest, at 29,028 feet, is the tallest mountain on the Earth. What is its height in kilometers? (Assume that 1 kilometer equals 3,281 feet.)

8. The speed of sound is measured to be 342 m/s on a certain day. What is this in km/h?

9. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. (a) What distance does it move in 1 s at this speed? (b) What is its speed in kilometers per million years?

I UNIT 1: MECHANICS I - MOTION AND FORCES

2 KINEMATICS

Figure 2.1 The motion of an American kestrel through the air can be described by the bird's displacement, speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Chapter Outline

2.1. Displacement
- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

2.2. Vectors, Scalars, and Coordinate Systems
- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

2.3. Time, Velocity, and Speed
- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Derive a graph of velocity vs. time given a graph of position vs. time.
- Interpret a graph of velocity vs. time.

2.4. Acceleration
- Define and distinguish between velocity and acceleration, and between instantaneous and average acceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

2.5. Motion Equations for Constant Acceleration in One Dimension
- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

2.6. Falling Objects
- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

2.7. Projectile Motion
- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, and trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

2.8. Centripetal Acceleration
- Explain what centripetal acceleration is.
- Use the formula for centripetal acceleration in simple situations.

Introduction to One-Dimensional Kinematics

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion.

2.1 Displacement

Figure 2.2 These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor’s position could be described in terms of where she is in relation to the nearby white board. (See **Figure 2.3**.) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See **Figure 2.4**.)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object’s position changes. This change in position is known as **displacement**. The word “displacement” implies that an object has moved, or has been displaced.

Displacement

Displacement is the *change in position* of an object:

$$\Delta x = x_{\text{f}} - x_0, \tag{2.1}$$

where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

In this text the upper case Greek letter Δ (delta) always means “change in” whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_{f} .

Note that the SI unit for displacement is the meter (m), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.

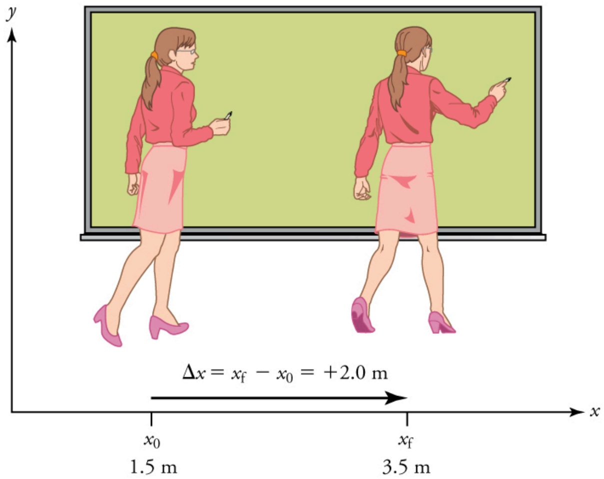


Figure 2.3 A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0 \text{ m}$ displacement of the professor relative to Earth is represented by an arrow pointing to the right.

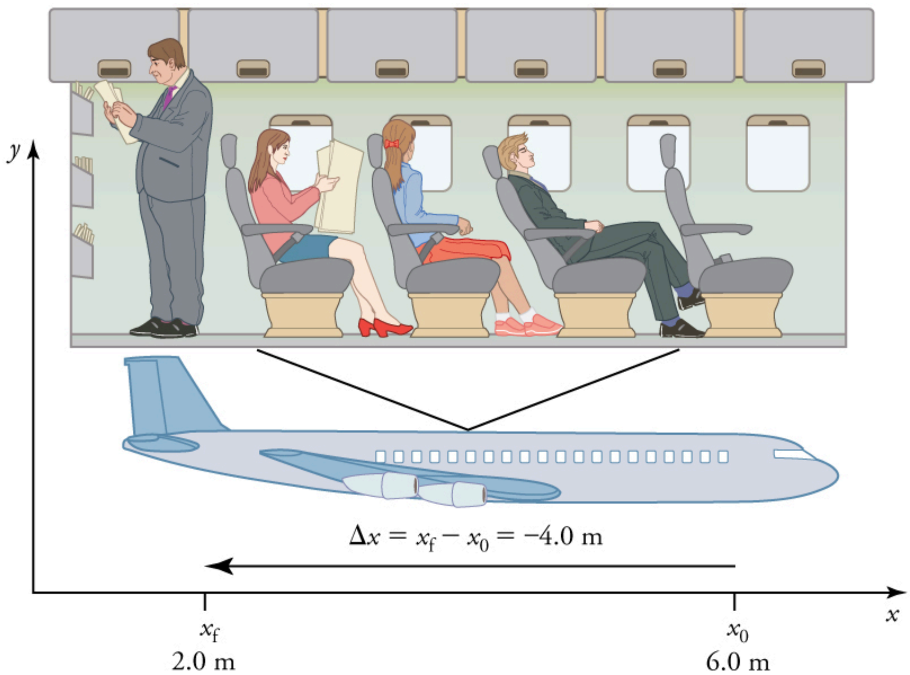


Figure 2.4 A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in **Figure 2.3**.

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5 \text{ m}$ and her final position is $x_\text{f} = 3.5 \text{ m}$. Thus her displacement is

$$\Delta x = x_\text{f} - x_0 = 3.5 \text{ m} - 1.5 \text{ m} = +2.0 \text{ m}. \tag{2.2}$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0 \text{ m}$ and his final position is $x_\text{f} = 2.0 \text{ m}$, so his displacement is

$$\Delta x = x_\text{f} - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}. \tag{2.3}$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Misconception Alert: Distance Traveled vs. Magnitude of Displacement

It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Check Your Understanding

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution

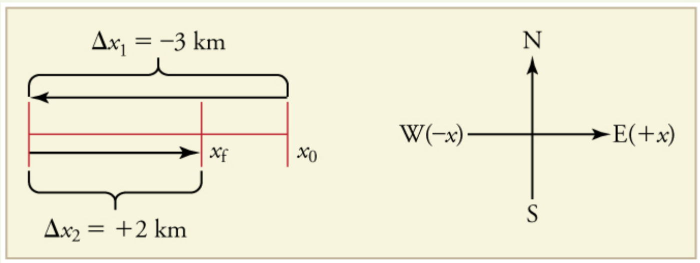


Figure 2.5

(a) The rider's displacement is $\Delta x = x_{\mathrm{f}} - x_0 = -1\text{ km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\text{ km} + 2\text{ km} = 5\text{ km}$.

(c) The magnitude of the displacement is 1 km .

2.2 Vectors, Scalars, and Coordinate Systems

Figure 2.6 The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x -coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in Figure 2.6, it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.

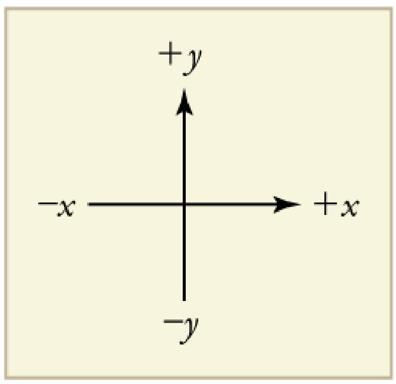


Figure 2.7 It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(-)$.

Check Your Understanding

A person's speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution
Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

2.3 Time, Velocity, and Speed

Figure 2.8 The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

The most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics, the definition of time is simple— **time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** $\boldsymbol{\Delta t}$ is the difference between the ending time and beginning time,

$$\Delta t = t_\text{f} - t_0, \tag{2.4}$$

where Δt is the change in time or elapsed time, t_f is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel*,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0}, \tag{2.5}$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively.

Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4\text{ m}}{5\text{ s}} = -0.8\text{ m/s}. \tag{2.6}$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.

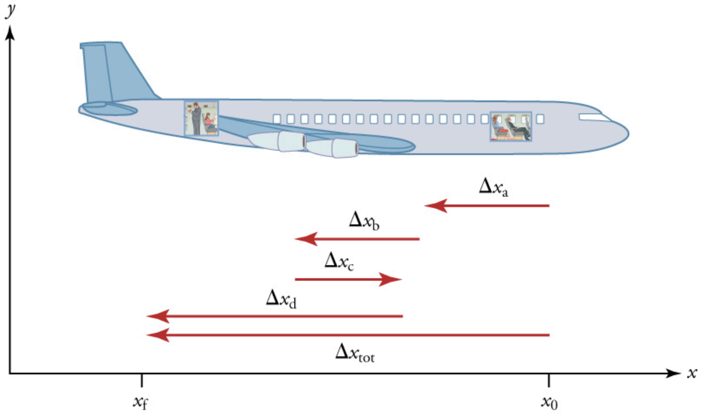


Figure 2.9 A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v , at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.

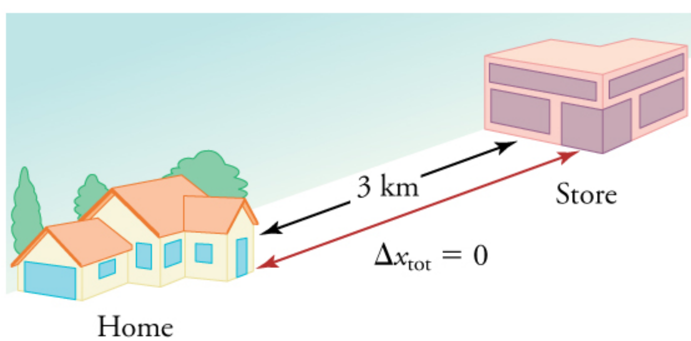


Figure 2.10 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in **Figure 2.11**. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)

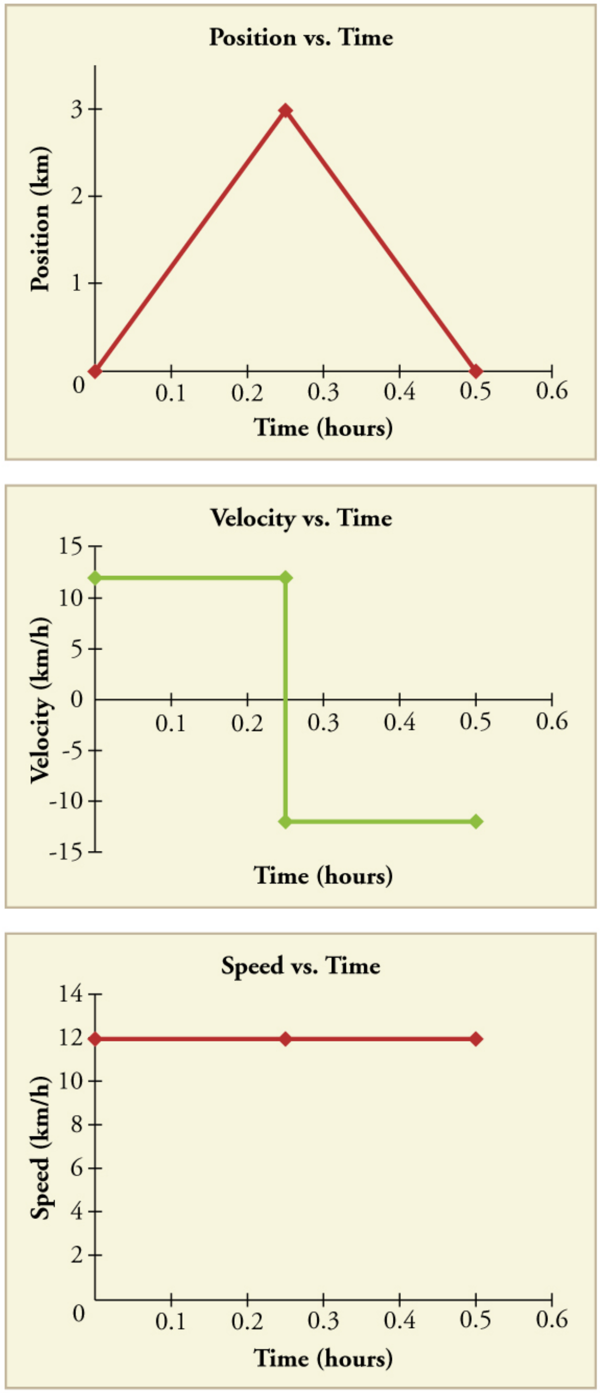


Figure 2.11 Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Check Your Understanding

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution

(a) The average velocity of the train is zero because $x_\text{f} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

$$\frac{\text{distance}}{\text{time}} = \frac{80 \text{ miles}}{105 \text{ minutes}} \tag{2.7}$$

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s} \tag{2.8}$$

2.4 Acceleration

Figure 2.12 A plane slows down as it comes in for landing in St. Maarten. It is accelerating in a direction opposite to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Average Acceleration

Average Acceleration is *the rate at which velocity changes*,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_\text{f} - v_0}{t_\text{f} - t_0}, \tag{2.9}$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are m/s^2 , meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Acceleration as a Vector

Acceleration is a vector in the same direction as the *change* in velocity, Δv . Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. If acceleration is in a direction opposite to the direction of motion, the object slows down.

Figure 2.13 A subway train in Sao Paulo, Brazil, slows down as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Example 2.1 Calculating Acceleration: A Racehorse Leaves the Gate

A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

Figure 2.14 (credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.

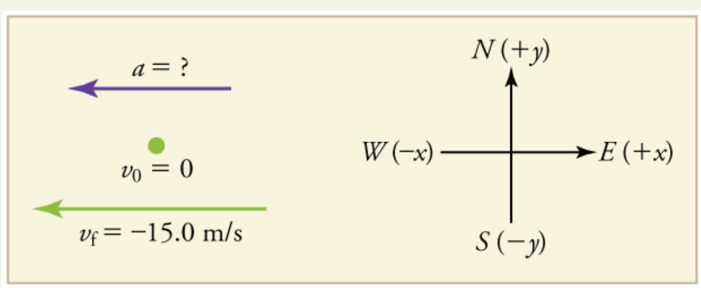


Figure 2.15

We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final

velocity: $\Delta v = v_{\text{f}} = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2. \tag{2.10}$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2 . This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

Instantaneous acceleration a , or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. **Figure 2.16** shows graphs of instantaneous acceleration versus time for two very different motions. In **Figure 2.16**(a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In **Figure 2.16**(b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and -2.0 m/s^2 , respectively.

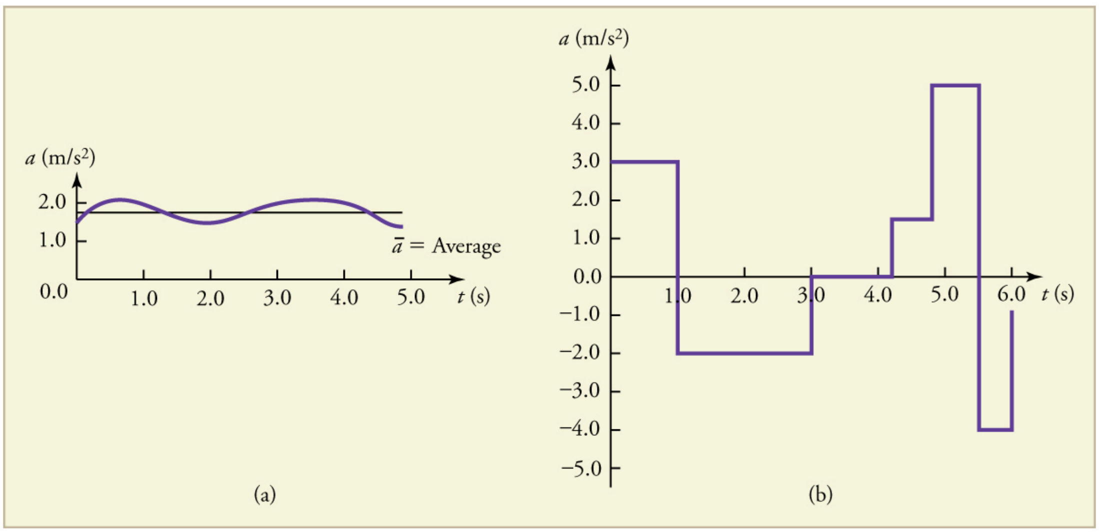


Figure 2.16 Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in **Figure 2.17**. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.

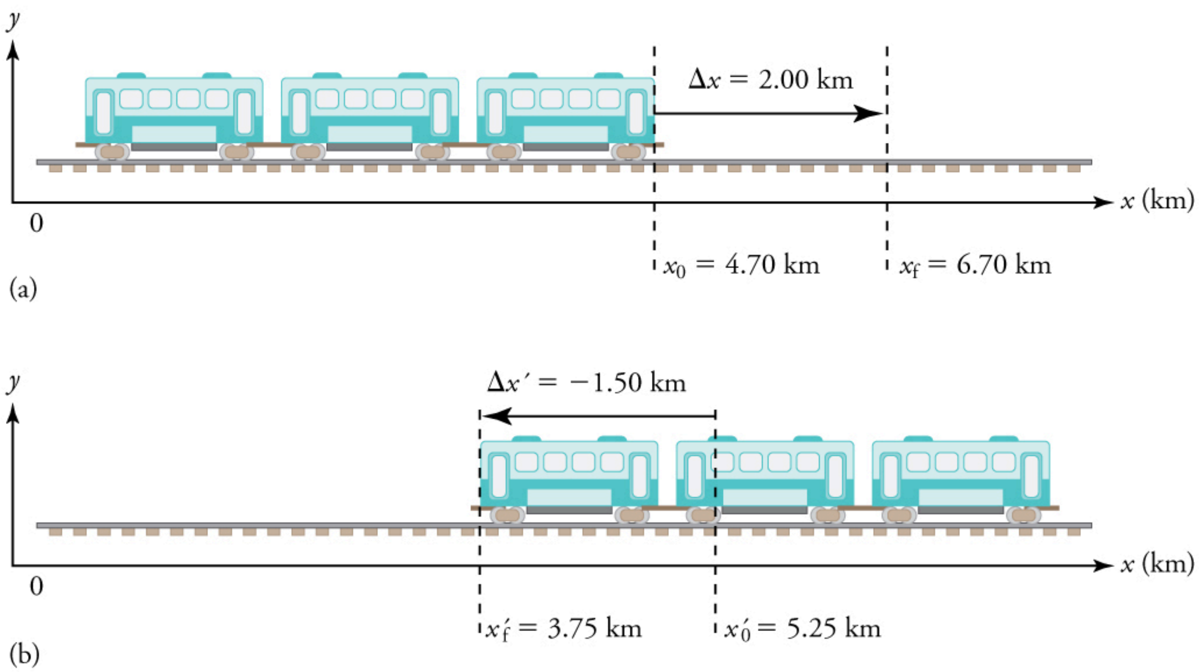


Figure 2.17 One-dimensional motion of a subway train considered in **Example 2.2**, **Example 2.3**, **Example 2.4**, **Example 2.5**, **Example 2.6**, and **Example 2.7**. Here we have chosen the x -axis so that + means to the right and $-$ means to the left for displacements, velocities, and accelerations. (a) The subway train moves to the right from x_0 to x_f . Its displacement Δx is +2.0 km. (b) The train moves to the left from x'_0 to x'_f . Its displacement $\Delta x'$ is -1.5 km . (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example 2.2 Calculating Displacement: A Subway Train

What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of **Figure 2.17**?

Strategy

A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_\text{f} - x_0$. This is straightforward since the initial and final positions are given.

Solution

1. Identify the knowns. In the figure we see that $x_\text{f} = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x'_\text{f} = 3.75\text{ km}$ and $x'_0 = 5.25\text{ km}$ for part (b).

2. Solve for displacement in part (a).

$$\Delta x = x_\text{f} - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km} \tag{2.11}$$

3. Solve for displacement in part (b).

$$\Delta x' = x'_\text{f} - x'_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km} \tag{2.12}$$

Discussion

The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and thus has a negative sign.

Example 2.3 Comparing Distance Traveled with Displacement: A Subway Train

What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in **Figure 2.17**?

Strategy

To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in **Example 2.2**. Distance traveled is the total length of the path traveled between the two positions. In the case of the subway train shown in **Figure 2.17**, the distance traveled is the same as the distance between the initial and final positions of the train.

Solution

1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.

2. The displacement for part (b) was $-1.5\text{ km}.$ Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.

Discussion

Distance is a scalar. It has magnitude but no sign to indicate direction.

Example 2.4 Calculating Acceleration: A Subway Train Speeding Up

Suppose the train in **Figure 2.17**(a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?

Strategy

It is worth it at this point to make a simple sketch:

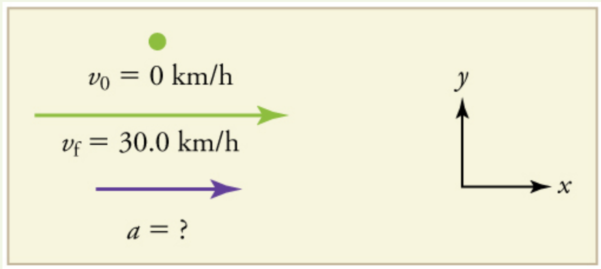


Figure 2.18

This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_\text{f} = 30.0\text{ km/h}$, and $\Delta t = 20.0\text{ s}$.

2. Calculate Δv. Since the train starts from rest, its change in velocity is $\Delta v = +30.0\text{ km/h}$, where the plus sign means velocity to the right.

3. Plug in known values and solve for the unknown, $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\text{ km/h}}{20.0\text{ s}} \tag{2.13}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds.

$$\bar{a} = \left(\frac{+30\text{ km/h}}{20.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 0.417\text{ m/s}^2 \tag{2.14}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example 2.5 Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in **Figure 2.17**(a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy

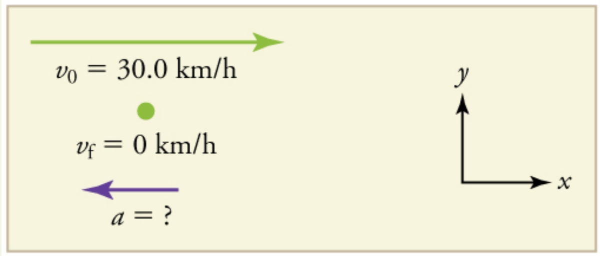


Figure 2.19

In this case, the train is slowing down and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\text{ km/h}$, $v_\text{f} = 0\text{ km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\text{ s}$.

2. Solve for the change in velocity, Δv .

$$\Delta v = v_\text{f} - v_0 = 0 - 30.0\text{ km/h} = -30.0\text{ km/h} \tag{2.15}$$

3. Plug in the knowns, Δv and Δt , and solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\text{ km/h}}{8.00\text{ s}} \tag{2.16}$$

4. Convert the units to meters and seconds.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\text{ km/h}}{8.00\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = -1.04\text{ m/s}^2. \tag{2.17}$$

Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion.

The graphs of position, velocity, and acceleration vs. time for the trains in **Example 2.4** and **Example 2.5** are displayed in **Figure 2.20**. (We have taken the velocity to remain constant from 20 to 40 s, after which the train slows down.)

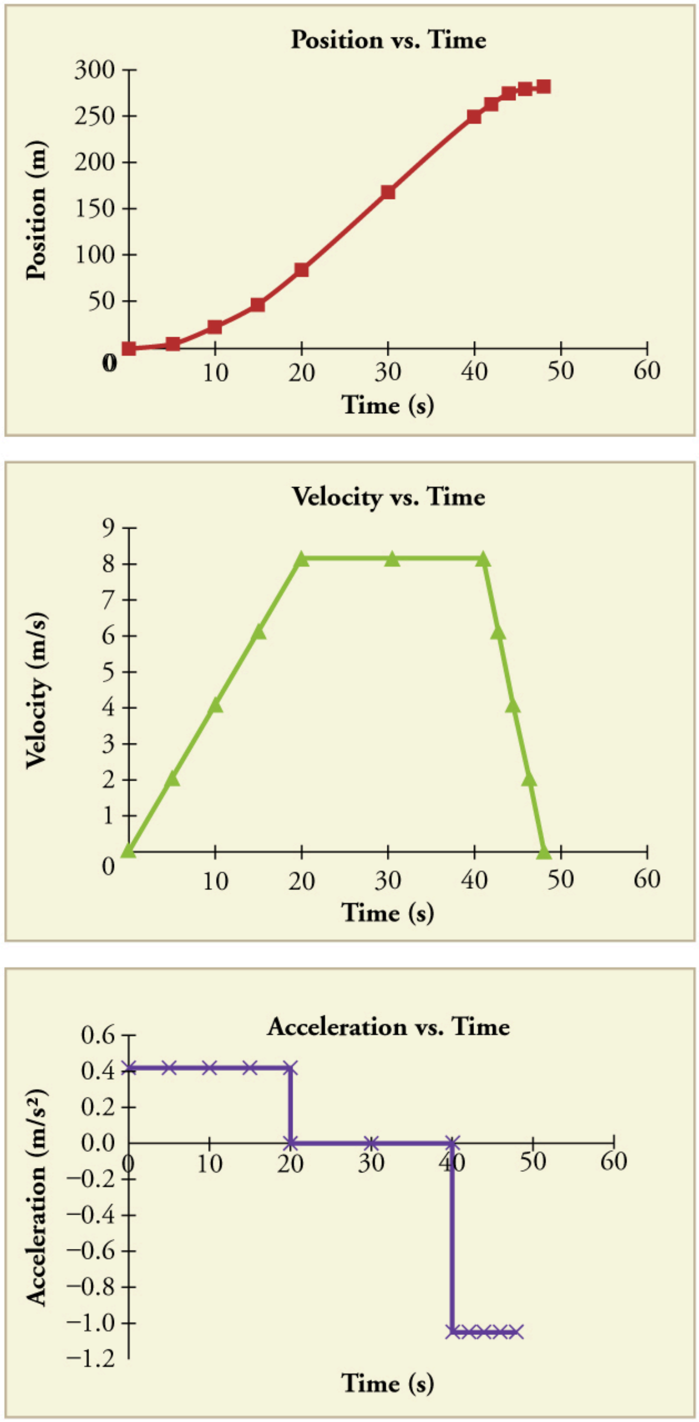


Figure 2.20 (a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train slows down at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example 2.6 Calculating Average Velocity: The Subway Train

What is the average velocity of the train in part b of **Example 2.2**, and shown again below, if it takes 5.00 min to make its trip?

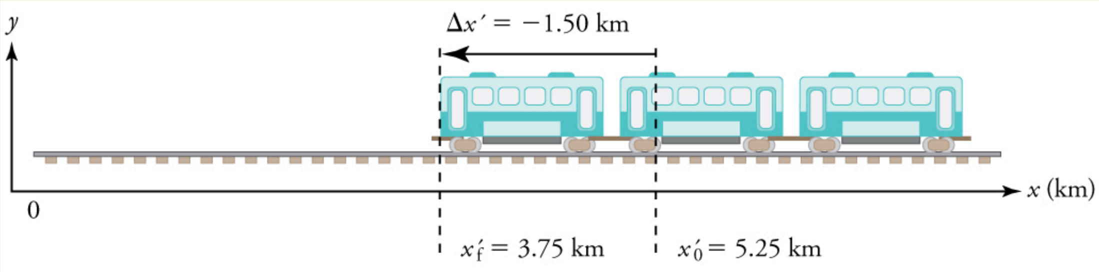


Figure 2.21

Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x'_f = 3.75\ \text{km}$, $x'_0 = 5.25\ \text{km}$, $\Delta t = 5.00\ \text{min}$.

2. Determine displacement, $\Delta x'$. We found $\Delta x'$ to be $-1.5\ \text{km}$ in Example 2.2.

3. Solve for average velocity.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \frac{-1.50\ \text{km}}{5.00\ \text{min}} \tag{2.18}$$

4. Convert units.

$$\bar{v} = \frac{\Delta x'}{\Delta t} = \left(\frac{-1.50\ \text{km}}{5.00\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = -18.0\ \text{km/h} \tag{2.19}$$

Discussion

The negative velocity indicates motion to the left.

Example 2.7 Calculating Deceleration: The Subway Train

Finally, suppose the train in Figure 2.21 slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:

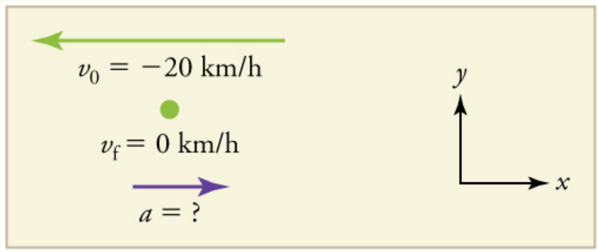


Figure 2.22

As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20\ \text{km/h}$, $v_f = 0\ \text{km/h}$, $\Delta t = 10.0\ \text{s}$.

2. Calculate Δv. The change in velocity here is actually positive, since

$$\Delta v = v_f - v_0 = 0 - (-20\ \text{km/h}) = +20\ \text{km/h}. \tag{2.20}$$

3. Solve for $\bar{a}$.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0\ \text{km/h}}{10.0\ \text{s}} \tag{2.21}$$

4. Convert units.

$$\bar{a} = \left(\frac{+20.0\text{ km/h}}{10.0\text{ s}}\right)\left(\frac{10^3\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = +0.556\text{ m/s}^2 \tag{2.22}$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right).

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in **Example 2.7**, where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in **Figure 2.21** is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the change in velocity, the object is speeding up. If acceleration has the opposite sign of the change in velocity, the object is slowing down.

Check Your Understanding

An airplane lands on a runway traveling east. Describe its acceleration.

Solution

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also slowing down: its acceleration is opposite in direction to its velocity.

2.5 Motion Equations for Constant Acceleration in One Dimension

Figure 2.23 Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_f - t_0$, taking $t_0 = 0$ means that $\Delta t = t_f$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

$$\left.\begin{aligned} \Delta t &= t \\ \Delta x &= x - x_0 \\ \Delta v &= v - v_0 \end{aligned}\right\} \tag{2.23}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

$$\bar{a} = a = \text{constant}, \tag{2.24}$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant

To get our first two new equations, we start with the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}. \tag{2.25}$$

Substituting the simplified notation for Δx and Δt yields

$$\bar{v} = \frac{x - x_0}{t}. \tag{2.26}$$

Solving for x yields

$$x = x_0 + \bar{v}t, \tag{2.27}$$

where the average velocity is

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a). \tag{2.28}$$

The equation $\bar{v} = \frac{v_0 + v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h}, \tag{2.29}$$

which seems logical.

Example 2.8 Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.

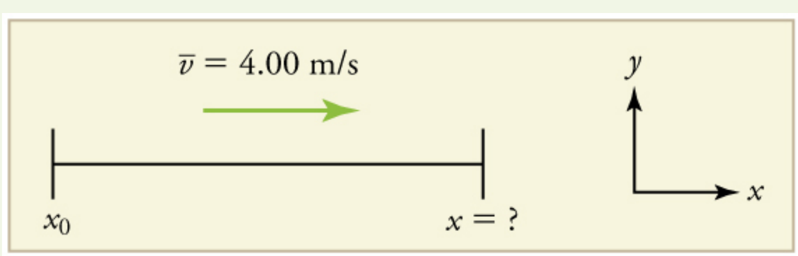


Figure 2.24

The final position x is given by the equation

$$x = x_0 + \bar{v}t. \tag{2.30}$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.

2. Enter the known values into the equation.

$$x = x_0 + \bar{v}t = 0 + (4.00\text{ m/s})(120\text{ s}) = 480\text{ m} \tag{2.31}$$

Discussion

Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.

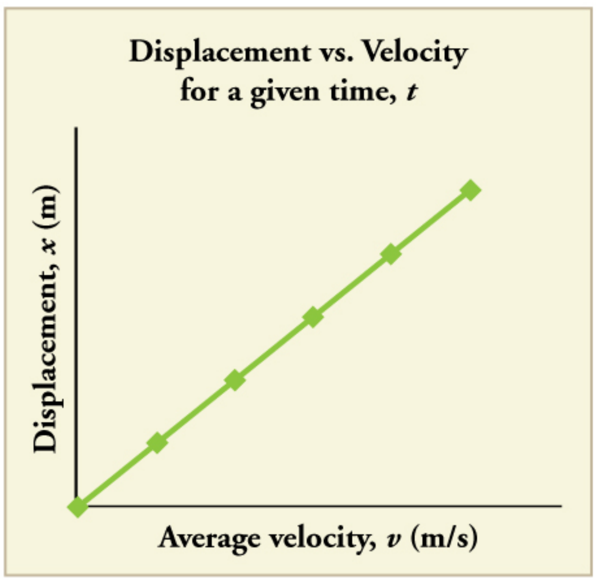


Figure 2.25 There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will move twice as far as the other object.

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

$$a = \frac{\Delta v}{\Delta t} \tag{2.32}$$

Substituting the simplified notation for Δv and Δt gives us

$$a = \frac{v - v_0}{t}\ (\text{constant } a). \tag{2.33}$$

Solving for v yields

$$v = v_0 + at\ (\text{constant } a). \tag{2.34}$$

Example 2.9 Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then slows down at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is slowing down.

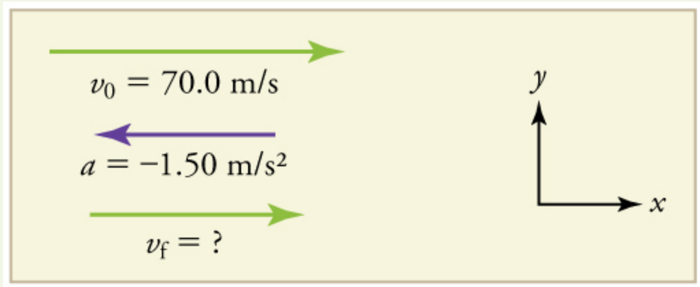


Figure 2.26

Solution

1. Identify the knowns. $\Delta v = 70.0 \text{ m/s}$, $a = -1.50 \text{ m/s}^2$, $t = 40.0 \text{ s}$.

2. Identify the unknown. In this case, it is final velocity, v_f .

3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + at$.

4. Plug in the known values and solve.

$$v = v_0 + at = 70.0 \text{ m/s} + \left(-1.50 \text{ m/s}^2\right)(40.0 \text{ s}) = 10.0 \text{ m/s} \tag{2.35}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.

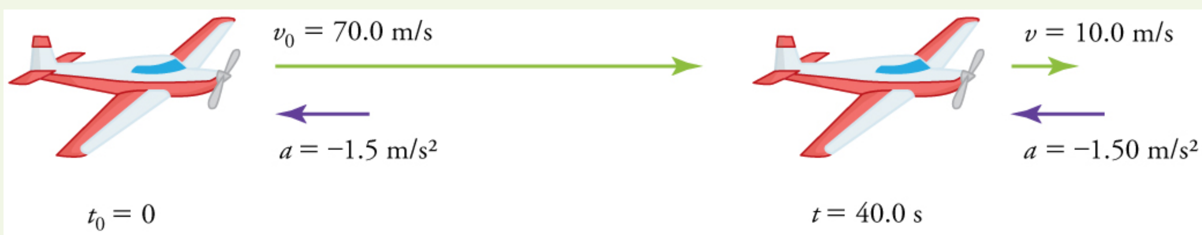


Figure 2.27 The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

(All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.)

Making Connections: Real-World Connection

Figure 2.28 The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Solving for Final Position When Velocity is Not Constant ($a \neq 0$)

We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with

$$v = v_0 + at. \tag{2.36}$$

Adding v_0 to each side of this equation and dividing by 2 gives

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}at. \tag{2.37}$$

Since $\frac{v_0 + v}{2} = \bar{v}$ for constant acceleration, then

$$\bar{v} = v_0 + \frac{1}{2}at. \tag{2.38}$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}. \tag{2.39}$$

Example 2.10 Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of 26.0 m/s^2. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

Figure 2.29 U.S. Army Top Fuel pilot Tony "The Sarge" Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.

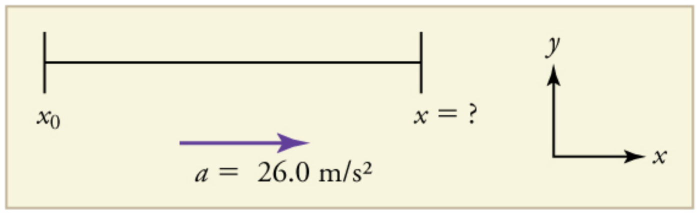


Figure 2.30

We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as $26.0\ \text{m/s}^2$ and t is given as 5.56 s.

2. Plug the known values into the equation to solve for the unknown x:

$$x = x_0 + v_0 t + \frac{1}{2}at^2. \tag{2.40}$$

Since the initial position and velocity are both zero, this simplifies to

$$x = \frac{1}{2}at^2. \tag{2.41}$$

Substituting the identified values of a and t gives

$$x = \frac{1}{2}\left(26.0\ \text{m/s}^2\right)(5.56\ \text{s})^2, \tag{2.42}$$

yielding

$$x = 402\ \text{m}. \tag{2.43}$$

Discussion

If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In **Example 2.10**, the dragster covers only one fourth of the total distance in the first half of the elapsed time

- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + at$ for t , we get

$$t = \frac{v - v_0}{a}. \tag{2.44}$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

$$v^2 = v_0^2 + 2a(x - x_0)\ (\text{constant}\ a). \tag{2.45}$$

Example 2.11 Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in **Example 2.10** without using information about time.

Strategy

Draw a sketch.

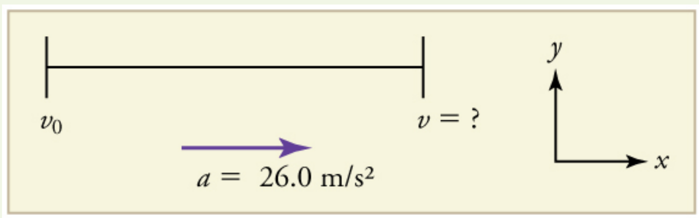


Figure 2.31

The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution

1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402\ \text{m}$ (this was the answer in **Example 2.10**). Finally, the average acceleration was given to be $a = 26.0\ \text{m/s}^2$.

2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0\ \text{m/s}^2\right)(402\ \text{m}). \tag{2.46}$$

Thus

$$v^2 = 2.09\times10^4\ \text{m}^2/\text{s}^2. \tag{2.47}$$

To get v , we take the square root:

$$v = \sqrt{2.09\times10^4\ \text{m}^2/\text{s}^2} = 145\ \text{m/s}. \tag{2.48}$$

Discussion

145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed magnitude of acceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes

much further to stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Summary of Kinematic Equations (constant a)

$$x = x_0 + \bar{v}t \tag{2.49}$$

$$\bar{v} = \frac{v_0 + v}{2} \tag{2.50}$$

$$v = v_0 + at \tag{2.51}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2.52}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{2.53}$$

Example 2.12 Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, a car can slows down at a rate of 7.00 m/s^2 , whereas on wet concrete the maximum magnitude of acceleration is only 5.00 m/s^2 . Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.

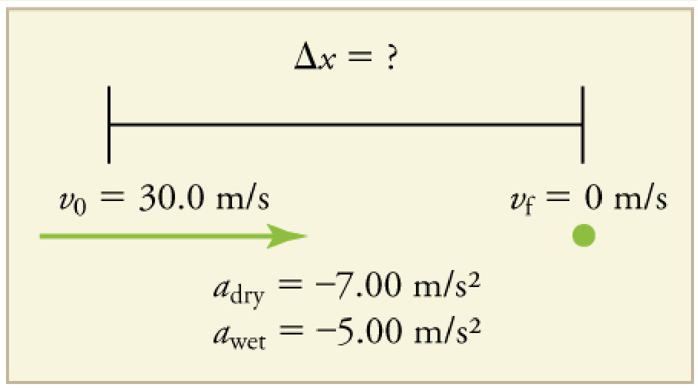


Figure 2.32

In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0 \text{ m/s}$; $v = 0$; $a = -7.00 \text{ m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx , or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

$$v^2 = v_0^2 + 2a(x - x_0). \tag{2.54}$$

This equation is best because it includes only one unknown, x . We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x , but they require us to know the stopping time, t , which we do not know. We could use them but it would entail additional calculations.)

3. Rearrange the equation to solve for x .

$$x - x_0 = \frac{v^2 - v_0^2}{2a} \tag{2.55}$$

4. Enter known values.

$$x - 0 = \frac{0^2 - (30.0\text{ m/s})^2}{2\left(-7.00\text{ m/s}^2\right)} \tag{2.56}$$

Thus,

$$x = 64.3\text{ m on dry concrete.} \tag{2.57}$$

Solution for (b)

This part can be solved in exactly the same manner as Part A. The only difference is that the acceleration is -5.00 m/s^2. The result is

$$x_{\text{wet}} = 90.0\text{ m on wet concrete.} \tag{2.58}$$

Solution for (c)

Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.

1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\text{ m/s}$; $t_{\text{reaction}} = 0.500\text{ s}$; $a_{\text{reaction}} = 0$. We take $x_{0-\text{reaction}}$ to be 0. We are looking for x_{reaction}.

2. Identify the best equation to use.

$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.

3. Plug in the knowns to solve the equation.

$$x = 0 + (30.0\text{ m/s})(0.500\text{ s}) = 15.0\text{ m.} \tag{2.59}$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.

4. Add the displacement during the reaction time to the displacement when braking.

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}} \tag{2.60}$$

a. 64.3 m + 15.0 m = 79.3 m when dry

b. 90.0 m + 15.0 m = 105 m when wet

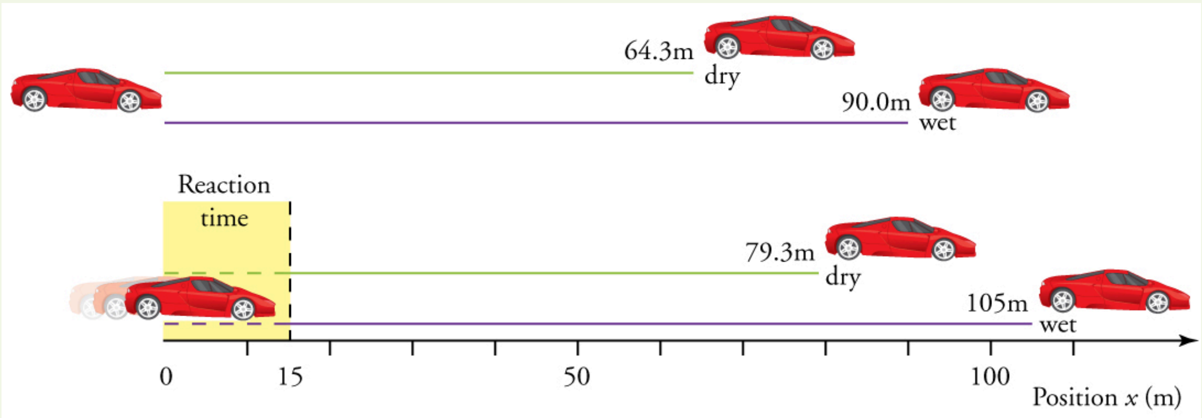


Figure 2.33 The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships.

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking acceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the acceleration in miles per hour per second. Convert this to meters per second squared and compare with other accelerations mentioned in this chapter. Calculate the distance traveled in braking.

Check Your Understanding

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket reach a velocity of 400 m/s?

Solution

To answer this, choose an equation that allows you to solve for time t , given only a , v_0 , and v .

$$v = v_0 + at \tag{2.61}$$

Rearrange to solve for t .

$$t = \frac{v - v}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s} \tag{2.62}$$

2.6 Falling Objects

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration*, *independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.

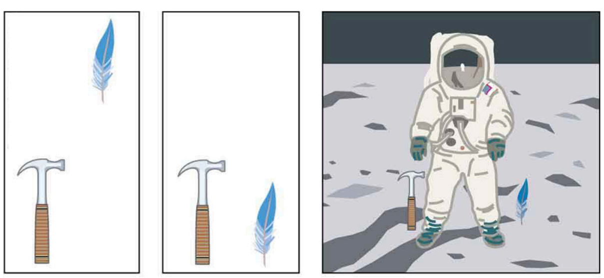


Figure 2.34 A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the acceleration due to gravity is only 1.67 m/s^2 .

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, g . It is constant at any given location on Earth and has the average value

$$g = 9.80 \text{ m/s}^2. \tag{2.63}$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2 , depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80 \text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80 \text{ m/s}^2$.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g . We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Kinematic Equations for Objects in Free-Fall where Acceleration = -*g*

$$v = v_0 - gt \tag{2.64}$$

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{2.65}$$

$$v^2 = v_0^2 - 2g(y - y_0) \tag{2.66}$$

Example 2.13 Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.

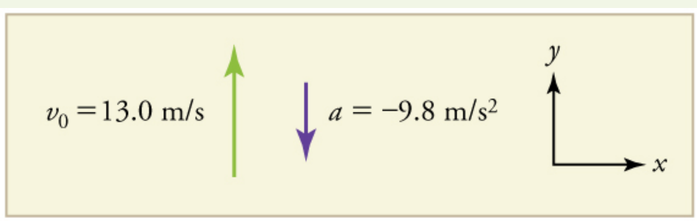


Figure 2.35

We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1 ; y_2 and v_2 ; and y_3 and v_3 .

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0 \text{ m/s}$; $a = -g = -9.80 \text{ m/s}^2$; and $t = 1.00 \text{ s}$.

2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

$$y = 0 + (13.0\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2 = 8.10\ \text{m} \tag{2.67}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.

2. Identify the best equation to use. The most straightforward is $v = v_0 - gt$ (from $v = v_0 + at$, where $a = \text{gravitational acceleration} = -g$).

3. Plug in the knowns and solve.

$$v_1 = v_0 - gt = 13.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)(1.00\ \text{s}) = 3.20\ \text{m/s} \tag{2.68}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\ \text{s}$. However, it has slowed from its original 13.0 m/s, as expected.

Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\ \text{s}$ and $3.00\ \text{s}$ are the same as those above. The results are summarized in **Table 2.1** and illustrated in **Figure 2.36**.

Table 2.1 Results

Time, *t*	Position, *y*	Velocity, *v*	Acceleration, *a*
1.00 s	8.10 m	3.20 m/s	$-9.80\ \text{m/s}^2$
2.00 s	6.40 m	−6.60 m/s	$-9.80\ \text{m/s}^2$
3.00 s	−5.10 m	−16.4 m/s	$-9.80\ \text{m/s}^2$

Graphing the data helps us understand it more clearly.

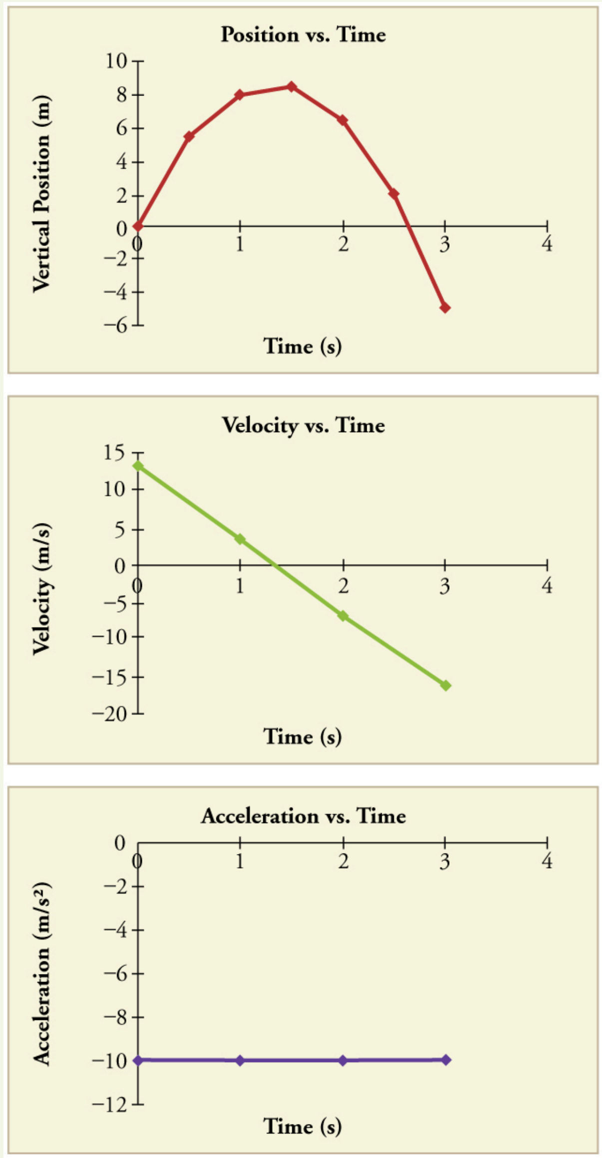


Figure 2.36 Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still -9.80 m/s^2. Its acceleration is -9.80 m/s^2 for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example 2.14 Calculating Velocity of a Falling Object: A Rock Thrown Down

What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.

Strategy

Draw a sketch.

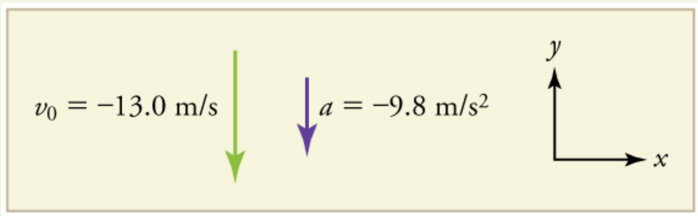


Figure 2.37

Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$.

Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.

Solution

1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.

2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)

3. Enter the known values

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2, \tag{2.69}$$

where we have retained extra significant figures because this is an intermediate result.

Taking the square root, and noting that a square root can be positive or negative, gives

$$v = \pm 16.4\text{ m/s}. \tag{2.70}$$

The negative root is chosen to indicate that the rock is still heading down. Thus,

$$v = -16.4\text{ m/s}. \tag{2.71}$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See **Example 2.13** and **Figure 2.38**(a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from **Example 2.13**) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20\text{ m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.

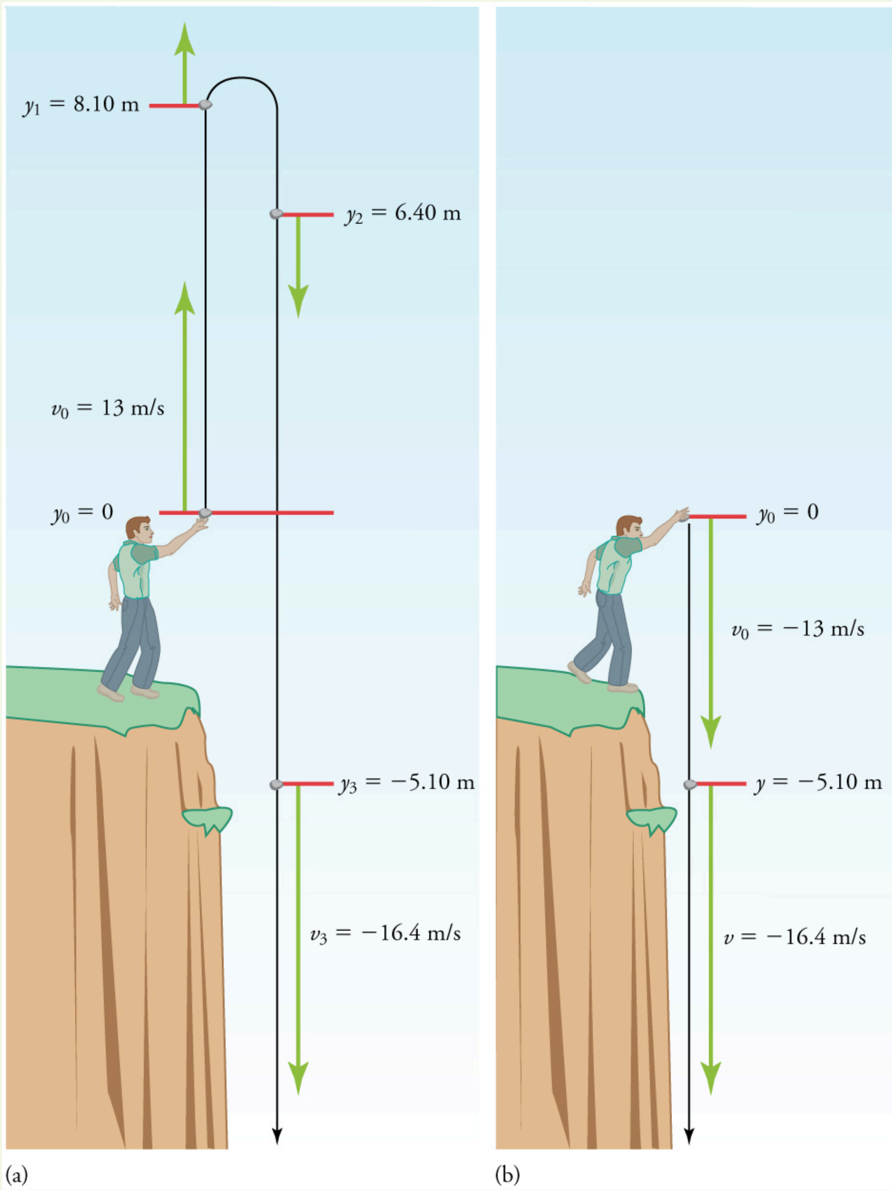
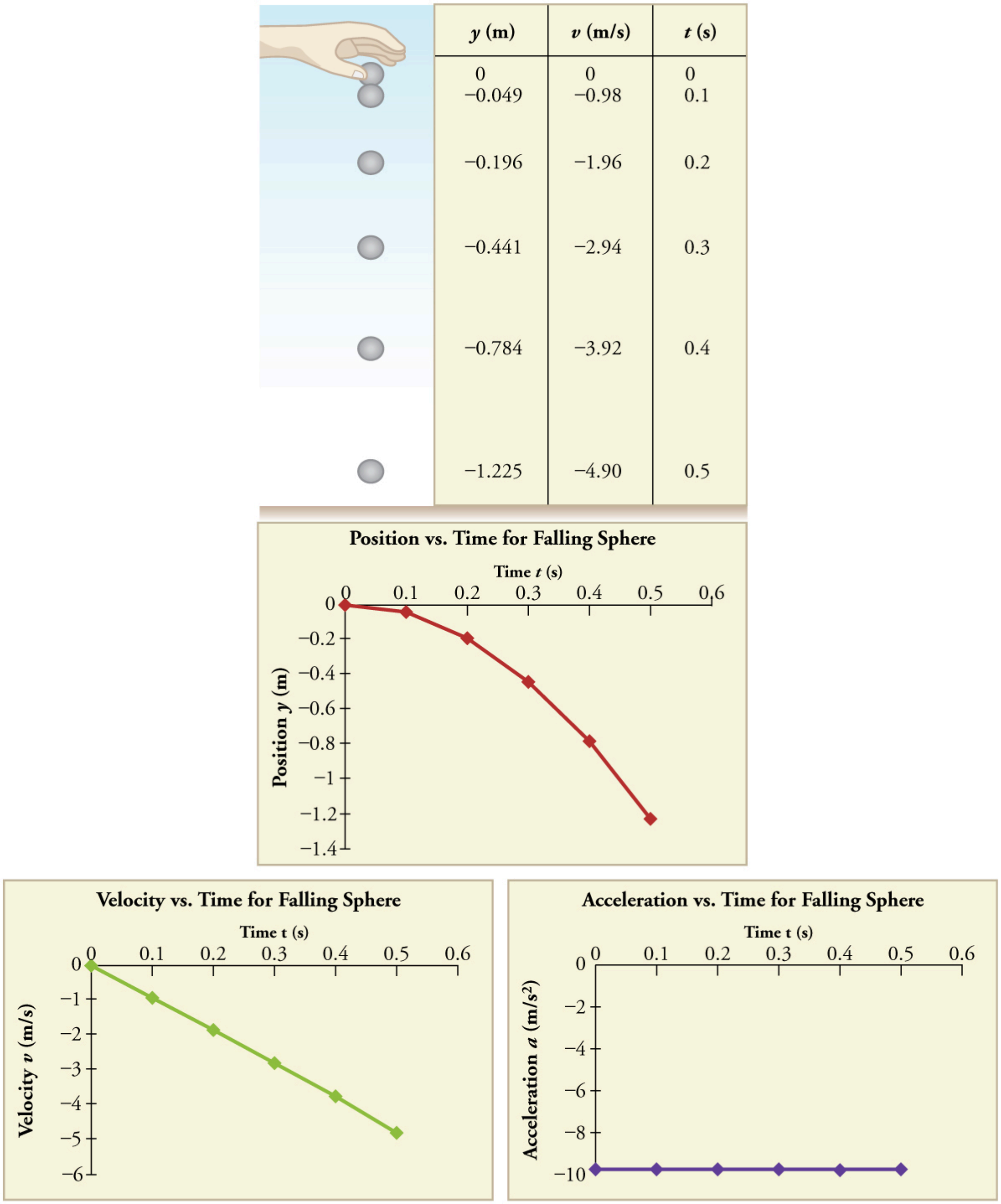


Figure 2.38 (a) A person throws a rock straight up, as explored in **Example 2.13**. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in **Example 2.14**. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In **Example 2.13**, the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its position is $y = 0$ on its way back down, its velocity is -13.0 m/s. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10\text{ m}$ to be the same whether we have thrown it upwards at $+13.0\text{ m/s}$ or thrown it downwards at -13.0 m/s. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example 2.15 Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, **Figure 2.39**. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

Figure 2.39 Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.

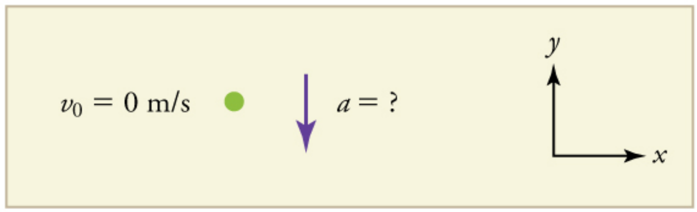


Figure 2.40

We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = -1.0000$ m; $t = 0.45173$; $v_0 = 0$.

2. Choose the equation that allows you to solve for a using the known values.

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \tag{2.72}$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

$$y = y_0 + \frac{1}{2}at^2. \tag{2.73}$$

Solving for a gives

$$a = \frac{2(y - y_0)}{t^2}. \tag{2.74}$$

4. Substitute known values yields

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2, \tag{2.75}$$

so, because $a = -g$ with the directions we have chosen,

$$g = 9.8010 \text{ m/s}^2. \tag{2.76}$$

Discussion

The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of 9.80 m/s^2, so 9.8010 m/s^2 makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of 9.80 m/s^2; it represents the local value for the acceleration due to gravity.

Check Your Understanding

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution

We know that initial position $y_0 = 0$, final position $y = -30.0$ m, and $a = -g = -9.80 \text{ m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain

$$\begin{aligned} y &= 0 + 0 - \frac{1}{2}gt^2 \\ t^2 &= \frac{2y}{-g} \\ t &= \pm\sqrt{\frac{2y}{-g}} = \pm\sqrt{\frac{2(-30.0 \text{ m})}{-9.80 \text{ m/s}^2}} = \pm\sqrt{6.12 \text{ s}^2} = 2.47 \text{ s} \approx 2.5 \text{ s} \end{aligned} \tag{2.77}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

2.7 Projectile Motion

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

The most important fact to know here is that *motions along perpendicular axes are independent* and thus can be considered separately. In particular, it is often convenient to consider vertical and horizontal motions separately, or independently. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. And we will assume all forces except gravity are negligible (that is, we ignore forces due to air resistance, wind, friction, etc.). The vertical component of acceleration is, then, $a_y = -g = -9.80 \text{ m/s}^2$, where we take upward direction as the positive y direction. And because there is no force in the horizontal direction, $a_x = 0$. **Figure 2.41** illustrates this approach to analyzing projectile motion.

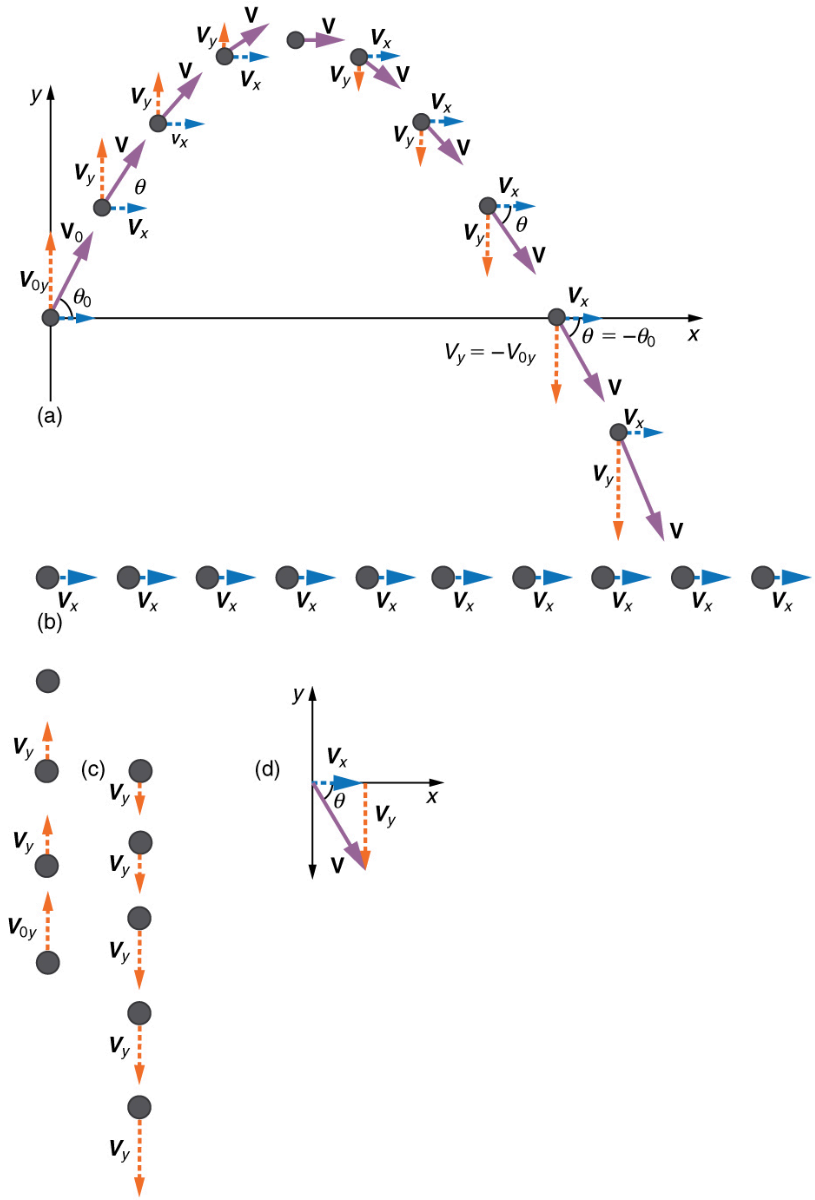


Figure 2.41 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude. This is exactly same as in the case of falling objects (or objects thrown directly upward). (d) The *x* - and *y* -motions can be recombined to give the total velocity at any given point on the trajectory.

One of the conceptual aspects of projectile motion we can discuss without a detailed analysis is the range. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. The range of projectiles describes everyday phenomena such as how far a football can be thrown or kicked. Also, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.

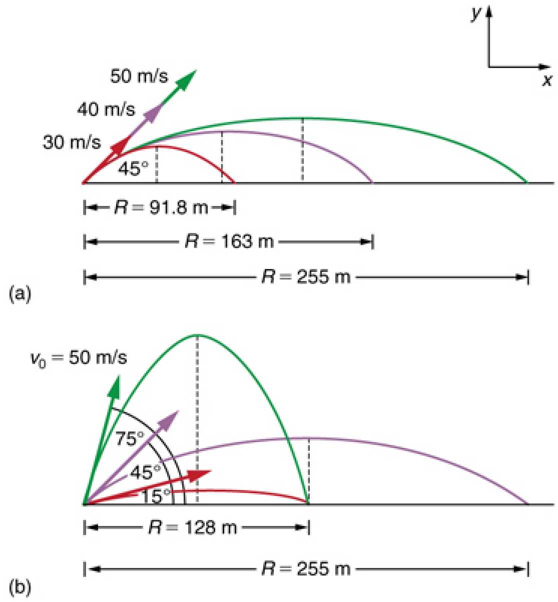


Figure 2.42 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that the range is the same for $15°$ and $75°$, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? It is intuitive to guess that, the greater the initial speed v_0, the greater the range, as shown in **Figure 2.42**(a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in **Figure 2.42**(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45°$. Interestingly, for every initial angle except $45°$, there are two angles that give the same range. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}, \tag{2.78}$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. This formula (which can be derived using algebra and trigonometry) fits the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See **Figure 2.43**.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface.

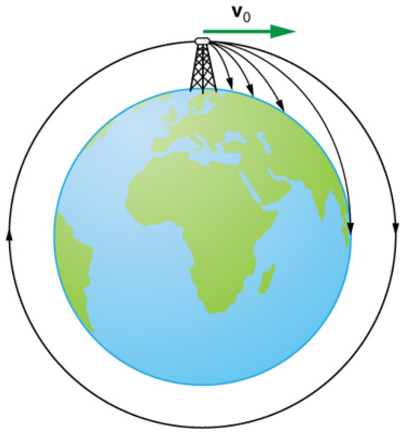


Figure 2.43 Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

2.8 Centripetal Acceleration

We defined acceleration as a change in velocity, either in its magnitude or in its direction, or both. When an object moves along a circular path, the direction of its velocity changes constantly, so there is always an associated acceleration, even if the speed of the object is constant. You experience this acceleration yourself when you turn a corner in your car. What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we briefly examine the direction and magnitude of that acceleration.

Figure 2.44 shows an object moving in a circular path at constant speed. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**(a_c); centripetal means "toward the center" or "center seeking."

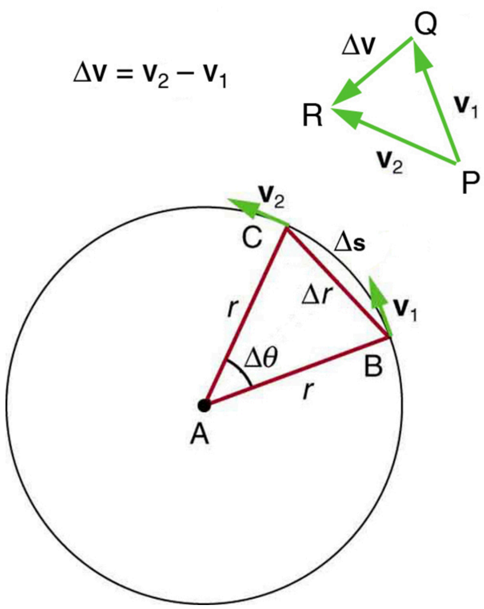


Figure 2.44 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta\mathbf{v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_c = \Delta\mathbf{v}/\Delta t$, the acceleration is also toward the center; $\mathbf{a}_c$ is called centripetal acceleration. (Because $\Delta\theta$ is very small, the arc length Δs is equal to the chord length Δr for small time differences.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? If we use the geometry shown in **Figure 2.44** along with some kinematics equations, we can obtain (detailed derivation skipped)

$$a_c = \frac{v^2}{r}, \tag{2.79}$$

which is the acceleration of an object in a circle of radius r at a speed v . Verify for yourself that a_c has unit of $\mathrm{m/s^2}$, as expected for acceleration.

We see in **Equation 2.79** that centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_{c} is proportional to speed squared, implying, for example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_{c} is greater for tighter turns, as you have probably noticed.

Glossary

acceleration: the rate of change in velocity; the change in velocity over time

acceleration due to gravity: acceleration of an object as a result of gravity

air resistance: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

average acceleration: the change in velocity divided by the time over which it changes

average speed: distance traveled divided by time during which motion occurs

average velocity: displacement divided by time over which displacement occurs

centripetal acceleration: the acceleration of an object moving in a circle, directed toward the center

displacement: the change in position of an object

distance: the magnitude of displacement between two positions

distance traveled: the total length of the path traveled between two positions

elapsed time: the difference between the ending time and beginning time

free-fall: the state of movement that results from gravitational force only

instantaneous acceleration: acceleration at a specific point in time

instantaneous speed: magnitude of the instantaneous velocity

instantaneous velocity: velocity at a specific instant, or the average velocity over an infinitesimal time interval

kinematics: the study of motion without considering its causes

kinematics: the study of motion without regard to mass or force

model: simplified description that contains only those elements necessary to describe the physics of a physical situation

motion: displacement of an object as a function of time

position: the location of an object at a particular time

projectile: an object that travels through the air and experiences only acceleration due to gravity

projectile motion: the motion of an object that is subject only to the acceleration of gravity

range: the maximum horizontal distance that a projectile travels

scalar: a quantity that is described by magnitude, but not direction

time: change, or the interval over which change occurs

trajectory: the path of a projectile through the air

ultracentrifuge: a centrifuge optimized for spinning a rotor at very high speeds

vector: a quantity that is described by both magnitude and direction

Section Summary

2.1 Displacement

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

- In symbols, displacement Δx is defined to be

$$\Delta x = x_{\text{f}} - x_0,$$

where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

2.2 Vectors, Scalars, and Coordinate Systems

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

2.3 Time, Velocity, and Speed

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is

$$\Delta t = t_{\text{f}} - t_0,$$

where t_{f} is the final time and t_0 is the initial time.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

2.4 Acceleration

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\text{f}} - v_0}{t_{\text{f}} - t_0}.$$

- The SI unit for acceleration is m/s^2.
- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- When an acceleration is in a direction opposite to that of the velocity of an object, the object slows down.

2.5 Motion Equations for Constant Acceleration in One Dimension

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,

$$\left.\begin{array}{rcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\}$$

- The following kinematic equations for motion with constant a are useful:

$$x = x_0 + \bar{v}\, t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

2.6 Falling Objects

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages

$$g = 9.80\ \text{m/s}^2.$$

- Whether the acceleration *a* should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \text{m/s}^2$ is negative. In the opposite case, $a = +\text{g} = 9.80\ \text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

2.7 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- The most important fact regarding projectile motion is that motions along vertical direction and the horizontal direction are independent.
- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

2.8 Centripetal Acceleration

- Centripetal acceleration a_c is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude

$$a_\text{c} = \frac{v^2}{r}.$$

- The unit of centripetal acceleration is m/s^2.

Conceptual Questions

2.1 Displacement

1. Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

2. Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

3. Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}\ \left(50\times10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

2.2 Vectors, Scalars, and Coordinate Systems

4. A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

5. What is the speed of the bird in **Exercise 2.4**?

6. Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

7. A weather forecast states that the temperature is predicted to be -5^oC the following day. Is this temperature a vector or a scalar quantity? Explain.

2.3 Time, Velocity, and Speed

8. Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

9. There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

10. Does a car's odometer measure position or displacement? Does its speedometer measure speed or velocity?

11. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

12. How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

2.4 Acceleration

13. Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

14. Is it possible for velocity to be constant while acceleration is not zero? Explain.

15. Give an example in which velocity is zero yet acceleration is not.

16. If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

17. Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

2.6 Falling Objects

18. What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

19. An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

20. Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

21. If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

22. The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

23. How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

2.7 Projectile Motion

24. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0^o nor 90^o): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t=0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t=0$?

25. Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0^o nor 90^o): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

26. For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

27. During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

2.8 Centripetal Acceleration

28. Can centripetal acceleration change the speed of circular motion? Explain.

Problems & Exercises

2.1 Displacement

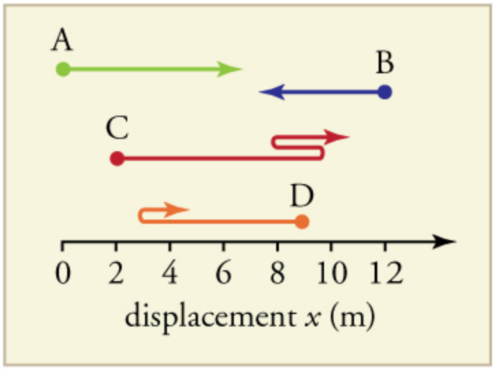


Figure 2.45

1. Find the following for path A in Figure 2.45: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2. Find the following for path B in Figure 2.45: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

3. Find the following for path C in Figure 2.45: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

4. Find the following for path D in Figure 2.45: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

2.3 Time, Velocity, and Speed

5. (a) Calculate Earth's average speed relative to the Sun. (b) What is its average velocity over a period of one year?

6. A helicopter blade spins at exactly 100 revolutions per minute. Its tip is 5.00 m from the center of rotation. (a) Calculate the average speed of the blade tip in the helicopter's frame of reference. (b) What is its average velocity over one revolution?

7. The North American and European continents are moving apart at a rate of about 3 cm/y. At this rate how long will it take them to drift 500 km farther apart than they are at present?

8. Land west of the San Andreas fault in southern California is moving at an average velocity of about 6 cm/y northwest relative to land east of the fault. Los Angeles is west of the fault and may thus someday be at the same latitude as San Francisco, which is east of the fault. How far in the future will this occur if the displacement to be made is 590 km northwest, assuming the motion remains constant?

9. On May 26, 1934, a streamlined, stainless steel diesel train called the Zephyr set the world's nonstop long-distance speed record for trains. Its run from Denver to Chicago took 13 hours, 4 minutes, 58 seconds, and was witnessed by more than a million people along the route. The total distance traveled was 1633.8 km. What was its average speed in km/h and m/s?

10. Tidal friction is slowing the rotation of the Earth. As a result, the orbit of the Moon is increasing in radius at a rate of approximately 4 cm/year. Assuming this to be a constant rate, how many years will pass before the radius of the Moon's orbit increases by $3.84\times10^{6}\ \text{m}$ (1%)?

11. A student drove to the university from her home and noted that the odometer reading of her car increased by 12.0 km. The trip took 18.0 min. (a) What was her average speed? (b) If the straight-line distance from her home to the university is 10.3 km in a direction 25.0° south of east, what was her average velocity? (c) If she returned home by the same path 7 h 30 min after she left, what were her average speed and velocity for the entire trip?

12. The speed of propagation of the action potential (an electrical signal) in a nerve cell depends (inversely) on the diameter of the axon (nerve fiber). If the nerve cell connecting the spinal cord to your feet is 1.1 m long, and the nerve impulse speed is 18 m/s, how long does it take for the nerve signal to travel this distance?

13. Conversations with astronauts on the lunar surface were characterized by a kind of echo in which the earthbound person's voice was so loud in the astronaut's space helmet that it was picked up by the astronaut's microphone and transmitted back to Earth. It is reasonable to assume that the echo time equals the time necessary for the radio wave to travel from the Earth to the Moon and back (that is, neglecting any time delays in the electronic equipment). Calculate the distance from Earth to the Moon given that the echo time was 2.56 s and that radio waves travel at the speed of light $(3.00\times10^{8}\ \text{m/s})$.

14. A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.

15. The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit $1.06\times10^{-10}\ \text{m}$ in diameter. (a) If the average speed of the electron in this orbit is known to be $2.20\times10^{6}\ \text{m/s}$, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?

2.4 Acceleration

16. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

17. Professional Application

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his (a) acceleration and (b) deceleration. Express each in multiples of g $(9.80\ \text{m/s}^2)$ by taking its ratio to the acceleration of gravity.

18. A commuter backs her car out of her garage with an acceleration of $1.40\ \text{m/s}^2$. (a) How long does it take her to reach a speed of 2.00 m/s? (b) If she then brakes to a stop in 0.800 s, what is her acceleration?

19. Assume that an intercontinental ballistic missile goes from rest to a suborbital speed of 6.50 km/s in 60.0 s (the actual speed and time are classified). What is its average acceleration in m/s^2 and in multiples of g $(9.80\ \text{m/s}^2)$?

2.5 Motion Equations for Constant Acceleration in One Dimension

20. An Olympic-class sprinter starts a race with an acceleration of $4.50\ \text{m/s}^2$. (a) What is her speed 2.40 s later? (b) Sketch a graph of her position vs. time for this period.

21. A well-thrown ball is caught in a well-padded mitt. If the magnitude of acceleration of the ball is $2.10\times10^4\ \text{m/s}^2$, and 1.85 ms $(1\ \text{ms} = 10^{-3}\ \text{s})$ elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?

22. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of $6.20\times10^5\ \text{m/s}^2$ for $8.10\times10^{-4}\ \text{s}$. What is its muzzle velocity (that is, its final velocity)?

23. (a) A light-rail commuter train accelerates at a rate of $1.35\ \text{m/s}^2$. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily slows down at a rate of $1.65\ \text{m/s}^2$. How long does it take to come to a stop from its top speed? (c) In emergencies the train can come to a stop more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency braking acceleration in m/s^2?

24. Professional Application:

Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?

25. In a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes $3.33\times10^{-2}\ \text{s}$, calculate the distance over which the puck accelerates.

26. A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time?

27. Freight trains can produce only relatively small accelerations. (a) What is the final velocity of a freight train that accelerates at a rate of $0.0500\ \text{m/s}^2$ for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of $0.550\ \text{m/s}^2$, how long will it take to come to a stop from this velocity? (c) How far will it travel in each case?

28. A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) Calculate the acceleration. (b) How long did the acceleration last?

29. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of $0.350\ \text{m/s}^2$, how far will it travel before becoming airborne? (b) How long does this take?

30. An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is the magnitude of his acceleration? (b) How long does the collision last?

31. A bicycle racer sprints at the end of a race to clinch a victory. The racer has an initial velocity of 11.5 m/s and accelerates at the rate of $0.500\ \text{m/s}^2$ for 7.00 s. What is his final velocity?

2.6 Falling Objects

Assume air resistance is negligible unless otherwise stated.

32. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be $y_0 = 0$.

33. Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, (d) 2.00, and (e) 2.50 s for a rock thrown straight down with an initial velocity of 14.0 m/s from the Verrazano Narrows Bridge in New York City. The roadway of this bridge is 70.0 m above the water.

34. A basketball referee tosses the ball straight up for the starting tip-off. At what velocity must a basketball player leave the ground to rise 1.25 m above the floor in an attempt to get the ball?

35. A rescue helicopter is hovering over a person whose boat has sunk. One of the rescuers throws a life preserver straight down to the victim with an initial velocity of 1.40 m/s and observes that it takes 1.8 s to reach the water. (a) List the knowns in this problem. (b) How high above the water was the preserver released? Note that the downdraft of the helicopter reduces the effects of air resistance on the falling life preserver, so that an acceleration equal to that of gravity is reasonable.

36. A dolphin in an aquatic show jumps straight up out of the water at a velocity of 13.0 m/s. (a) List the knowns in this problem. (b) How high does his body rise above the water? To solve this part, first note that the final velocity is now a known and identify its value. Then identify the unknown, and discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking units, and discuss whether the answer is reasonable. (c) How long is the dolphin in the air? Neglect any effects due to his size or orientation.

37. Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s.

38. A kangaroo can jump over an object 2.50 m high. (a) Calculate its vertical speed when it leaves the ground. (b) How long is it in the air?

39. An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

2.7 Projectile Motion

40. Verify the ranges for the projectiles in **Figure 2.42**(a) for $\theta = 45^{\text{o}}$ and the given initial velocities.

41. Verify the ranges shown for the projectiles in **Figure 2.42**(b) for an initial velocity of 50 m/s at the given initial angles.

42. The world long jump record is 8.95 m (Mike Powell, USA, 1991). Treated as a projectile, what is the maximum range obtainable by a person if he has a take-off speed of 9.5 m/s? State your assumptions.

43. Gun sights are adjusted to aim high to compensate for the effect of gravity, effectively making the gun accurate only for a specific range. (a) If a gun is sighted to hit targets that are at the same height as the gun and 100.0 m away, how low will the bullet hit if aimed directly at a target 150.0 m away? The muzzle velocity of the bullet is 275 m/s. (b) Discuss qualitatively how a larger muzzle velocity would affect this problem and what would be the effect of air resistance.

44. An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.

45. Can a goalkeeper at her/ his goal kick a soccer ball into the opponent's goal without the ball touching the ground? The distance will be about 95 m. A goalkeeper can give the ball a speed of 30 m/s.

46. In 2007, Michael Carter (U.S.) set a world record in the shot put with a throw of 24.77 m. What was the initial speed of the shot if he released it at a height of 2.10 m and threw it at an angle of 38.0^{o} above the horizontal?

47. A basketball player is running at 5.00 m/s directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?

48. A football player punts the ball at a 45.0^{o} angle. Without an effect from the wind, the ball would travel 60.0 m horizontally. (a) What is the initial speed of the ball? (b) When the ball is near its maximum height it experiences a brief gust of wind that reduces its horizontal velocity by 1.50 m/s. What distance does the ball travel horizontally?

49. Unreasonable Results (a) Find the maximum range of a super cannon that has a muzzle velocity of 4.0 km/s. (b) What is unreasonable about the range you found? (c) Is the premise unreasonable or is the available equation inapplicable? Explain your answer. (d) If such a muzzle velocity could be obtained, discuss the effects of air resistance, thinning air with altitude, and the curvature of the Earth on the range of the super cannon.

50. Construct Your Own Problem Consider a ball tossed over a fence. Construct a problem in which you calculate the ball's needed initial velocity to just clear the fence. Among the things to determine are; the height of the fence, the distance to the fence from the point of release of the ball, and the height at which the ball is released. You should also consider whether it is possible to choose the initial speed for the ball and just calculate the angle at which it is thrown. Also examine the possibility of multiple solutions given the distances and heights you have chosen.

2.8 Centripetal Acceleration

51. A runner taking part in the 200 m dash must run around the end of a track that has a circular arc with a radius of curvature of 30 m. If he completes the 200 m dash in 23.2 s and runs at constant speed throughout the race, what is the magnitude of his centripetal acceleration as he runs the curved portion of the track?

52. An ordinary workshop grindstone has a radius of 7.50 cm and rotates at 6500 rev/min.

(a) Calculate the magnitude of the centripetal acceleration at its edge in meters per second squared and convert it to multiples of g.

(b) What is the linear speed of a point on its edge?

53. Helicopter blades withstand tremendous stresses. In addition to supporting the weight of a helicopter, they are spun at rapid rates and experience large centripetal accelerations, especially at the tip.

(a) Calculate the magnitude of the centripetal acceleration at the tip of a 4.00 m long helicopter blade that rotates at 300 rev/min.

(b) Compare the linear speed of the tip with the speed of sound (taken to be 340 m/s).

54. Olympic ice skaters are able to spin at about 5 rev/s.

(a) What is the centripetal acceleration of the skater's nose if it is 0.120 m from the axis of rotation?

(b) An exceptional skater named Dick Button was able to spin much faster in the 1950s than anyone since—at about 9 rev/s. What was the centripetal acceleration of the tip of his nose, assuming it is at 0.120 m radius?

(c) Comment on the magnitudes of the accelerations found. It is reputed that Button ruptured small blood vessels during his spins.

55. A rotating space station is said to create "artificial gravity"—a loosely-defined term used for an acceleration that would be crudely similar to gravity. The outer wall of the rotating space station would become a floor for the astronauts, and centripetal acceleration supplied by the floor would allow astronauts to exercise and maintain muscle and bone strength more naturally than in non-rotating space environments. If the space station is 200 m in diameter, what rotating speed would produce an "artificial gravity" of 9.80 m/s^2 at the rim?

56. Unreasonable Results

A mother pushes her child on a swing so that his speed is 9.00 m/s at the lowest point of his path. The swing is suspended 2.00 m above the child's center of mass.

(a) What is the magnitude of the centripetal acceleration of the child at the low point?

(b) What is the magnitude of the force the child exerts on the seat if his mass is 18.0 kg?

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent?

3 DYNAMICS

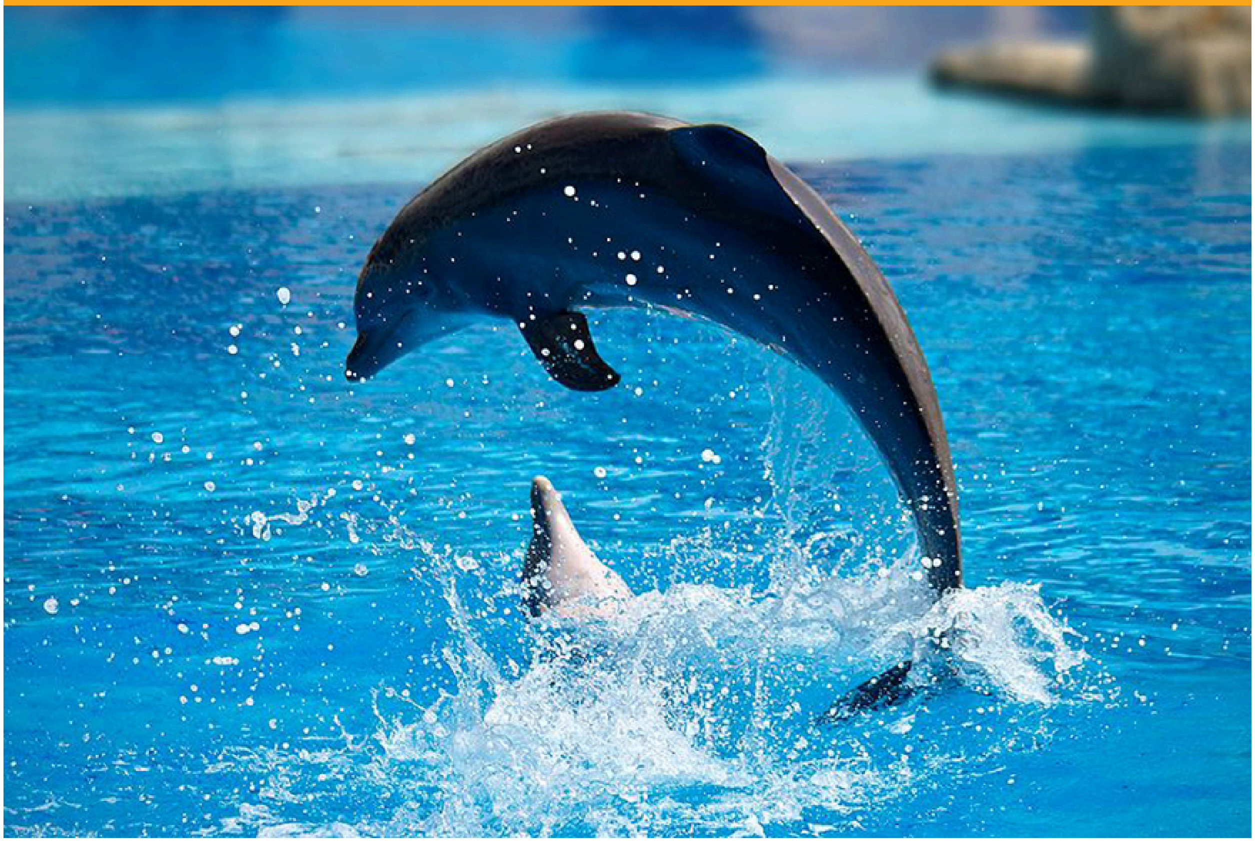


Figure 3.1 Newton’s laws of motion describe the motion of the dolphin’s path. (credit: Jin Jang)

Chapter Outline

3.1. Development of Force Concept
- Understand the definition of force.

3.2. Newton’s First Law of Motion: Inertia
- Define mass and inertia.
- Understand Newton's first law of motion.

3.3. Newton’s Second Law of Motion: Concept of a System
- Define net force, external force, and system.
- Understand Newton’s second law of motion.
- Apply Newton’s second law to determine the weight of an object.

3.4. Newton’s Third Law of Motion: Symmetry in Forces
- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

3.5. Normal Force and Tension
- Define normal and tension forces.
- Apply Newton's laws of motion to solve problems involving a variety of forces.

3.6. Spring Force: Hooke’s Law
- Describe the restoring force and displacement.
- Explain oscillatory motion under a spring force.

3.7. Friction
- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Calculate the magnitude of static and kinetic friction.

3.8. Newton’s Universal Law of Gravitation
- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Discuss weightlessness in space.

3.9. Centripetal Force
- Calculate centripetal force and acceleration for simple situations.
- Calculate coefficient of friction on a car tire.

Introduction to Dynamics: Newton's Laws of Motion

Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Issac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.

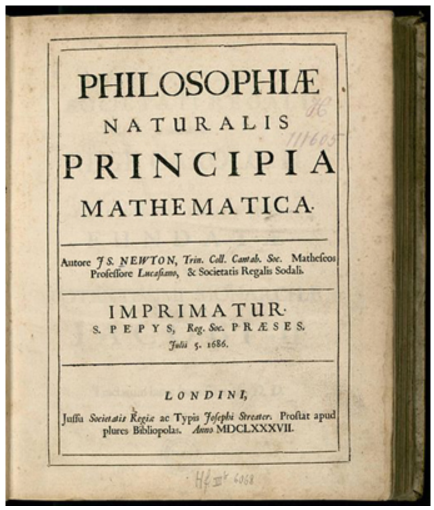


Figure 3.2 Issac Newton's monumental work, *Philosophiae Naturalis Principia Mathematica*, was published in 1687. It proposed scientific laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than "logical" argument. Galileo's use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton's first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton's laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the size of most molecules (about $10^{-9}\ \text{m}$ in diameter). These constraints define the realm of classical mechanics. At the beginning of the 20^{th} century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter are in the realm of classical physics.

Making Connections: Past and Present Philosophy

The importance of observation and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

3.1 Development of Force Concept

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in Figure 3.3, and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. Some alternative definitions of force will be given later in this chapter.

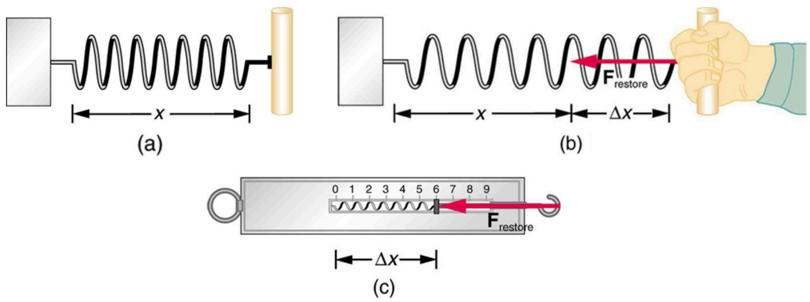


Figure 3.3 The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a magnitude of 6 units in the force standard being employed.

Take-Home Experiment: Force Standards

To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

3.2 Newton's First Law of Motion: Inertia

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton's first law of motion** states, however, is the following:

Newton's First Law of Motion

A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton's first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton's first law). The object would not slow down at all if friction were completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton's first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, "What is the cause?" Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, "That is the nature of the beast." True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of "stuff" (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Check Your Understanding

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Solution

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

3.3 Newton's Second Law of Motion: Concept of a System

Newton's second law of motion is closely related to Newton's first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton's second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton's second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton's first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in **Figure 3.4**(a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at **Figure 3.4**(a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton's first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton's laws. This concept will be revisited many times on our journey through physics.

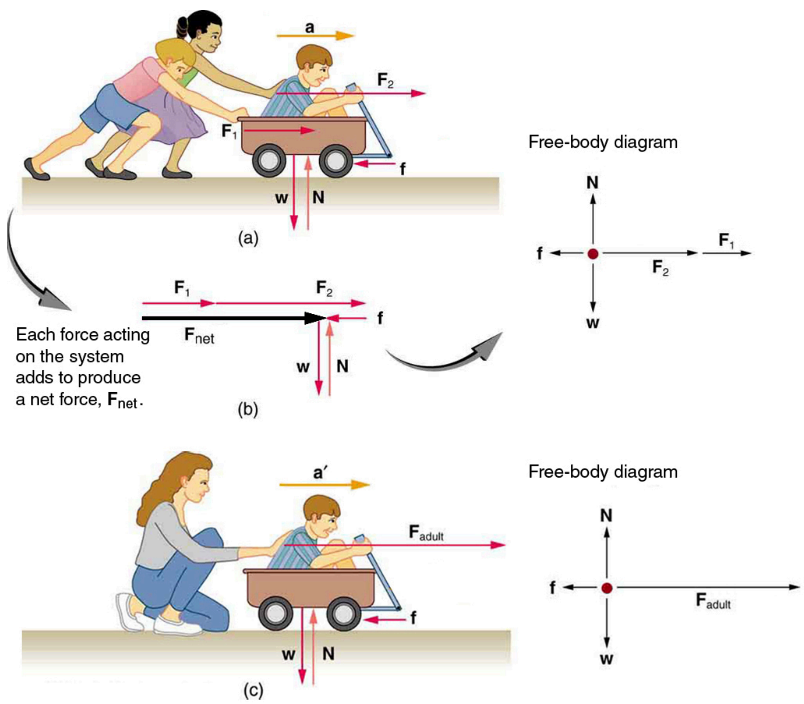


Figure 3.4 Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight $\mathbf{w}$ of the system and the support of the ground $\mathbf{N}$ are also shown for completeness and are assumed to cancel. The vector $\mathbf{f}$ represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, $\mathbf{F}_{\text{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger acceleration ($\mathbf{a}' > \mathbf{a}$) when an adult pushes the child.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in **Figure 3.4**. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\mathbf{w}$ and the support of the ground $\mathbf{N}$, and the horizontal force $\mathbf{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will define **friction** as a force that opposes the motion past each other of objects that are touching. **Figure 3.4**(b) shows how vectors representing the external forces add together to produce a net force, $\mathbf{F}_{\text{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{3.1}$$

where the symbol $\propto$ means "proportional to," and $\mathbf{F}_{\text{net}}$ is the **net external force**. (The net external force is the vector sum of all external forces. This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces and ignore the internal ones. It is a tremendous simplification not to have to consider the numerous internal forces acting between objects within the system, such as muscular forces within the child's body, let alone the myriad of forces between atoms in the objects, but by doing so, we can easily solve some very complex problems with only minimal error due to our simplification

Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. And indeed, as illustrated in **Figure 3.5**, the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

$$\mathbf{a} \propto \frac{1}{m} \tag{3.2}$$

where m is the mass of the system. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is exactly linearly proportional to the net external force.

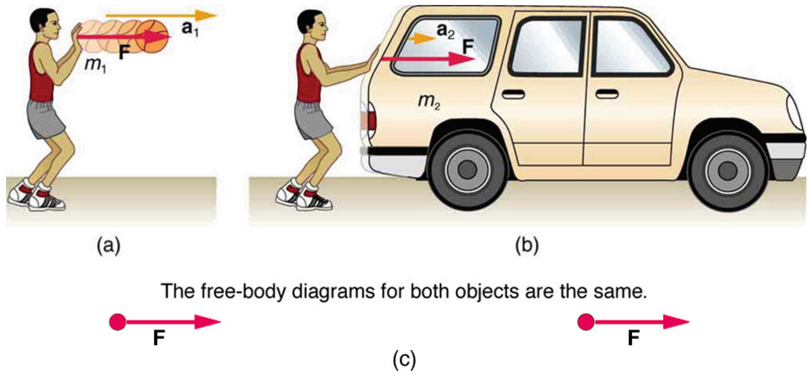


Figure 3.5 The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

It has been found that the acceleration of an object depends *only* on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

In equation form, Newton's second law of motion is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \tag{3.3}$$

This is often written in the more familiar form

$$\mathbf{F}_{\text{net}} = m\mathbf{a}. \tag{3.4}$$

When only the magnitude of force and acceleration are considered, this equation is simply

$$F_{\text{net}} = ma. \tag{3.5}$$

Although these last two equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities that is not simply based on their definitions. The validity of the second law is completely based on experimental verification.

Units of Force

$\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1m/s^2. That is, since $\mathbf{F}_{\text{net}} = m\mathbf{a}$,

$$1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2. \tag{3.6}$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\mathbf{w}$. Weight can be denoted as a vector $\mathbf{w}$ because it has a direction; *down* is, by definition, the direction of gravity, and hence weight is a downward force. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w. Newton's second law states that the magnitude of the net external force on an object is $F_{\text{net}} = ma$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

Weight

This is the equation for *weight*—the gravitational force on a mass m :

$$w = mg. \quad (3.7)$$

Since $g = 9.80 \text{ m/s}^2$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:

$$w = mg = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}. \quad (3.8)$$

Recall that g can take a positive or negative value, depending on the positive direction in the coordinate system. Be sure to take this into consideration when solving problems with weight.

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth's surface. On the Moon, for example, the acceleration due to gravity is only 1.67 m/s^2 . A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are really referring to the phenomenon we call "free-fall" in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much "stuff") and does not vary in classical physics, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our "weight" in kilograms, but never in the correct units of newtons.

Common Misconceptions: Mass vs. Weight

Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the "slug" in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units).

Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is 1.67 m/s^2 (which is much less than the acceleration due to gravity on Earth, 9.80 m/s^2). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

Example 3.1 What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?

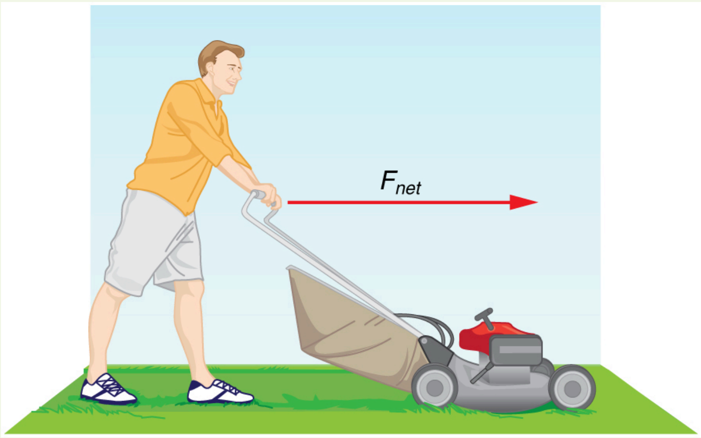


Figure 3.6 The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy

Since $\mathbf{F}_{\text{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law as stated in $\mathbf{F}_{\text{net}} = m\mathbf{a}$.

Solution

The magnitude of the acceleration a is $a = \frac{F_{\text{net}}}{m}$. Entering known values gives

$$a = \frac{51\ \text{N}}{24\ \text{kg}} \tag{3.9}$$

Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$a = \frac{51\ \text{kg} \cdot \text{m/s}^2}{24\ \text{kg}} = 2.1\ \text{m/s}^2. \tag{3.10}$$

Discussion

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moves forward), and the vertical forces must cancel if there is to be no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long because the person's top speed would soon be reached.

Example 3.2 What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust $\mathbf{T}$, for the four-rocket propulsion system shown in Figure 3.7. The sled's initial acceleration is $49\ \text{m/s}^2,$ the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.

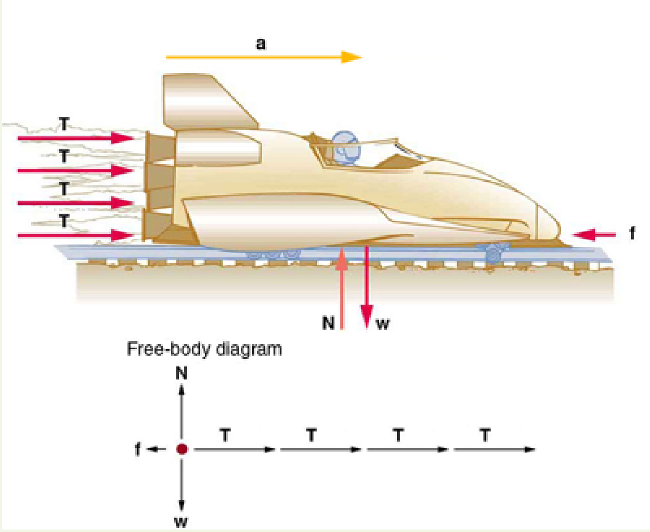


Figure 3.7 A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust $\mathbf{T}$. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force $\mathbf{N}$ on the system that is equal in magnitude and opposite in direction to its weight, $\mathbf{w}$. The system here is the sled, its rockets, and rider, so none of the forces *between* these objects are considered. The arrow representing friction ($\mathbf{f}$) is drawn larger than scale.

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = ma, \tag{3.11}$$

where F_{net} is the net force along the horizontal direction. We can see from **Figure 3.7** that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f. \tag{3.12}$$

Substituting this into Newton's second law gives

$$F_{\text{net}} = ma = 4T - f. \tag{3.13}$$

Using a little algebra, we solve for the total thrust 4*T*:

$$4T = ma + f. \tag{3.14}$$

Substituting known values yields

$$4T = ma + f = (2100\text{ kg})(49\text{ m/s}^2) + 650\text{ N}. \tag{3.15}$$

So the total thrust is

$$4T = 1.0\times10^5\text{ N}, \tag{3.16}$$

and the individual thrusts are

$$T = \frac{1.0\times10^5\text{ N}}{4} = 2.6\times10^4\text{ N}. \tag{3.17}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g 's. (Recall that g, the acceleration due to gravity, is

$9.80\ \text{m/s}^2$. When we say that an acceleration is 45 g's, it is $45 \times 9.80\ \text{m/s}^2$, which is approximately $440\ \text{m/s}^2$.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law of motion is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

3.4 Newton's Third Law of Motion: Symmetry in Forces

Whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton's third law of motion**.

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 3.8. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer or the wall. If we select the swimmer to be the system of interest, as in the figure, then $\mathbf{F}_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of $\mathbf{F}_{\text{wall on feet}}$. In contrast, the force $\mathbf{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\mathbf{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathbf{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to that in which she wishes to move. The reaction to her push is thus in the desired direction.

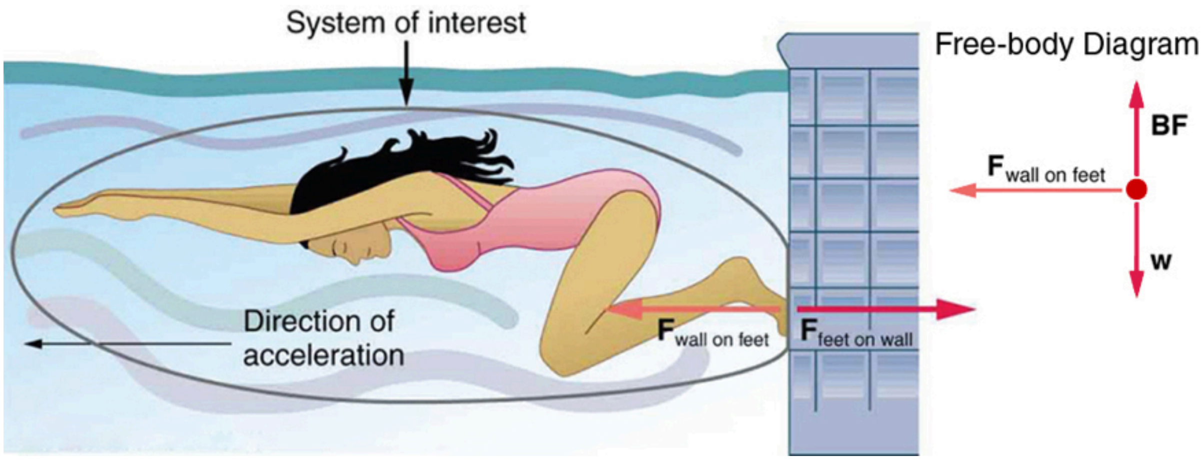


Figure 3.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\mathbf{F}_{\text{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\mathbf{F}_{\text{wall on feet}}$. Thus the free-body diagram shows only $\mathbf{F}_{\text{wall on feet}}$, $\mathbf{w}$, the gravitational force, and $\mathbf{BF}$, the buoyant force of the water supporting the swimmer's weight. The vertical forces $\mathbf{w}$ and $\mathbf{BF}$ cancel since there is no vertical motion.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases. Helicopters similarly create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example 3.3 Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in Figure 3.9. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

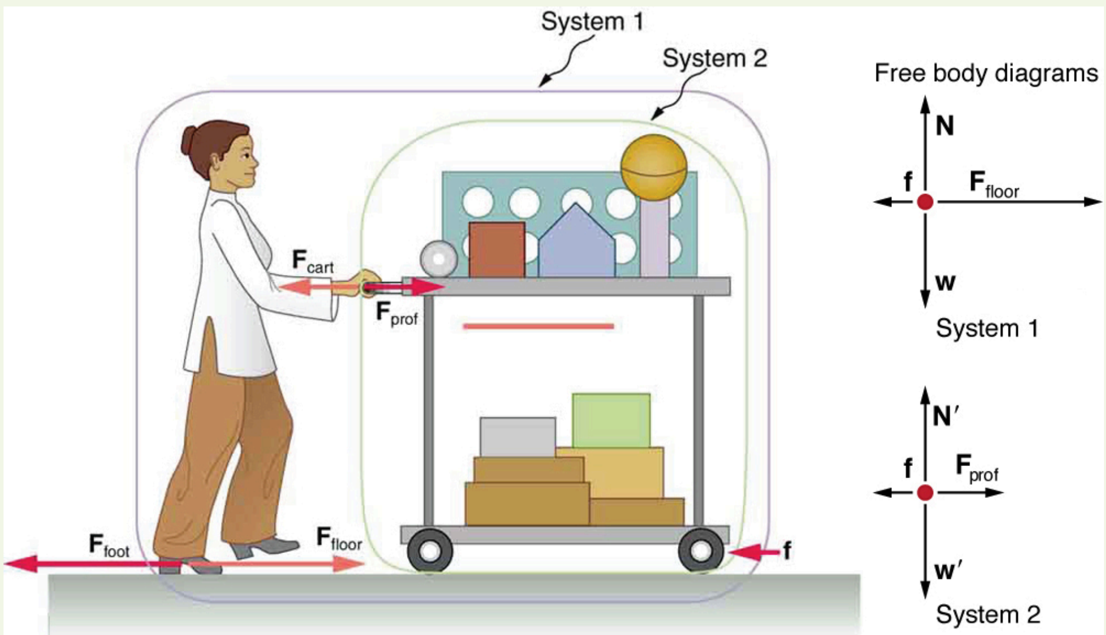


Figure 3.9 A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\mathbf{f}$, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for Example 3.4, since it asks for the acceleration of the entire group of objects. Only $\mathbf{F}_{\text{floor}}$ and $\mathbf{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for this example so that $\mathbf{F}_{\text{prof}}$ will be an external force and enter into Newton's second law. Note that the free-body diagrams, which allow us to apply Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in Figure 3.9. The professor pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\mathbf{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. The problem is therefore one-dimensional along the horizontal direction. As noted, $\mathbf{f}$ opposes the motion and is thus in the opposite direction of $\mathbf{F}_{\text{floor}}$. Note that we do not include the forces $\mathbf{F}_{\text{prof}}$ or $\mathbf{F}_{\text{cart}}$ because these are internal forces, and we do not include $\mathbf{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{\text{net}}}{m}. \tag{3.18}$$

The net external force on System 1 is deduced from **Figure 3.9** and the discussion above to be

$$F_{\text{net}} = F_{\text{floor}} - f = 150\text{ N} - 24.0\text{ N} = 126\text{ N}. \tag{3.19}$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0)\text{ kg} = 84\text{ kg}. \tag{3.20}$$

These values of F_{net} and m produce an acceleration of

$$\begin{aligned} a &= \frac{F_{\text{net}}}{m}, \\ a &= \frac{126\text{ N}}{84\text{ kg}} = 1.5\text{ m/s}^2. \end{aligned} \tag{3.21}$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 3.4 Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in **Figure 3.9** using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in **Figure 3.9**), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F}_{\text{prof}}$, is an external force acting on System 2. $\mathbf{F}_{\text{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\mathbf{F}_{\text{prof}}$. Starting with

$$a = \frac{F_{\text{net}}}{m} \tag{3.22}$$

and noting that the magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f, \tag{3.23}$$

we solve for F_{prof}, the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f. \tag{3.24}$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

$$F_{\text{net}} = ma, \tag{3.25}$$

where the mass of System 2 is 19.0 kg (m = 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5\text{ m/s}^2$ in the previous example. Thus,

$$F_{\text{net}} = ma, \tag{3.26}$$

$$F_{\text{net}} = (19.0\text{ kg})(1.5\text{ m/s}^2) = 29\text{ N}. \tag{3.27}$$

Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f, \tag{3.28}$$

$$F_{\text{prof}} = 29\text{ N}+24.0\text{ N} = 53\text{ N}. \tag{3.29}$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

3.5 Normal Force and Tension

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in **Figure 3.10**(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in **Figure 3.10**(b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

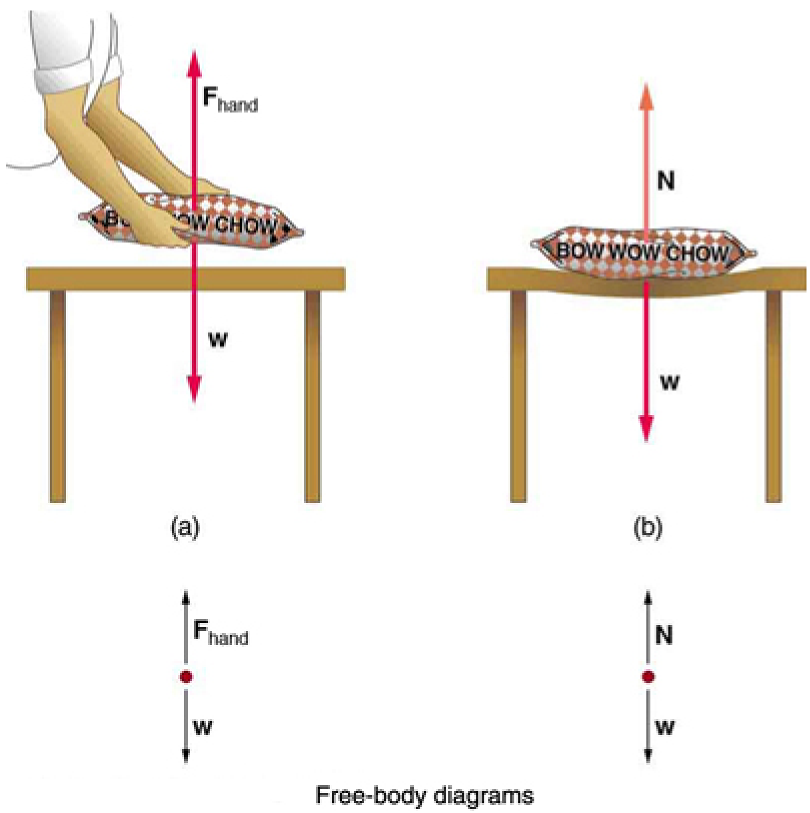


Figure 3.10 (a) The person holding the bag of dog food must supply an upward force $\mathbf{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food $\mathbf{w}$. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\mathbf{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting a load is perpendicular to the surface of contact between the load and its support, this force is defined to be a **normal force** and here is given the symbol $\mathbf{N}$. (This is not the unit for force N.) The word *normal* means perpendicular to a surface. The normal force can be less than the object's weight if the object is on an incline, as you will see in the next example.

Common Misconception: Normal Force (N) vs. Newton (N)

In this section we have introduced the quantity normal force, which is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. These symbols are particularly important to distinguish because the units of a normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force $\mathbf{N}$ that the floor exerts on a chair might be $\mathbf{N} = 100\text{ N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations! You will encounter more similarities among variables and units as you proceed in physics. Another example of this is the quantity work (W) and the unit watts (W).

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word "tension" comes from a Latin word meaning "to stretch." Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: "You can't push a rope." The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in Figure 3.11.

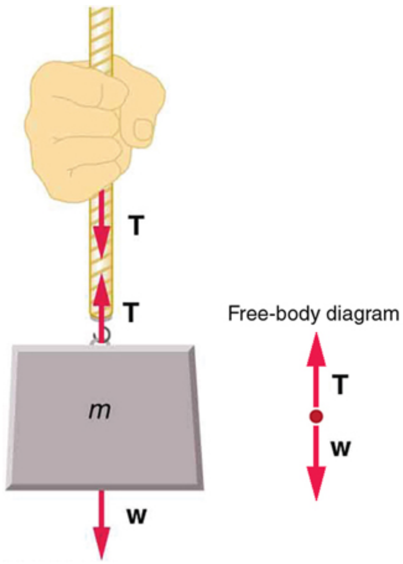


Figure 3.11 When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\mathbf{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\mathbf{w}$ and the tension $\mathbf{T}$ supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0, \tag{3.30}$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

$$T = w = mg. \tag{3.31}$$

For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

$$T = mg = (5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}. \tag{3.32}$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in **Figure 3.12** (a) and (b).

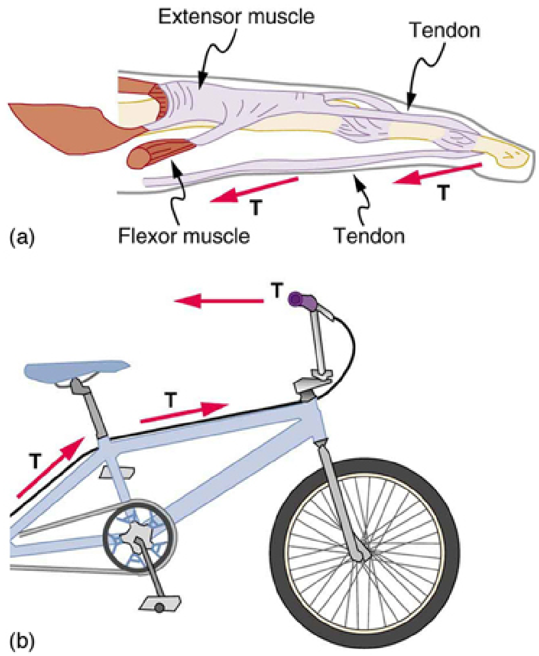


Figure 3.12 (a) Tendons in the finger carry force $\mathbf{T}$ from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\mathbf{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\mathbf{T}$ is changed.

Figure 3.13 Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

3.6 Spring Force: Hooke's Law

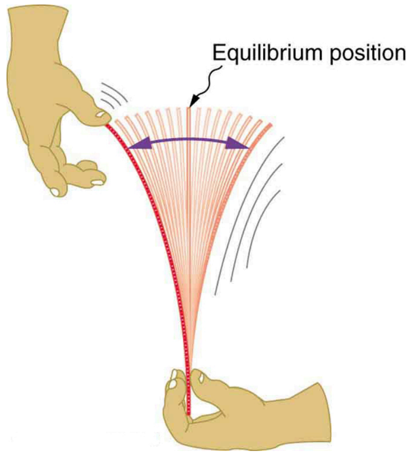


Figure 3.14 When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in **Figure 3.14**. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. This is called Hooke's law force, or spring force:

$$F = -kx. \tag{3.33}$$

Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

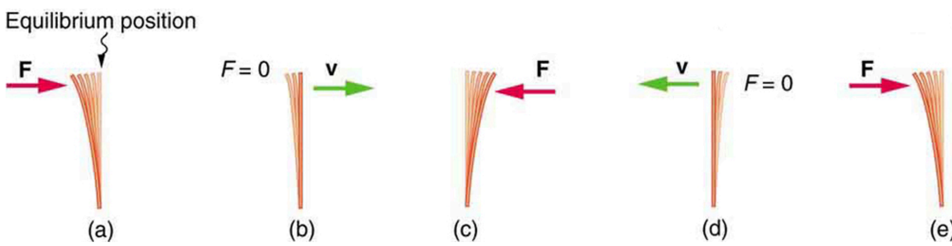
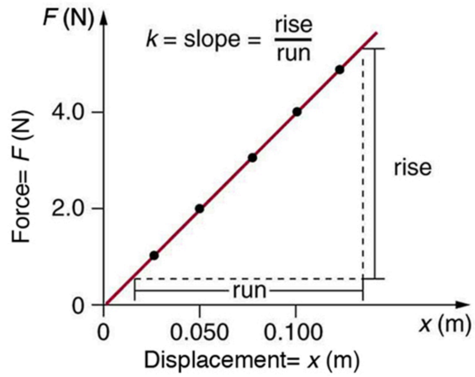


Figure 3.15 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). **Figure 3.16** shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter.

a)



m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

b)

m (kg)	w (N)	x (m)
0.000	0.00	0.000
0.100	0.98	0.025
0.200	1.96	0.050
0.300	2.94	0.076
0.400	3.92	0.099
0.500	4.90	0.127

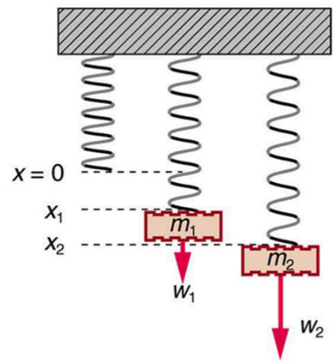


Figure 3.16 (a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the force constant k . (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

Example 3.5 How Stiff Are Car Springs?

Figure 3.17 The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. (credit: exfordy on Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20\times10^{-2}\text{ m}$. At that point, the springs supply a restoring force F equal to the person's weight $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$. We take this force to be F in Hooke's law. Knowing F and

x , we can then solve the force constant k .

Solution

1. Solve Hooke's law, $F = -kx$, for k :

$$k = -\frac{F}{x}. \tag{3.34}$$

Substitute known values and solve k :

$$\begin{aligned} k &= -\frac{784 \text{ N}}{-1.20\times10^{-2} \text{ m}} \\ &= 6.53\times10^{4} \text{ N/m.} \end{aligned} \tag{3.35}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

3.7 Friction

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

Friction

Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Kinetic Friction

If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

Figure 3.18 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

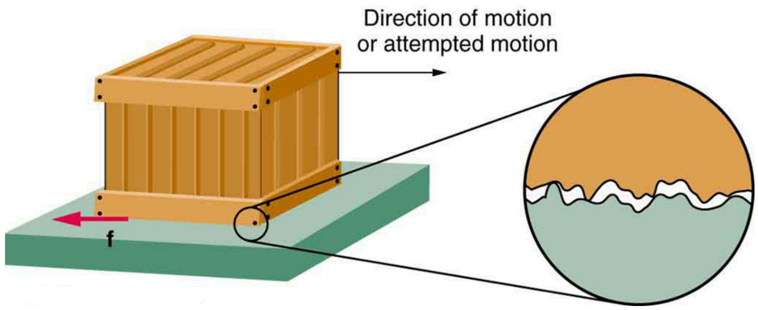


Figure 3.18 Frictional forces, such as f , always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction** $\mathbf{f_s}$ is

$$f_{\text{s}} \leq \mu_{\text{s}} N, \tag{3.36}$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Magnitude of Static Friction

Magnitude of static friction f_{s} is

$$f_{\text{s}} \leq \mu_{\text{s}} N, \tag{3.37}$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_{\text{s}} N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{\text{s(max)}}$, the object will move. Thus

$$f_{\text{s(max)}} = \mu_{\text{s}} N. \tag{3.38}$$

Once an object is moving, the **magnitude of kinetic friction** $\mathbf{f_k}$ is given by

$$f_{\text{k}} = \mu_{\text{k}} N, \tag{3.39}$$

where μ_{k} is the coefficient of kinetic friction. A system in which $f_{\text{k}} = \mu_{\text{k}} N$ is described as a system in which *friction behaves simply*.

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_{k} is given by

$$f_{\text{k}} = \mu_{\text{k}} N, \tag{3.40}$$

where μ_{k} is the coefficient of kinetic friction.

As seen in **Table 3.1**, the coefficients of kinetic friction are less than their static counterparts. That values of μ in **Table 3.1** are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

Table 3.1 Coefficients of Static and Kinetic Friction

System	Static friction μ_s	Kinetic friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100\text{ kg})(9.80\text{ m/s}^2) = 980\text{ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\text{s(max)}} = \mu_\text{s}N = (0.45)(980\text{ N}) = 440\text{ N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_\text{k} = \mu_\text{k}N = (0.30)(980\text{ N}) = 290\text{ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Take-Home Experiment

Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table, simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (**Figure 3.19**). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

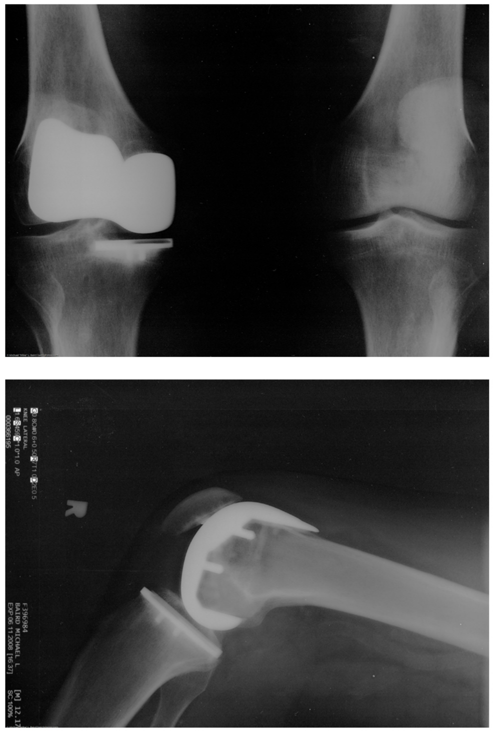


Figure 3.19 Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op x rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor’s clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

Figure 3.20 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.

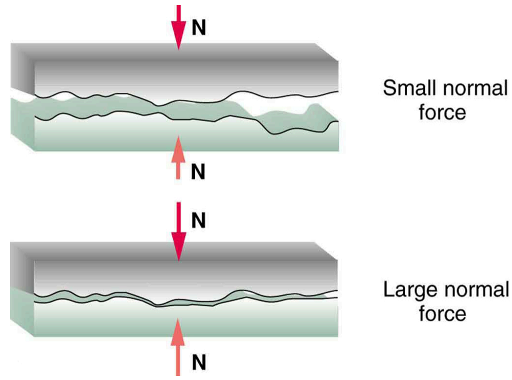


Figure 3.20 Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. **Figure 3.21** shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

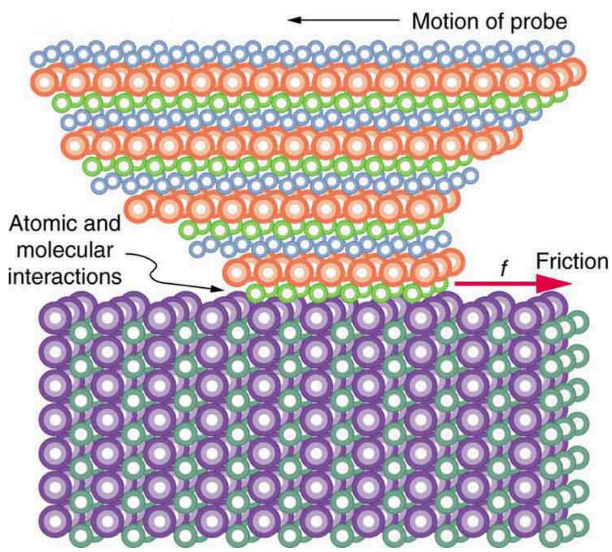


Figure 3.21 The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

3.8 Newton's Universal Law of Gravitation

What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth's gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth's surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See **Figure 3.22**. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

Figure 3.22 According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

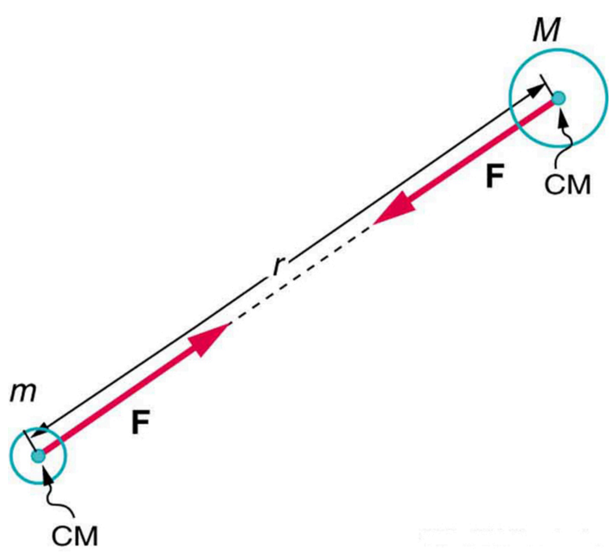


Figure 3.23 Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

Misconception Alert

The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM). For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$F = G\frac{mM}{r^2}, \tag{3.41}$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be

$$G = 6.673\times10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}} \tag{3.42}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{mM}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of $6.673\times10^{-11}\ \mathrm{N}$. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of $5.98\times10^{24}\ \mathrm{kg}$.

Recall that the acceleration due to gravity g is about $9.80\ \mathrm{m/s^2}$ on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

$$mg = G\frac{mM}{r^2}, \tag{3.43}$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See **Figure 3.24**. The mass m of the object cancels, leaving an equation for g:

$$g = G\frac{M}{r^2}. \tag{3.44}$$

Substituting known values for Earth's mass and radius (to three significant figures),

$$g = \left(6.67\times10^{-11}\frac{\mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)\times\frac{5.98\times10^{24}\ \mathrm{kg}}{(6.38\times10^{6}\ \mathrm{m})^2}, \tag{3.45}$$

and we obtain a value for the acceleration of a falling body:

$$g = 9.80\ \mathrm{m/s^2}. \tag{3.46}$$

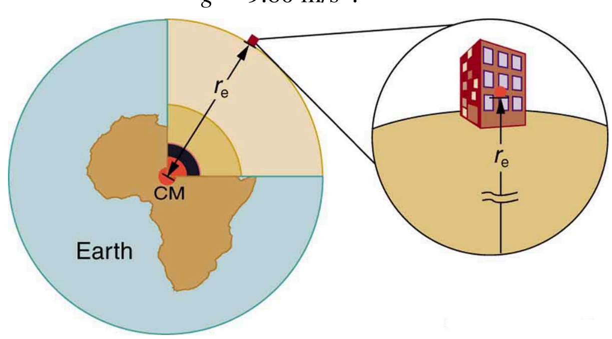


Figure 3.24 The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body's mass*. Newton's law of gravitation takes Galileo's observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Take-Home Experiment

Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Example 3.6 Earth's Gravitational Force on a Mass

(a) Determine the weight of a 5.00 kg rock when on Earth's surface.

(b) Determine the weight of a 5.00 kg rock when 3620 km above the surface of Earth.

Strategy for (a)

Use acceleration due to gravity near Earth's surface and Newton's second law.

Solution for (a)

$$F = mg = 5.00\,\text{kg} \times 9.80\,\text{m}/\text{s}^2 = 49.0\,\text{N} \tag{3.47}$$

Strategy for (b)

Use Newton's universal law of gravitation. Remember that distance is from the center of Earth. In this case the distance is 6380 km + 3620 km = 10,000 km = $1.00 \times 10^7\,\text{m}$.

Solution for (b)

$$F = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\,\frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\frac{(5.00\text{kg})\left(5.98 \times 10^{24}\,\text{kg}\right)}{\left(1.00 \times 10^7\,\text{m}\right)^2} = 20.0\text{N} \tag{3.48}$$

Tides

Ocean tides are one very observable result of the Moon's gravity acting on Earth. **Figure 3.25** is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).

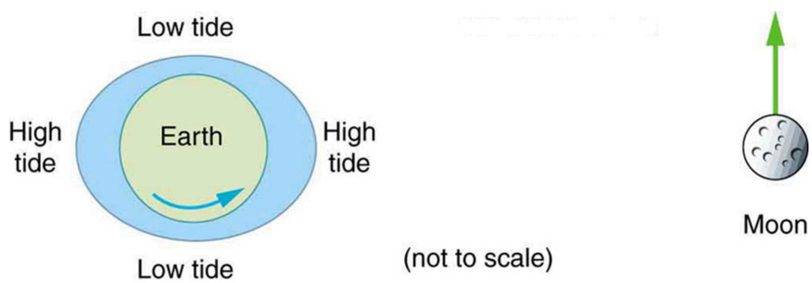


Figure 3.25 The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day at any location, because Earth rotates under the tidal bulge.

The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90^{o} angle to the Earth-Moon alignment.

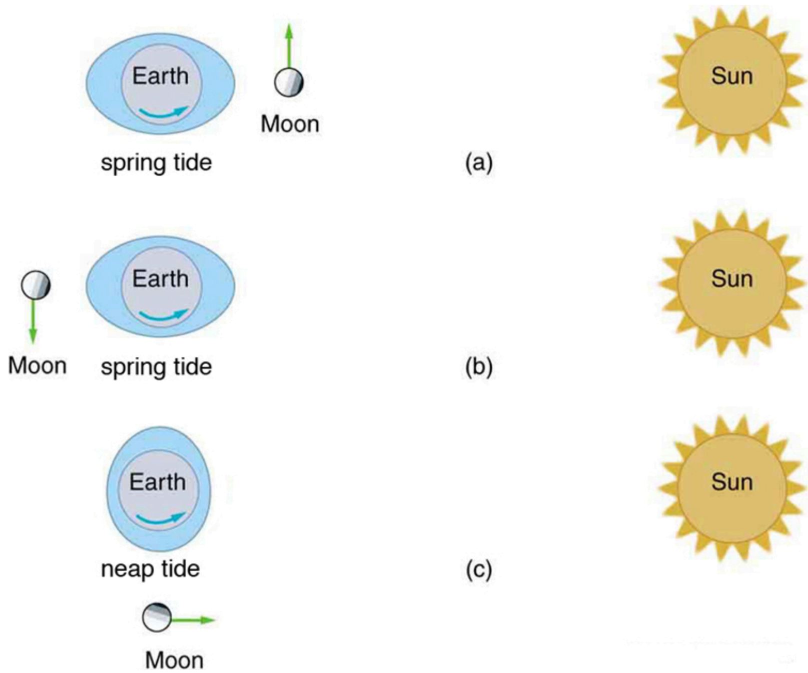


Figure 3.26 (a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90° to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see **Figure 3.27**). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

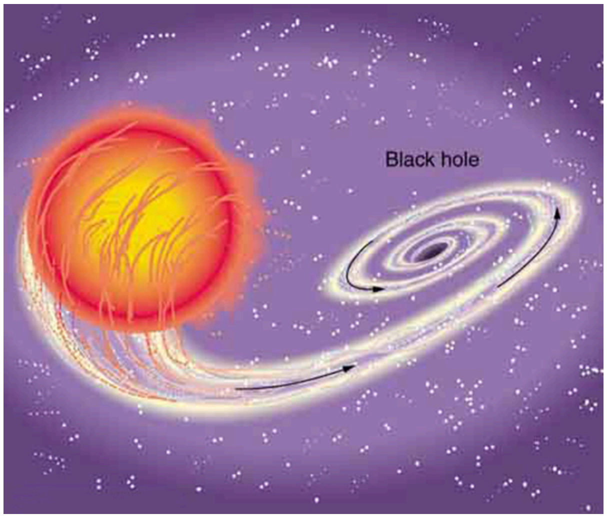


Figure 3.27 A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

"Weightlessness" and Microgravity

In contrast to the tremendous gravitational force near black holes is the apparent gravitational field experienced by astronauts orbiting Earth. What is the effect of "weightlessness" upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn't mean that an astronaut is not being acted upon by the gravitational force. There is no "zero gravity" in an astronaut's orbit. The term just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks.

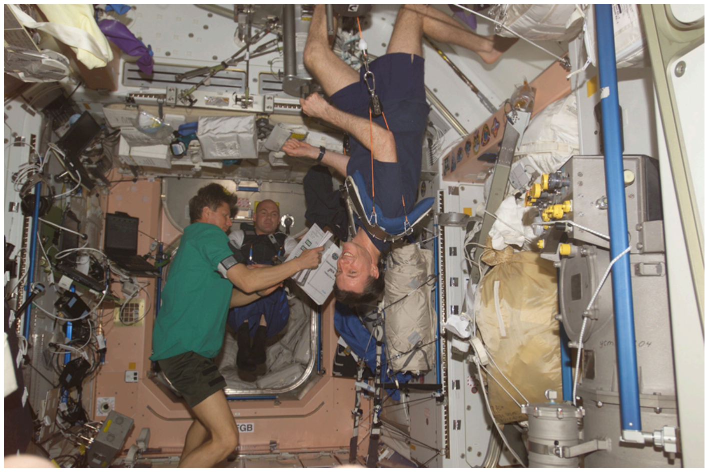


Figure 3.28 Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

Microgravity refers to an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

3.9 Centripetal Force

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: net $\mathrm{F} = ma$. For uniform circular motion, the acceleration is the centripetal acceleration— $a = a_c$. Thus, the magnitude of centripetal force $\mathrm{F_c}$ is

$$\mathrm{F_c} = ma_\mathrm{c}. \tag{3.49}$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r}$, we get an expression for the centripetal force $\mathrm{F_c}$ in terms of mass, velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r}. \tag{3.50}$$

Note that if you solve the first expression for r , you get

$$r = \frac{mv^2}{F_c}. \tag{3.51}$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

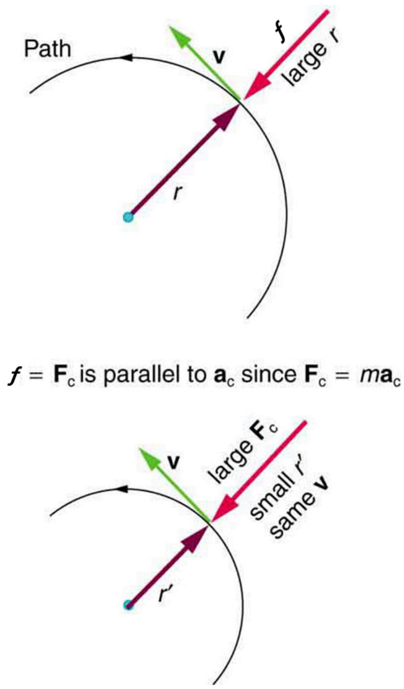


Figure 3.29 The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the $\mathrm{F_c}$, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v , but a larger $\mathrm{F_c}$ produces a smaller r' .

Example 3.7 What Coefficient of Friction Do Car Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see **Figure 3.30**).

Strategy and Solution for (a)

We know that $F_{\mathrm{c}} = \frac{mv^2}{r}$. Thus,

$$F_{\mathrm{c}} = \frac{mv^2}{r} = \frac{(900\ \mathrm{kg})(25.0\ \mathrm{m/s})^2}{(500\ \mathrm{m})} = 1125\ \mathrm{N}. \tag{3.52}$$

Strategy for (b)

Figure 3.30 shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_{\mathrm{s}}N$, where μ_{s} is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that $N = mg$. Thus the centripetal force in this situation is

$$F_{\mathrm{c}} = f = \mu_{\mathrm{s}}N = \mu_{\mathrm{s}}mg. \tag{3.53}$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the expression for F_{c}

$$F_c = m\frac{v^2}{r}, \tag{3.54}$$

$$m\frac{v^2}{r} = \mu_{\mathrm{s}}mg. \tag{3.55}$$

We solve this for μ_{s} , noting that mass cancels, and obtain

$$\mu_{\mathrm{s}} = \frac{v^2}{rg}. \tag{3.56}$$

Solution for (b)

Substituting the knowns,

$$\mu_s = \frac{(25.0 \text{ m/s})^2}{(500 \text{ m})(9.80 \text{ m/s}^2)} = 0.13. \tag{3.57}$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.

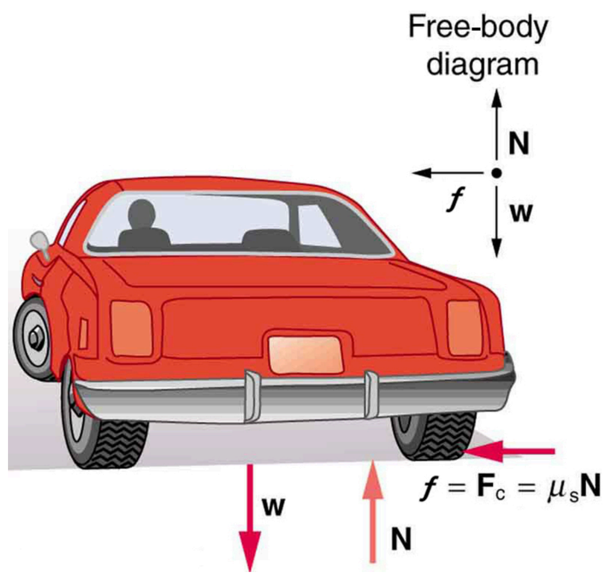


Figure 3.30 This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

In the case of **banked curves**, where the slope of the road helps you negotiate the curve, some or all of the necessary centripetal force is provided by the normal force. See **Figure 3.31**. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. Conceptually, for **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force $\mathbf{N}$ in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively.

Figure 3.31 shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight $\mathbf{w}$ and the normal force of the road $\mathbf{N}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. We omit detailed calculations, which require trigonometry.

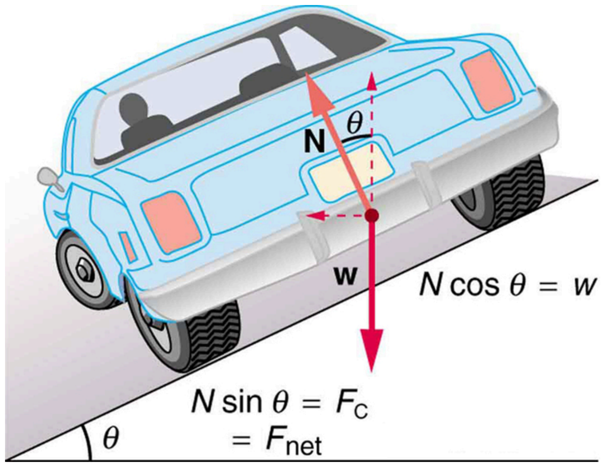


Figure 3.31 The car on this banked curve is moving away and turning to the left.

Take-Home Experiment

Ask a friend or relative to swing a golf club or a tennis racquet. Take appropriate measurements to estimate the centripetal acceleration of the end of the club or racquet. You may choose to do this in slow motion.

Glossary

acceleration: the rate at which an object's velocity changes over a period of time

banked curve: the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

center of mass: the point where the entire mass of an object can be thought to be concentrated

centripetal force: any net force causing uniform circular motion

deformation: displacement from equilibrium

dynamics: the study of how forces affect the motion of objects and systems

external force: a force acting on an object or system that originates outside of the object or system

force: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

force constant: a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by *k*

free-body diagram: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

free-fall: a situation in which the only force acting on an object is the force due to gravity

friction: a force past each other of objects that are touching; examples include rough surfaces and air resistance

friction: a force that opposes relative motion or attempts at motion between systems in contact

gravitational constant, *G*: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

ideal banking: the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

inertia: the tendency of an object to remain at rest or remain in motion

inertial frame of reference: a coordinate system that is not accelerating; all forces acting in an inertial frame of reference are real forces, as opposed to fictitious forces that are observed due to an accelerating frame of reference

kinetic friction: a force that opposes the motion of two systems that are in contact and moving relative to one another

law of inertia: see Newton's first law of motion

magnitude of kinetic friction: $f_k = \mu_k N$, where μ_k is the coefficient of kinetic friction

magnitude of static friction: $f_s \leq \mu_s N$, where μ_s is the coefficient of static friction and N is the magnitude of the normal force

mass: the quantity of matter in a substance; measured in kilograms

microgravity: an environment in which the apparent net acceleration of a body is small compared with that produced by Earth at its surface

net external force: the vector sum of all external forces acting on an object or system; causes a mass to accelerate

Newton's first law of motion: a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion: the net external force $\mathbf{F}_{\text{net}}$ on an object with mass m is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and inversely proportional to the mass; defined mathematically as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$

Newton's third law of motion: whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts

Newton's universal law of gravitation: every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

normal force: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

restoring force: force acting in opposition to the force caused by a deformation

static friction: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

system: defined by the boundaries of an object or collection of objects being observed; all forces originating from outside of the system are considered external forces

tension: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

thrust: a reaction force that pushes a body forward in response to a backward force; rockets, airplanes, and cars are pushed forward by a thrust reaction force

weight: the force $\mathbf{w}$ due to gravity acting on an object of mass m; defined mathematically as: $\boldsymbol{w} = m\boldsymbol{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

Section Summary

3.1 Development of Force Concept

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.
- **External forces** are any outside forces that act on a body. A **free-body diagram** is a drawing of all external forces acting on a body.

3.2 Newton's First Law of Motion: Inertia

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

3.3 Newton's Second Law of Motion: Concept of a System

- Acceleration, $\mathbf{a}$, is defined as a change in velocity, meaning a change in its magnitude or direction, or both.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.

- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$.
- This is often written in the more familiar form: $\mathbf{F}_{\text{net}} = m\mathbf{a}$.
- The weight $\mathbf{w}$ of an object is defined as the force of gravity acting on an object of mass m . The object experiences an acceleration due to gravity $\mathbf{g}$:

$$\mathbf{w} = m\mathbf{g}.$$

- If the only force acting on an object is due to gravity, the object is in free fall.
- Friction is a force that opposes the motion past each other of objects that are touching.

3.4 Newton's Third Law of Motion: Symmetry in Forces

- **Newton's third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

3.5 Normal Force and Tension

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, $\mathbf{N}$.
- When objects rest on a non-accelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object:

$$N = mg.$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\mathbf{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:

$$T = mg.$$

3.6 Spring Force: Hooke's Law

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:

$$F = -kx,$$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.

3.7 Friction

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_{s} between systems stationary relative to one another is given by

$$f_{\text{s}} \leq \mu_{\text{s}}N,$$

  where μ_{s} is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_{k} between systems moving relative to one another is given by

$$f_{\text{k}} = \mu_{\text{k}}N,$$

  where μ_{k} is the coefficient of kinetic friction, which also depends on both materials.

3.8 Newton's Universal Law of Gravitation

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is

$$F = G\frac{mM}{r^2},$$

  where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

- Newton's law of gravitation applies universally.

3.9 Centripetal Force

- Centripetal force $\mathrm{F_c}$ is any force causing uniform circular motion. It is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude

$$F_\mathrm{c} = ma_\mathrm{c},$$

which can also be expressed as

$$F_c = m\frac{v^2}{r}.$$

Conceptual Questions

3.1 Development of Force Concept

1. Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

2. What properties do forces have that allow us to classify them as vectors?

3.2 Newton's First Law of Motion: Inertia

3. How are inertia and mass related?

4. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

3.3 Newton's Second Law of Motion: Concept of a System

5. Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

6. Why can we neglect forces such as those holding a body together when we apply Newton's second law of motion?

7. Explain how the choice of the "system of interest" affects which forces must be considered when applying Newton's second law of motion.

8. Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

9. A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

10. A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

11. (a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

12. If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

13. If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

14. The gravitational force on the basketball in **Figure 3.5** is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

3.4 Newton's Third Law of Motion: Symmetry in Forces

15. When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

16. A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the "ballistocardiograph." What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

17. Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton's laws of motion apply?

18. Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton's third law applies when one is fired. Can you safely stand close behind one when it is fired?

19. An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

20. Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

3.6 Spring Force: Hooke's Law

21. Describe a system which undergoes an oscillation under a Hooke's law force.

3.7 Friction

22. Define normal force. What is its relationship to friction when friction behaves simply?

23. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

24. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

25. When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

3.8 Newton's Universal Law of Gravitation

26. Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

27. Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not 9.80 m/s^2. Who do you agree with and why?

28. Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

3.9 Centripetal Force

29. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

30. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

31. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

32. Race car drivers routinely cut corners as shown in Figure 3.32. Explain how this allows the curve to be taken at the greatest speed.

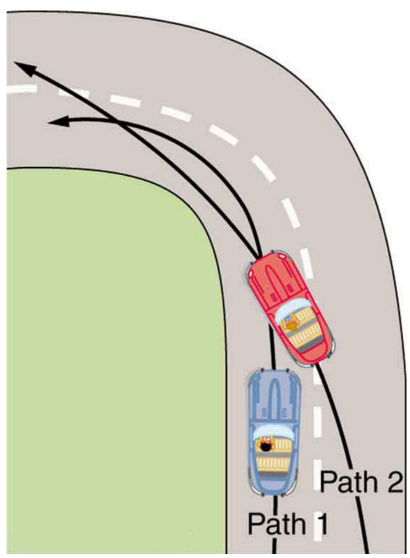


Figure 3.32 Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

33. A number of amusement parks have rides that make vertical loops like the one shown in **Figure 3.33**. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?

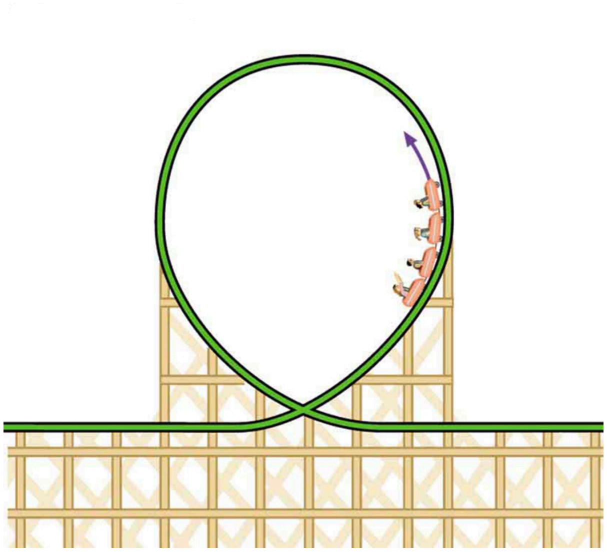


Figure 3.33 Amusement rides with a vertical loop are an example of a form of curved motion.

34. What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in **Figure 3.33** under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

35. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in **Figure 3.34** will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.

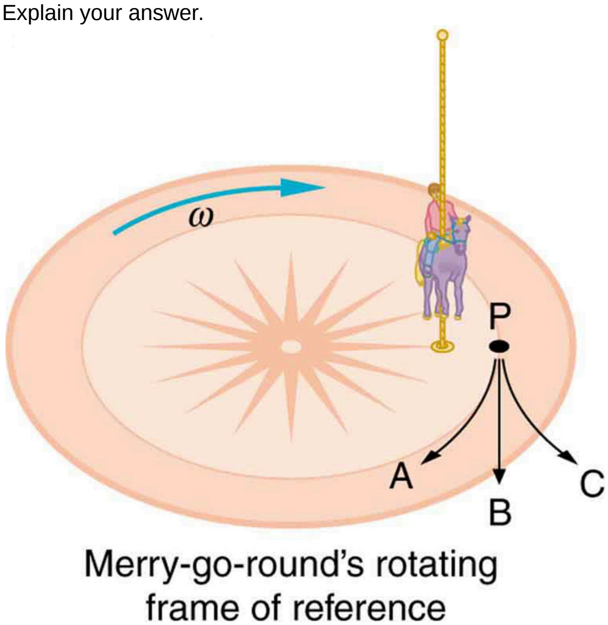


Figure 3.34 A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

36. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

37. Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.

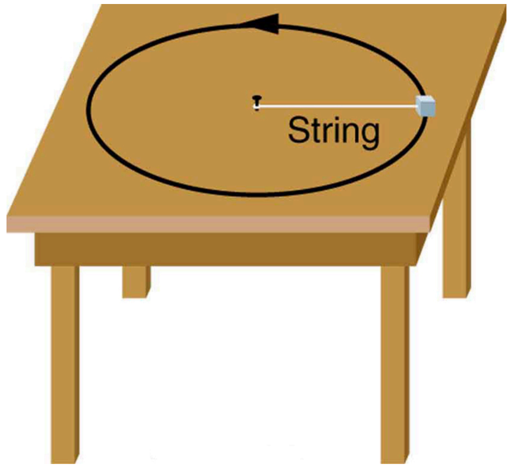


Figure 3.35 A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

Problems & Exercises

3.3 Newton's Second Law of Motion: Concept of a System

You may assume data taken from illustrations is accurate to three digits.

1. A 63.0-kg sprinter starts a race with an acceleration of 4.20 m/s^2. What is the net external force on him?

2. If the sprinter from the previous problem accelerates at that rate for 20 m, and then maintains that velocity for the remainder of the 100-m dash, what will be his time for the race?

3. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of its acceleration.

4. Since astronauts in orbit are apparently weightless, a clever method of measuring their masses is needed to monitor their mass gains or losses to adjust diets. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted and the astronaut's acceleration is measured to be 0.893 m/s^2. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which they orbit experiences an equal and opposite force. Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method in which recoil of the vehicle is avoided.

5. In **Figure 3.6**, the net external force on the 24-kg mower is stated to be 51 N. If the force of friction opposing the motion is 24 N, what force F (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force F is removed. How far will the mower go before stopping?

6. The same rocket sled drawn in **Figure 3.36** is decelerated at a rate of 196 m/s^2. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2100 kg.

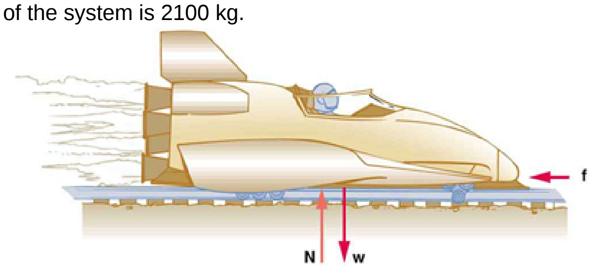


Figure 3.36

7. (a) If the rocket sled shown in **Figure 3.37** starts with only one rocket burning, what is the magnitude of its acceleration? Assume that the mass of the system is 2100 kg, the thrust T is 2.4×10^4 N, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

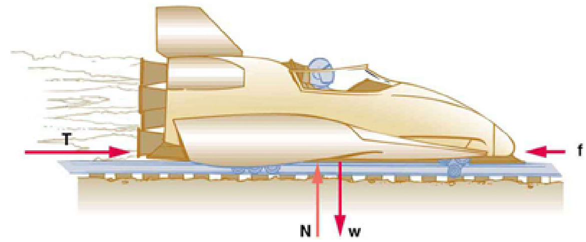


Figure 3.37

8. What is the deceleration of the rocket sled if it comes to rest in 1.1 s from a speed of 1000 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

9. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (b) Draw a free-body diagram, including all forces acting on the system. (c) Calculate the acceleration. (d) What would the acceleration be if friction were 15.0 N?

10. A powerful motorcycle can produce an acceleration of 3.50 m/s^2 while traveling at 90.0 km/h. At that speed the forces resisting motion, including friction and air resistance, total 400 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force the motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

11. The rocket sled shown in **Figure 3.38** accelerates at a rate of 49.0 m/s^2. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight by using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

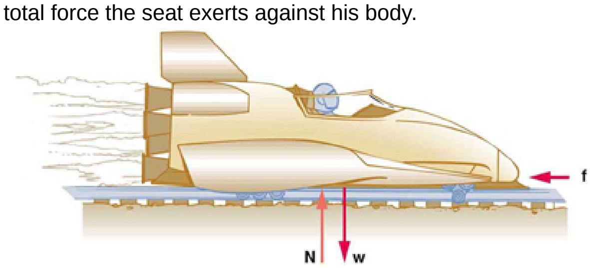


Figure 3.38

12. Repeat the previous problem for the situation in which the rocket sled decelerates at a rate of 201 m/s^2. In this problem, the forces are exerted by the seat and restraining belts.

13. The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?

14. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 10,000 kg. The thrust of its engines is 30,000 N. (a) Calculate its the magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

3.4 Newton's Third Law of Motion: Symmetry in

Forces

15. What net external force is exerted on a 1100-kg artillery shell fired from a battleship if the shell is accelerated at $2.40\times10^4\ \mathrm{m/s}^2$? What is the magnitude of the force exerted on the ship by the artillery shell?

16. A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating at $1.20\ \mathrm{m/s}^2$ backward. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110 kg? (c) Draw a sketch of the situation showing the system of interest used to solve each part. For this situation, draw a free-body diagram and write the net force equation.

3.5 Normal Force and Tension

17. Two teams of nine members each engage in a tug of war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams? (b) What is the tension in the section of rope between the teams?

18. What force does a trampoline have to apply to a 45.0-kg gymnast to accelerate her straight up at $7.50\ \mathrm{m/s}^2$? Note that the answer is independent of the velocity of the gymnast—she can be moving either up or down, or be stationary.

19. Calculate the tension in a vertical strand of spider web if a spider of mass $8.00\times10^{-5}\ \mathrm{kg}$ hangs motionless on it.

20. Suppose a 60.0-kg gymnast climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of $1.50\ \mathrm{m/s}^2$?

21. Consider the baby being weighed in Figure 3.39. (a) What is the mass of the child and basket if a scale reading of 55 N is observed? (b) What is the tension T_1 in the cord attaching the baby to the scale? (c) What is the tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Draw a sketch of the situation indicating the system of interest used to solve each part. The masses of the cords are negligible.

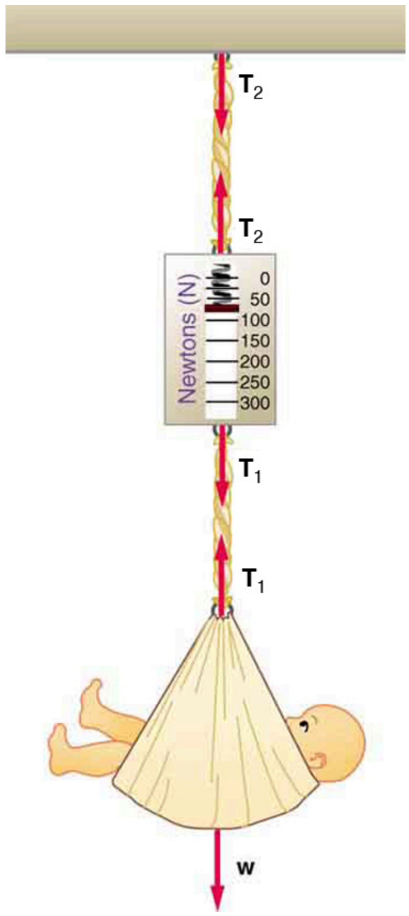


Figure 3.39 A baby is weighed using a spring scale.

3.6 Spring Force: Hooke's Law

22. Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass).

(a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load?

(b) What is the mass of a fish that stretches the spring 5.50 cm?

(c) How far apart are the half-kilogram marks on the scale?

23. It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

24. One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

25. (a) The springs of a pickup truck act like a single spring with a force constant of $1.30\times10^5\ \mathrm{N/m}$. By how much will the truck be depressed by its maximum load of 1000 kg?

(b) If the pickup truck has four identical springs, what is the force constant of each?

26. When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m.

(a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

27. A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

3.7 Friction

28. A physics major is cooking breakfast when he notices that the frictional force between his steel spatula and his Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, he quickly calculates the normal force. What is it?

29. (a) When rebuilding her car's engine, a physics major must exert 300 N of force to insert a dry steel piston into a steel cylinder. What is the magnitude of the normal force between the piston and cylinder? (b) What is the magnitude of the force would she have to exert if the steel parts were oiled?

30. (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise it is possible to exert forces to the joints that are easily ten times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

31. Suppose you have a 120-kg wooden crate resting on a wood floor. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will the magnitude of its acceleration then be?

32. (a) If half of the weight of a small $1.00\times10^3\ \text{kg}$ utility truck is supported by its two drive wheels, what is the magnitude of the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

33. A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the magnitude of the acceleration starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) What is the magnitude of the acceleration once the sled starts to move? (c) For both situations, calculate the magnitude of the force in the coupling between the dogs and the sled.

34. A freight train consists of two $8.00\times10^5\text{-kg}$ engines and 45 cars with average masses of $5.50\times10^5\ \text{kg}$. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00\times10^{-2}\ \text{m/s}^2$ if the force of friction is $7.50\times10^5\ \text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently trains are very energy-efficient transportation systems. (b) What is the magnitude of the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

3.8 Newton's Universal Law of Gravitation

35. (a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is $9.830\ \text{m/s}^2$ and the radius of the Earth is 6371 km at that location. **Hint:** use the expression for acceleration due to gravity, $g = G\frac{M}{r^2}$, to solve for mass.

(b) Compare this with the accepted value of $5.979\times10^{24}\ \text{kg}$.

36. (a) What is the acceleration due to gravity on the surface of the Moon?

(b) On the surface of Mars? The mass of Mars is $6.418\times10^{23}\ \text{kg}$ and its radius is $3.38\times10^6\ \text{m}$.

37. (a) Calculate the acceleration due to gravity on the surface of the Sun.

(b) By what factor would your weight increase if you could stand on the Sun? (Never mind that you cannot.)

38. Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational.

(a) Calculate the magnitude of the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.100 m away at birth (he is assisting, so he is close to the child).

(b) Calculate the magnitude of the force on the baby due to Jupiter if it is at its closest distance to Earth, some $6.29\times10^{11}\ \text{m}$ away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces.

39. Unreasonable Result

A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00% of his weight.

(a) Calculate the mass of the mountain.

(b) Compare the mountain's mass with that of Earth.

(c) What is unreasonable about these results?

(d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)

3.9 Centripetal Force

40. (a) A 22.0 kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force must she exert to stay on if she is 1.25 m from its center?

(b) What centripetal force does she need to stay on an amusement park merry-go-round that rotates at 3.00 rev/min if she is 8.00 m from its center?

(c) Compare each force with her weight.

41. Calculate the centripetal force on the end of a 100 m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

42. Modern roller coasters have vertical loops like the one shown in **Figure 3.40**. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g?

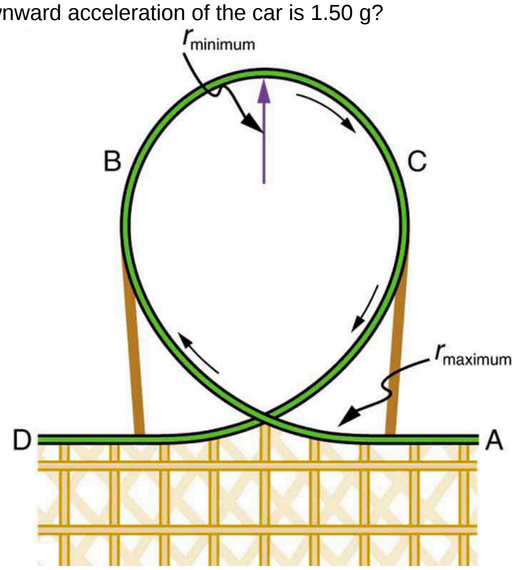


Figure 3.40 Teardrop-shaped loops are used in the latest roller coasters so that the radius of curvature gradually decreases to a minimum at the top. This means that the centripetal acceleration builds from zero to a maximum at the top and gradually decreases again. A circular loop would cause a jolting change in acceleration at entry, a disadvantage discovered long ago in railroad curve design. With a small radius of curvature at the top, the centripetal acceleration can more easily be kept greater than g so that the passengers do not lose contact with their seats nor do they need seat belts to keep them in place.

43. Unreasonable Results

(a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s.

(b) What is unreasonable about the result?

(c) Which premises are unreasonable or inconsistent?

II UNIT 2: MECHANICS II - ENERGY AND MOMENTUM, OSCILLATIONS AND WAVES, ROTATION, AND FLUIDS

4 WORK AND ENERGY

Figure 4.1 How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Chapter Outline

4.1. Work: The Scientific Definition
- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

4.2. Kinetic Energy and the Work-Energy Theorem
- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

4.3. Gravitational Potential Energy
- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\text{PE}_\text{g} = mgh$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

4.4. Conservative Forces, Potential Energy, and Conservation of Energy
- Define conservative force, potential energy, and mechanical energy.
- Apply conservation of mechanical energy to simple physical situations.
- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.

4.5. Spring Potential Energy
- Explain the work done in deforming a spring.
- Describe the potential energy stored in a deformed spring.

4.6. Power
- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

Introduction to Work and Energy

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = mc^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

4.1 Work: The Scientific Definition

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be motion or displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = F_{||}d \tag{4.1}$$

Where W is work, d is the distance the force acts, and $F_{||}$ is the force parallel to the direction of motion. We can also write this more simply as

$$W = Fd \tag{4.2}$$

as long as one keeps in mind that the force is in the same direction as the distance.

To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we would divide the motion into one-way one-dimensional segments and add up the work done over each segment.

What is Work?

The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as

$$W = Fd \tag{4.3}$$

Where W is work, d is the distance the force acts, F is the force parallel to the direction of motion.

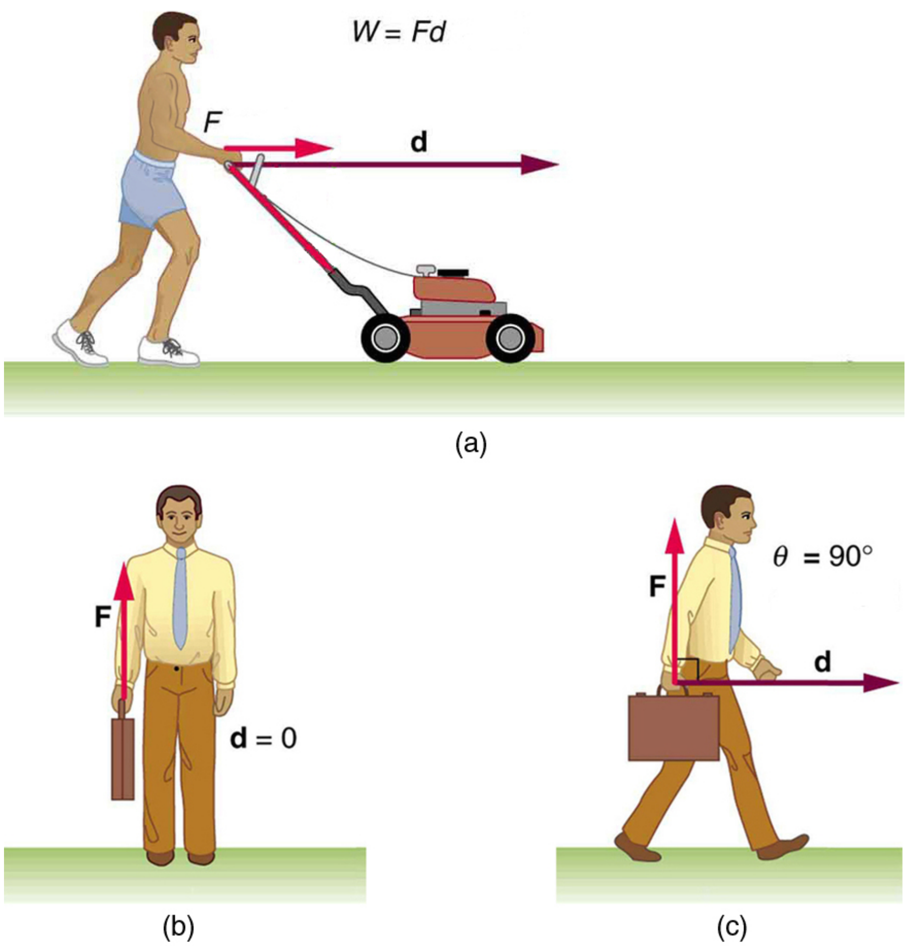


Figure 4.2 Examples of work. (a) Note that the force is in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. The force acting on the briefcase (against gravity) is perpendicular to the direction of motion.

To examine what the definition of work means, let us consider the other situations shown in **Figure 4.2**. The person holding the briefcase in **Figure 4.2**(b) does no work, for example. Here $d=0$, so $W=0$. Why is it you get tired just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the "briefcase-Earth system"). There must be motion for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in **Figure 4.2**(c) does no work on it, because the force is perpendicular to the motion.

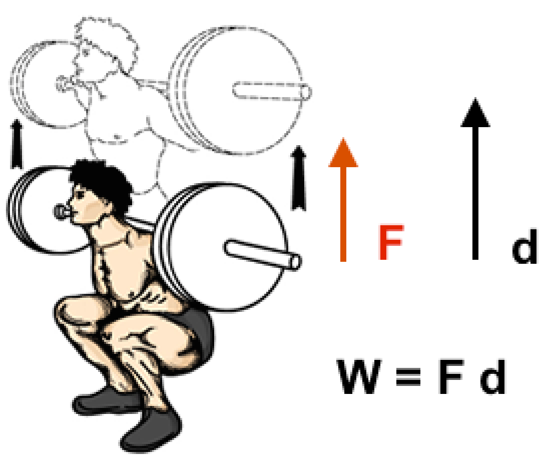


Figure 4.3 Work done when lifting a mass. Work is positive on the way up because force and displacement point in the same direction. On the way down work is negative due to force and displacement pointing in opposite directions (force up and displacement down).

Work can be positive or negative. In **Figure 4.3** work done lifting the mass is positive because both force and displacement are in the same direction. Likewise, when the mass is lowered the work done is negative because the force and displacement are in *opposite* directions. We will soon see that positive work adds energy to the system and negative work removes energy from a system.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example 4.1 Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in **Figure 4.2**(a) if he exerts a constant force of 75.0 N and pushes the mower 25.0 m on level ground? Compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd$. The force and displacement are given, so that only the work W is unknown. Note that force and displacement are in the same direction.

Solution

The equation for the work is

$$W = Fd. \tag{4.4}$$

Substituting the known values gives

$$W = (75.0\text{ N})(25.0\text{ m}) = 1875\text{ J} = 1.88\times10^3\text{ J}. \tag{4.5}$$

The ratio of the work done to the daily consumption is (4.6)

$$\frac{W}{1.00\times10^4\text{ kJ}} = \frac{1.88\times10^3\text{ J}}{1.00\times10^7\text{ J}} = 1.88\times10^{-4}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we "work" all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Example 4.2 Calculating the Work Done Lifting a Mass

The 60.0 kg mass shown in **Figure 4.3** is raised at a constant speed through a vertical distance of 0.800 m. What is the force that must be exerted? How much work is done lifting the mass?

Strategy

Constant speed tells us that the upward force must have the same magnitude as the downward force of gravity $F = mg$ (to have a zero net force). Using that information, along with the given values for mass and displacement, we can determine work using $W = Fd$.

Solution

The force is given by

$$F = mg = (60.0\text{ kg})(9.80\text{ m}/\text{s}^2) = 588\text{ N}.$$

The work done is

$$W = Fd = (588\text{ N})(0.800\text{ m}) = 470\text{ J}.$$

Discussion

Note that the work done lowering the mass back to its starting position would be - 407 J because force and displacement point in opposite directions. The total work done raising and lowering the mass is 407 J - 407 J = 0 J.

4.2 Kinetic Energy and the Work-Energy Theorem

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if a lawn mower is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on a briefcase by a person carrying it up stairs is stored in the briefcase-Earth system and can be recovered at any time. In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

Net Work and the Work-Energy Theorem

We know from the study of Newton's laws that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is, **net work** is the work done by the net external force $\mathbf{F_{net}}$. In equation form, this is $W_{\text{net}} = F_{\text{net}} d$.

Consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 4.4.

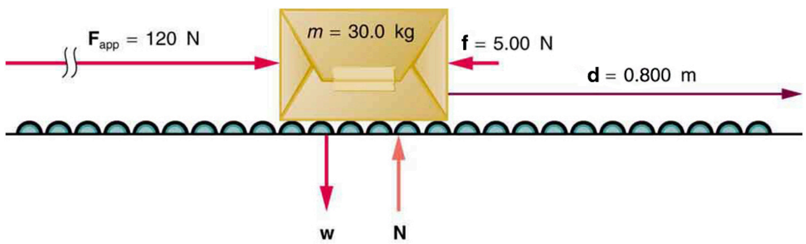


Figure 4.4 A package on a roller belt is pushed horizontally through a distance $\mathbf{d}$.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement and the net work is given by

$$W_{\text{net}} = F_{\text{net}} d. \tag{4.7}$$

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v . By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = ma$ from Newton's second law gives

$$W_{\text{net}} = mad. \tag{4.8}$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in "Motion Equations for Constant Acceleration in One Dimension" for the change in speed over a distance d if the acceleration has the constant value a ; namely, $v^2 = v_0{}^2 + 2ad$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - v_0{}^2}{2d}$. When a is substituted into the preceding expression for W_{net} , we obtain

$$W_{\text{net}} = m\left(\frac{v^2 - v_0{}^2}{2d}\right)d. \tag{4.9}$$

The d cancels, and we rearrange this to obtain

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 . \tag{4.10}$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \tag{4.11}$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. In equation form, the translational kinetic energy,

$$\text{KE} = \frac{1}{2}mv^2, \tag{4.12}$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together. The work-energy theorem can be compactly written as, $W_{\text{net}} = \Delta\text{KE}$ where Δ KE is understood to mean "change in KE."

We are aware that it takes energy to get an object, like a car or the package in **Figure 4.4**, up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 40 m/s has four times the kinetic energy it has at 20 m/s, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example 4.3 Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in **Figure 4.4** is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = \frac{1}{2}mv^2$.

Solution

The kinetic energy is given by

$$\text{KE} = \frac{1}{2}mv^2. \tag{4.13}$$

Entering known values gives

$$\text{KE} = 0.5(30.0\text{ kg})(0.500\text{ m/s})^2, \tag{4.14}$$

which yields

$$\text{KE} = 3.75\text{ kg}\cdot\text{m}^2/\text{s}^2 = 3.75\text{ J}. \tag{4.15}$$

Discussion

Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example 4.4 Determining the Work to Accelerate a Package

Suppose that you push on the 30.0-kg package in **Figure 4.4** with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.

(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.

Strategy and Concept for (a)

This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net force, while the applied force, friction, and the displacement are all horizontal. (See **Figure 4.4**.) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\text{ N} - 5.00\text{ N} = 115\text{ N}$. Thus the net work is

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115\text{ N})(0.800\text{ m}) \\ &= 92.0\text{ N}\cdot\text{m} = 92.0\text{ J}. \end{aligned} \tag{4.16}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

$$W_{\text{app}} = F_{\text{app}}d = (120\,\text{N})(0.800\,\text{m}) = 96.0\,\text{J} \tag{4.17}$$

The friction force and displacement are in opposite directions. The work done by friction is therefore negative.

$$W_{\text{fr}} = -F_{\text{fr}}d = -(5.00\,\text{N})(0.800\,\text{m}) = -4.00\,\text{J} \tag{4.18}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

$$\begin{aligned} W_{\text{gr}} &= 0, \\ W_{\text{N}} &= 0, \\ W_{\text{app}} &= 96.0\text{ J}, \\ W_{\text{fr}} &= -4.00\text{ J}. \end{aligned} \tag{4.19}$$

The total work done as the sum of the work done by each force is then seen to be

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\text{ J}. \tag{4.20}$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example 4.5 Determining Speed from Work and Energy

Find the speed of the package in **Figure 4.4** at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}mv_0^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{4.21}$$

Solving for $\frac{1}{2}mv^2$ gives

$$\frac{1}{2}mv^2 = W_{\text{net}} + \frac{1}{2}mv_0^2. \tag{4.22}$$

Thus,

$$\frac{1}{2}mv^2 = 92.0\text{ J}+3.75\text{ J} = 95.75\text{ J}. \tag{4.23}$$

Solving for the final speed as requested and entering known values gives

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg} \cdot \text{m}^2/\text{s}^2}{30.0\ \text{kg}}} \\ &= 2.53\ \text{m/s}. \end{aligned} \tag{4.24}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example 4.6 Work and Energy Can Reveal Distance, Too

How far does the package in **Figure 4.4** coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package's kinetic energy. The work done by friction can be expressed as the force of friction times the distance traveled **and** as the change in kinetic energy. Equating both expressions for work gives us a way of finding the distance traveled.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the displacement. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be equal to the change in kinetic energy.

$$W_{\text{fr}} = -F_{\text{fr}}d = \Delta\text{KE} \tag{4.25}$$

$$-F_{\text{fr}}d = \Delta\text{KE} \tag{4.26}$$

and so

$$-F_{\text{fr}}d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \tag{4.27}$$

Initial speed is the speed at the instant the push stops (the result determined in the previous example). The final speed is zero for this case and leads to the expression below.

$$F_{\text{fr}}d = \frac{1}{2}mv_0^2 \tag{4.28}$$

Solving for distance, we obtain the result shown below.

$$d = \frac{95.75\,\text{J}}{5.00\,\text{N}} = 19.2\,\text{m} \tag{4.29}$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

4.3 Gravitational Potential Energy

Work Done Against Gravity

Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in **Figure 4.5**. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $W = Fd = mgh$. We define this to be the **gravitational potential energy** (PE_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the PE_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word "system"? Potential energy is a property of a system rather than of a single object—due to its physical position. An object's gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth's surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathrm{PE_g}$ to KE without explicitly considering the intermediate step of work. (See **Example 4.8**.) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.

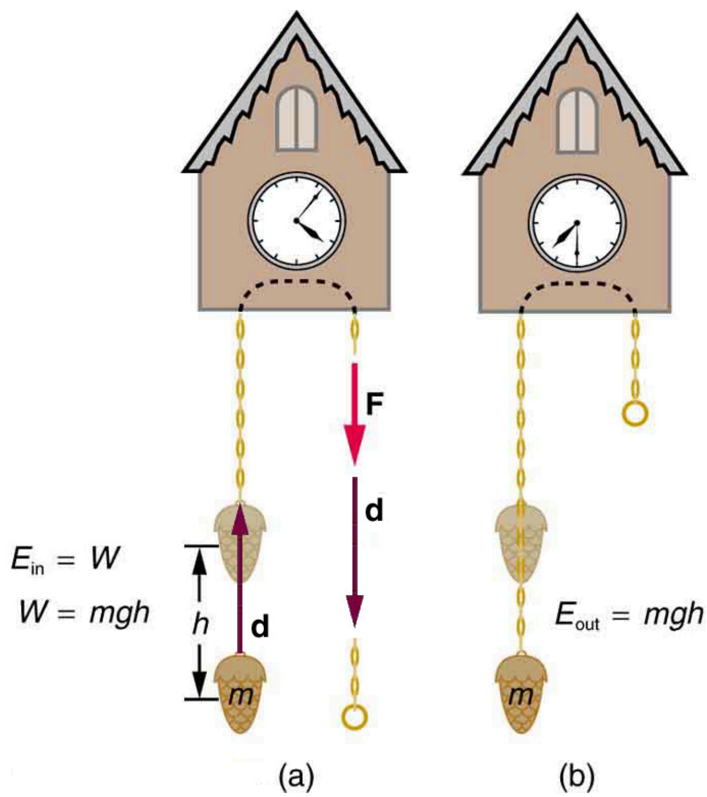


Figure 4.5 (a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy $\Delta\mathrm{PE_g}$ to be

$$\Delta\mathrm{PE_g} = \mathrm{PE_{gf}} - \mathrm{PE_{gi}} = mgh_\mathrm{f} - mgh_\mathrm{i} = mg(h_\mathrm{f} - h_\mathrm{i}) = mg\Delta h \tag{4.30}$$

Note that Δh is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

$$\begin{aligned} mg\Delta h &= (0.500\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(1.00\ \mathrm{m}) \\ &= 4.90\ \mathrm{kg\cdot m^2/s^2} = 4.90\ \mathrm{J.} \end{aligned} \tag{4.31}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta\mathrm{PE_g} = mg\Delta h$ applies for any path that has a change in height of $\Delta \mathrm{h}$, not just when the mass is lifted straight up. (See **Figure 4.6**.) It is much easier to calculate $mg\Delta h$ (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position $\Delta \mathrm{h}$ of a mass m is accompanied by a change in gravitational potential energy $mg\Delta h$, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.

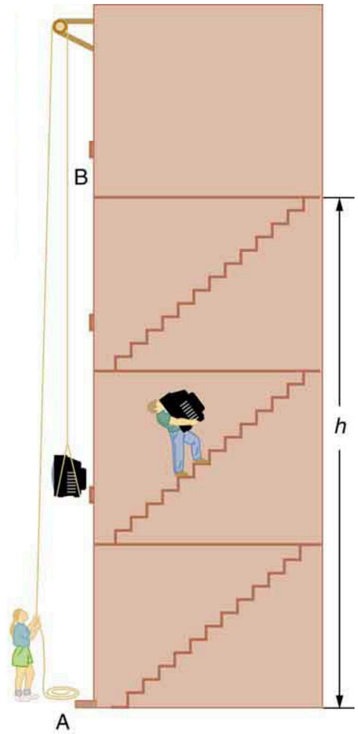


Figure 4.6 The change in gravitational potential energy $(\Delta \mathrm{PE}_\mathrm{g})$ between points A and B is independent of the path. $\Delta \mathrm{PE}_\mathrm{g} = mg\Delta h$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example 4.7 The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

$$W = -Fd, \tag{4.32}$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height $\Delta \mathrm{h}$:

$$\mathrm{KE} = -\Delta \mathrm{PE}_\mathrm{g} = -mg\Delta h, \tag{4.33}$$

The distance d that the person's knees bend is much smaller than the height $\Delta \mathrm{h}$ of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:

$$W = -\mathrm{KE} = mg\Delta h. \tag{4.34}$$

Combining this equation with the expression for W gives

$$-Fd = mg\Delta h. \tag{4.35}$$

Recalling that Δh is negative because the person fell *down*, the force on the knee joints is given by

$$F = -\frac{mg\Delta h}{d} = -\frac{(60.0\text{ kg})(9.80\text{ m/s}^2)(-3.00\text{ m})}{5.00\times10^{-3}\text{ m}} = 3.53\times10^{5}\text{ N.} \tag{4.36}$$

Discussion

Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See **Figure 4.7**.)

Figure 4.7 The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example 4.8 Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in **Figure 4.8** if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?

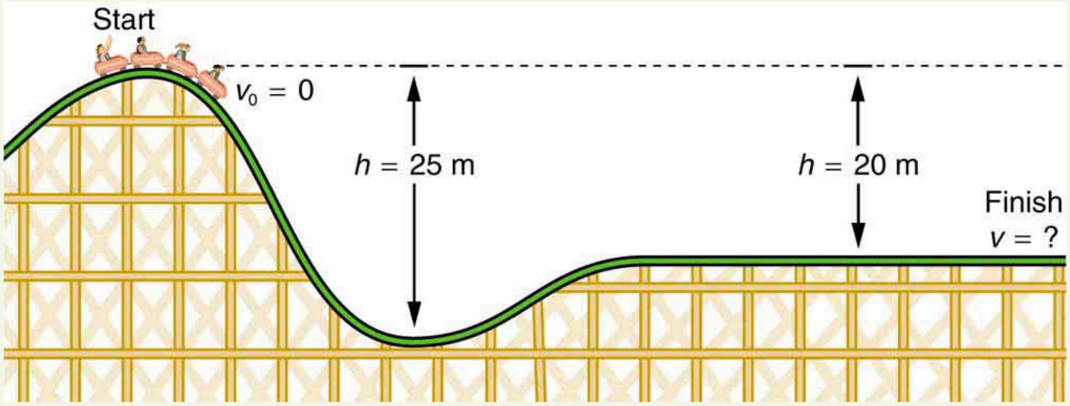


Figure 4.8 The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all ΔPE_g is converted to KE.

Strategy

The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. Using the equations for PE_g and KE, we can solve for the final speed v, which is the desired quantity.

Solution for (a)

$$-\Delta\text{PE}_\text{g} = \Delta\text{KE} \tag{4.37}$$

becomes

$$-(\text{PE}_\text{gf} - \text{PE}_\text{gi}) = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \tag{4.38}$$

Noting that the initial speed is zero, and using the formula for gravitational potential energy, we have

$$-(mgh_\text{f} - mgh_\text{i}) = \frac{1}{2}mv_\text{f}^2 \tag{4.39}$$

Taking the initial height as zero gives

$$-mgh_\text{f} = \frac{1}{2}mv_\text{f}^2 \tag{4.40}$$

Solving for the final speed, we find that mass cancels and that

$$v_\text{f} = \sqrt{-2g\,h_\text{f}} \tag{4.41}$$

Substituting known values,

$$v_\text{f} = \sqrt{-2g\,h_\text{f}} = \sqrt{(-2)(9.80\text{ m/s}^2)(-20.0\text{ m})} = 19.8\text{ m/s} \tag{4.42}$$

Please note that the height used above was negative. This is because we chose the initial height to be our reference level for gravitational potential energy. Our final height is 20 meters *below* that reference level.

Solution for (b)

Again $-\Delta\text{PE}_\text{g} = \Delta\text{KE}$. Thus,

$$-(\text{PE}_\text{gf} - \text{PE}_\text{gi}) = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \tag{4.43}$$

Taking the initial height as zero leads to

$$-mgh_\text{f} = \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \tag{4.44}$$

Rearranging gives

$$\frac{1}{2}mv_\text{f}^2 = -mgh_\text{f} + \frac{1}{2}mv_\text{i}^2 \tag{4.45}$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

$$v_\text{f} = \sqrt{-2gh_\text{f} + v_\text{i}^2} \tag{4.46}$$

Now, substituting known values gives

$$v_\text{f} = \sqrt{-2gh_\text{f} + v_\text{i}^2} = \sqrt{(-2)(9.80\text{ m/s}^2)(-20.0\text{ m}) + (5.00\text{ m/s})^2} = 20.4\text{ m/s}. \tag{4.47}$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

4.4 Conservative Forces, Potential Energy, and Conservation of Energy

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Potential Energy and Conservative Forces

Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.

A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.

We can define a potential energy (PE) for any conservative force. The work done against a conservative force to reach a final configuration depends on the configuration, not the path followed, and is the potential energy added.

Conservation of Mechanical Energy

Consider an object in a system. If it gains any kinetic energy, this is a result of a net work done on the object, according to the work-kinetic energy theorem. If only conservative forces, such as the gravitational force or a spring force, do work in this system, then as kinetic energy increases with the net work done by the conservative forces, the system loses potential energy. That is, $\Delta\mathrm{KE} = W_c = -\Delta\mathrm{PE}$. In other words,

$$\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0. \tag{4.48}$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,

$$\left.\begin{array}{c}\mathrm{KE} + \mathrm{PE} = \text{constant} \\ \text{or} \\ \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}\end{array}\right\} \text{(conservative forces only)}, \tag{4.49}$$

where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to be its **mechanical energy**, $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Example 4.9 Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in **Figure 4.9**. The car follows a track that rises 0.180 m above the starting point. The compressed spring has a potential energy of 0.200 J. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.

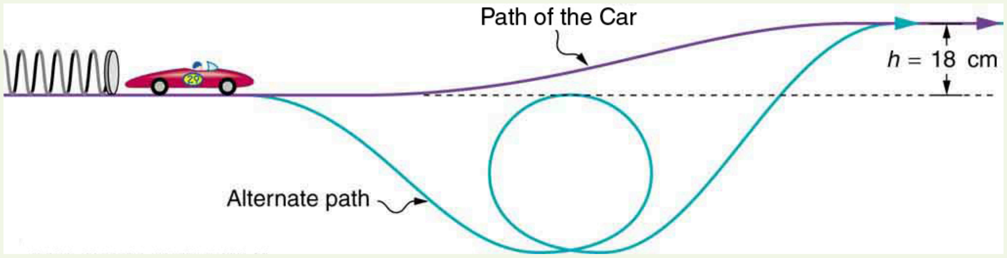


Figure 4.9 A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \tag{4.50}$$

or

$$\frac{1}{2}mv_\mathrm{i}^2 + mgh_\mathrm{i} + \mathrm{PE_{si}} = \frac{1}{2}mv_\mathrm{f}^2 + mgh_\mathrm{f} + \mathrm{PE_{sf}} \tag{4.51}$$

where h is the height (vertical position) and $\mathrm{PE_s}$ is the potential energy of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_i and h_f are zero. Furthermore, the initial speed v_i is zero and the final compression of the spring is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

$$\mathrm{PE_{si}} = \frac{1}{2}mv_\mathrm{f}^2. \tag{4.52}$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

$$v_\mathrm{f} = \sqrt{\frac{2\,\mathrm{PE_{si}}}{m}} = \sqrt{\frac{(2)(0.200\,\mathrm{J})}{0.100\,\mathrm{kg}}} = 2.00\,\mathrm{m/s}. \tag{4.53}$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

$$\mathrm{PE_{si}} = \frac{1}{2}mv_\mathrm{f}^2 + mgh_\mathrm{f}. \tag{4.54}$$

This form of the equation means that the spring's initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_f and substituting known values gives

$$v_\mathrm{f} = \sqrt{\frac{(2)(\mathrm{PE_{si}} - mgh_\mathrm{f})}{m}} = \sqrt{\frac{0.0472\,\mathrm{J}}{0.100\,\mathrm{kg}}} = 0.687\,\mathrm{m/s} \tag{4.55}$$

Discussion

Another way to solve this problem is to realize that the car's kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in **Example 4.9**. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

Conservation of Total Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another, in the case of work done by conservative forces transforming between potential energy and kinetic energy. Together, these make up the mechanical energy $(\mathrm{KE} + \mathrm{PE})$, and the mechanical energy is not always conserved, because it can be transformed into other forms of energy through work done by non-conservative forces. Here, we list some of the many forms energy can take.

Some of the Many Forms of Energy

Here are some of the many forms of energy. You probably have heard of some of these before; many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Problem-Solving Strategies for Energy Problems

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. Use $\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$ or $\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0$ as starting point and use formulas for kinetic energy and potential energies.

Step 4. If you know that the total mechanical energy changes (usually there will be some hint of work done by non-conservative forces, energy input from other forms of energy, or even an explicit statement of how much work is done on the system, changing its total mechanical energy), then you can use the information given in the problem to say that, the change of total mechanical energy is due to these non-conservative forces: $\Delta\mathrm{KE} + \Delta\mathrm{PE} = W_{\mathrm{nc}}$. Make sure W_{nc} does not include work done by conservative forces, since those are already accounted for in the potential energy.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that $\mathrm{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see **Figure 4.10**) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Figure 4.10 Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Problems & Exercises

Exercise 4.1

Using energy considerations and assuming negligible air resistance, show that a rock thrown from a bridge 20.0 m above water with an initial speed of 15.0 m/s strikes the water with a speed of 24.8 m/s independent of the direction thrown.

Solution

Equating $\Delta \mathrm{PE_g}$ and $\Delta \mathrm{KE}$, we obtain $v = \sqrt{2gh + {v_0}^2} = \sqrt{2(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m}) + (15.0\ \mathrm{m/s})^2} = 24.8\ \mathrm{m/s}$

4.5 Spring Potential Energy

Hooke's Law, $F = -kx$, describes force exerted by a spring being deformed. Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_{el}} = \frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke’s law. Hence,

$$\mathrm{PE_{el}} = \frac{1}{2}kx^2, \tag{4.56}$$

where $\mathrm{PE_{el}}$ is the **elastic potential energy** stored in any deformed system that obeys Hooke’s law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\mathrm{app}} = kx$. **Figure 4.11** shows a graph of the applied force versus deformation x for a system that can be described by Hooke’s law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the average force is $(1/2)kx$, the distance moved is x, and thus $W = F_{\mathrm{app}}d = [(1/2)kx](x) = (1/2)kx^2$ (Method B in the figure).

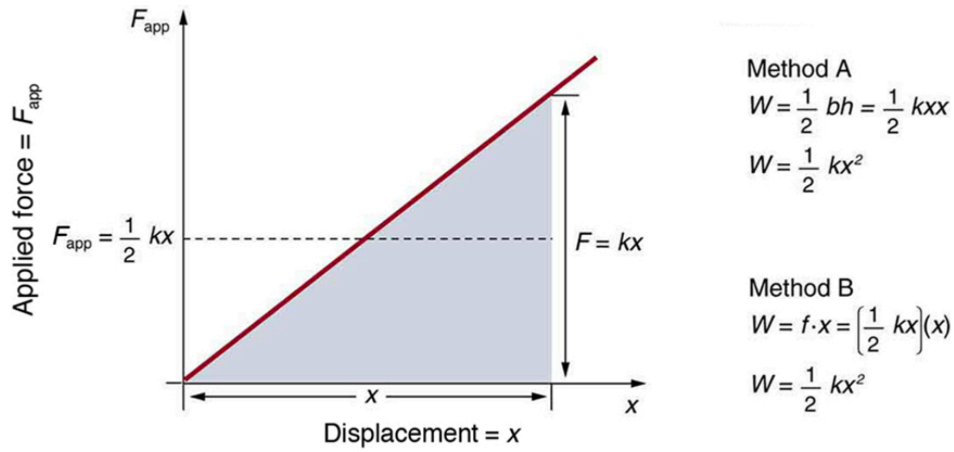


Figure 4.11 A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)kx^2$.

Example 4.10 Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?

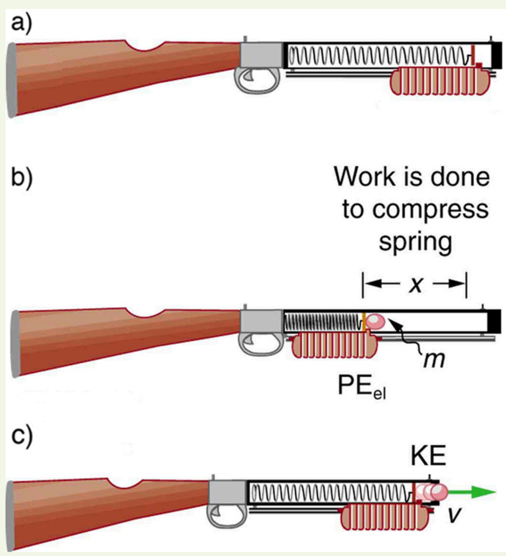


Figure 4.12 (a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

$$\begin{aligned} PE_{el} &= \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \text{N/m})(0.150\ \text{m})^2 = 0.563\ \text{N}\cdot\text{m} \\ &= 0.563\ \text{J} \end{aligned} \tag{4.57}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

$$\mathrm{KE_f} = \mathrm{PE_{el}} \text{ or } 1/2mv^2 = (1/2)kx^2 = \mathrm{PE_{el}} = 0.563\text{ J} \tag{4.58}$$

2. Solve for v:

$$v = \left[\frac{2\mathrm{PE_{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563\text{ J})}{0.002\text{ kg}}\right]^{1/2} = 23.7(\text{J/kg})^{1/2} \tag{4.59}$$

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Check your Understanding

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Check your Understanding

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution

It was stored in the object as potential energy.

4.6 Power

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in **Figure 4.13**.

Figure 4.13 This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Power

Power is the rate at which work is done.

$$P = \frac{W}{t} \tag{4.60}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1 \text{ W} = 1 \text{ J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example 4.11 Calculating the Power to Climb Stairs

What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See **Figure 4.14**.)

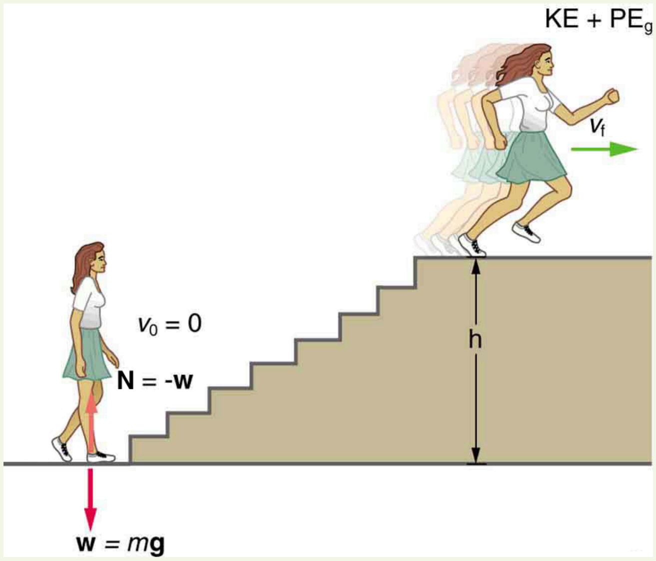


Figure 4.14 When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept

The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus, $W = \text{KE}_\text{f} + \text{PE}_\text{g} = \frac{1}{2}mv_\text{f}^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.

Solution

Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_\text{f}^2 + mgh}{t}. \tag{4.61}$$

Entering known values yields

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2 + (60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J} + 1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned} \tag{4.62}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1 \text{ hp} = 746 \text{ W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See **Table 4.1** for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter (kW/m^2). A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is $10^6\ \text{W}$ of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See **Figure 4.15**.)

Figure 4.15 Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings. The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Table 4.1 Power Output or Consumption

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

$$E = Pt. \tag{4.63}$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\text{kW} \cdot \text{h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example 4.12 Calculating Energy Costs

What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\text{kW} \cdot \text{h}$?

Strategy

Cost is based on energy consumed; thus, we must find E from $E = Pt$ and then calculate the cost. Because electrical energy is expressed in $\text{kW} \cdot \text{h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.

Solution

The energy consumed in $\text{kW} \cdot \text{h}$ is

$$\begin{aligned} E &= Pt = (0.200\ \text{kW})(6.00\ \text{h/d})(30.0\ \text{d}) \\ &= 36.0\ \text{kW} \cdot \text{h}, \end{aligned} \tag{4.64}$$

and the cost is simply given by

$$\text{cost} = (36.0\ \text{kW} \cdot \text{h})(\$0.120\ \text{per kW} \cdot \text{h}) = \$4.32\ \text{per month.} \tag{4.65}$$

Discussion

The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail later, the potential for energy to produce useful work has been "degraded" in the energy transformation.

Glossary

chemical energy: the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

conservation of mechanical energy: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

conservative force: a force that does the same work for any given initial and final configuration, regardless of the path followed

deformation: displacement from equilibrium

elastic potential energy: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

electrical energy: the energy carried by a flow of charge

energy: the ability to do work

gravitational potential energy: the energy an object has due to its position in a gravitational field

horsepower: an older non-SI unit of power, with $1\ \text{hp} = 746\ \text{W}$

joule: SI unit of work and energy, equal to one newton-meter

kilowatt-hour: $(\text{kW} \cdot \text{h})$ unit used primarily for electrical energy provided by electric utility companies

kinetic energy: the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

law of conservation of energy: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

mechanical energy: the sum of kinetic energy and potential energy

net work: work done by the net force, or vector sum of all the forces, acting on an object

nuclear energy: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

potential energy: energy due to position, shape, or configuration

power: the rate at which work is done

radiant energy: the energy carried by electromagnetic waves

thermal energy: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

watt: (W) SI unit of power, with $1\text{ W} = 1\text{ J/s}$

work: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

work-energy theorem: the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

Section Summary

4.1 Work: The Scientific Definition

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force parallel to the motion, times the magnitude d of the displacement. In symbols, $W = Fd$.
- The SI unit for work and energy is the joule (J), where $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

4.2 Kinetic Energy and the Work-Energy Theorem

- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy,
  $$W_{\text{net}} = \Delta\text{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

4.3 Gravitational Potential Energy

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, ΔPE_g, is $\Delta\text{PE}_\text{g} = mg\Delta h$, with Δh being the change in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, ΔPE_g, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that $\Delta\text{KE} = -\Delta\text{PE}_\text{g}$.

4.4 Conservative Forces, Potential Energy, and Conservation of Energy

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_g for the gravitational force.
- Mechanical energy is defined to be $\text{KE} + \text{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is conserved, $\Delta\text{KE} + \Delta\text{PE} = 0$.
- The law of conservation of energy states that the total energy, including the mechanical energy and other forms of energy, is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.

4.5 Spring Potential Energy

- Hooke's law describes force exerted by a spring being deformed,
  $$F = -kx,$$
  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the

system.
- Elastic potential energy $\mathrm{PE_{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by

$$\mathrm{PE_{el}} = (1/2)kx^2.$$

4.6 Power

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\ \mathrm{W} = 1\ \mathrm{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\ \mathrm{hp} = 746\ \mathrm{W}$.

Conceptual Questions

4.1 Work: The Scientific Definition

1. Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

2. Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

3. Describe a situation in which a force is exerted for a long time but does no work. Explain.

4.2 Kinetic Energy and the Work-Energy Theorem

4. The person in **Figure 4.16** does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?

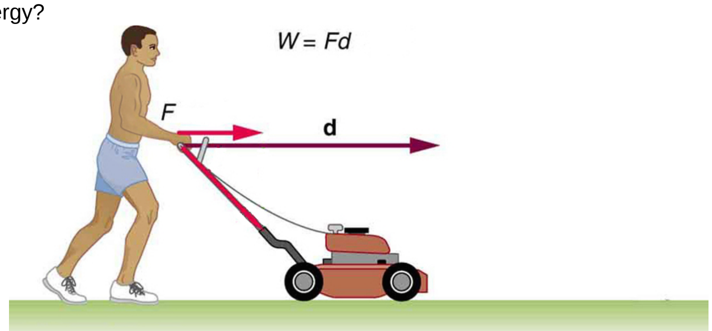


Figure 4.16

5. Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

6. When solving for speed in **Example 4.5**, we kept only the positive root. Why?

4.3 Gravitational Potential Energy

7. In **Example 4.8**, we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

8. Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

4.4 Conservative Forces, Potential Energy, and Conservation of Energy

9. What is a conservative force?

10. The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

11. Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

12. What is the relationship of potential energy to conservative force?

13. Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See **Figure 4.17**.)

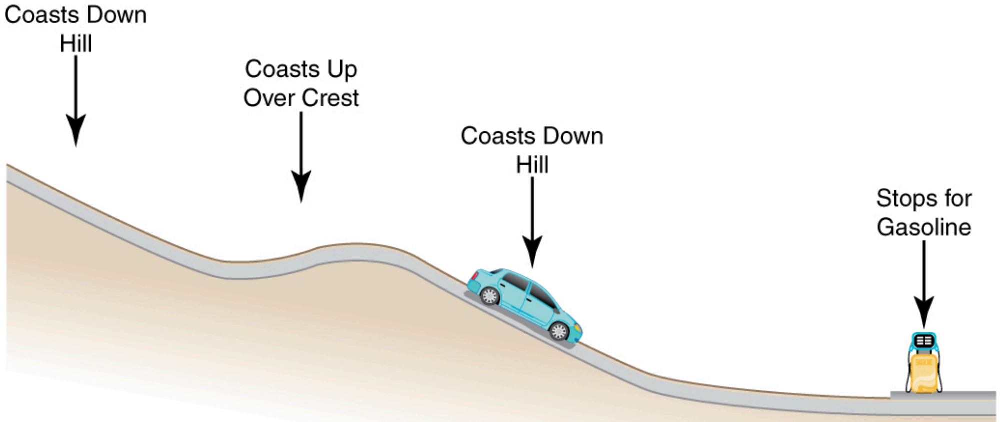


Figure 4.17 A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

14. Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

15. List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

16. List the energy conversions that occur when riding a bicycle.

4.5 Spring Potential Energy

17. Describe a system in which elastic potential energy is stored.

4.6 Power

18. Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

19. Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

20. A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Problems & Exercises

4.1 Work: The Scientific Definition

1. How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules.

2. A 75.0-kg person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.

3. (a) Calculate the work done on a 1500-kg elevator car by its cable to lift it 40.0 m at constant speed, assuming friction averages 100 N. (b) What is the work done on the lift by the gravitational force in this process? (c) What is the total work done on the lift?

4. Calculate the work done on a crate pushed 4.00 m up along a ramp (see **Figure 4.18**). The man exerts a force of 500 N on the crate parallel to the ramp and moves at a constant speed.

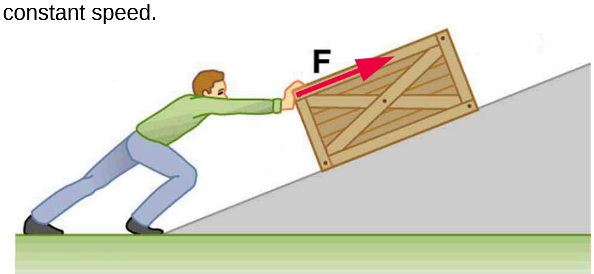


Figure 4.18 A man pushes a crate up a ramp.

5. A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts parallel to the direction of motion. (e) What is the total work done on the cart?

4.2 Kinetic Energy and the Work-Energy Theorem

6. Compare the kinetic energy of a 20,000-kg truck moving at 30.0 m/s with that of an 80.0-kg astronaut in orbit moving at 7,500 m/s.

7. (a) How fast must a 3000-kg elephant move to have the same kinetic energy as a 65.0-kg sprinter running at 10.0 m/s? (b) Discuss how the larger energies needed for the movement of larger animals would relate to metabolic rates.

8. (a) Calculate the force needed to bring a 950-kg car to rest from a speed of 25.0 m/s in a distance of 120 m (a fairly typical distance for a non-panic stop). (b) Suppose instead the car hits a concrete abutment at full speed and is brought to a stop in 2.00 m. Calculate the force exerted on the car and compare it with the force found in part (a).

9. A car's bumper is designed to withstand a 1.1 m/s collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.

10. Boxing gloves are padded to lessen the force of a blow. (a) Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 cm during a blow in which the 7.00-kg arm and glove are brought to rest from an initial speed of 10.0 m/s. (b) Calculate the force exerted by an identical blow in the gory old days when no gloves were used and the knuckles and face would compress only 2.00 cm. (c) Discuss the magnitude of the force with glove on. Does it seem high enough to cause damage even though it is lower than the force with no glove?

11. Using energy considerations, calculate the average force a 60.0-kg sprinter exerts backward on the track to accelerate from 2.00 to 8.00 m/s in a distance of 25.0 m, if he encounters a headwind that exerts an average force of 30.0 N against him.

4.3 Gravitational Potential Energy

12. A hydroelectric power facility (see **Figure 4.19**) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume $50.0\ \text{km}^3$ ($\text{mass} = 5.00{\times}10^{13}\ \text{kg})$, given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy released by a 9-megaton fusion bomb $(3.77{\times}10^{16}\ \text{J})$.

Figure 4.19 Hydroelectric facility (credit: Denis Belevich, Wikimedia Commons)

13. (a) How much gravitational potential energy (relative to the ground on which it is built) is stored in the Great Pyramid of Cheops, given that its mass is about $7 \times 10^9\ \text{kg}$ and its center of mass is 36.5 m above the surrounding ground? (b) How does this energy compare with the daily food intake of a person (10,000 kJ which is about 2400 kcal)?

14. Suppose a 350-g kookaburra (a large kingfisher bird) picks up a 75-g snake and raises it 2.5 m from the ground to a branch. (a) How much work did the bird do on the snake? (b) How much work did it do to raise its own center of mass to the branch?

15. In **Example 4.8**, we found that the speed of a roller coaster that had descended 20.0 m was only slightly greater when it had an initial speed of 5.00 m/s than when it started from rest. This implies that $\Delta\text{PE} >> \text{KE}_\text{i}$. Confirm this statement by taking the ratio of ΔPE to KE_i . (Note that mass cancels.)

16. A 100-g toy car is propelled by a compressed spring that starts it moving. The car follows the curved track in **Figure 4.20**. Show that the final speed of the toy car is 0.687 m/s if its initial speed is 2.00 m/s and it coasts up the frictionless slope, gaining 0.180 m in altitude.

Figure 4.20 A toy car moves up a sloped track. (credit: Leszek Leszczynski, Flickr)

17. Determine the maximum speed the roller coaster shown in **Figure 4.8** can obtain.

4.6 Power

18. The Crab Nebula (see **Figure 4.21**) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from **Table 4.1**, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.

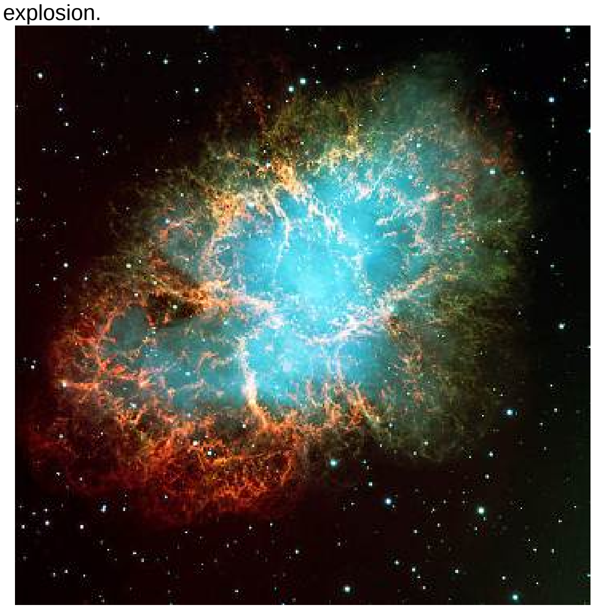


Figure 4.21 Crab Nebula (credit: ESO, via Wikimedia Commons)

19. Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from **Table 4.1**: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

20. A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

21. What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

22. A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $\text{kW} \cdot \text{h}$?

23. (a) What is the average power consumption in watts of an appliance that uses $5.00\ \text{kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

24. (a) What is the average useful power output of a person who does $6.00\times10^{6}\ \text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

25. A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

26. (a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

27. (a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\text{kW} \cdot \text{h}$?

28. (a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00\times10^{4}\ \text{J}$ run a pocket calculator that consumes energy at the rate of $1.00\times10^{-3}\ \text{W}$?

29. (a) How long would it take a 1.50×10^{5} -kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

30. (a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be 4.00×10^{26} W.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of $1.30\ \text{kW/m}^2$ reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05\times10^{20}\ \text{J})$? Australia's energy needs $(5.4\times10^{18}\ \text{J})$? China's energy needs $(6.3\times10^{19}\ \text{J})$? (These energy consumption values are from 2006.)

5 IMPULSE AND MOMENTUM

Figure 5.1 Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

Chapter Outline

5.1. Linear Momentum and Force
- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

5.2. Impulse
- Define impulse.
- Describe effects of impulses in everyday life.
- Determine the average effective force using graphical representation.
- Calculate average force and impulse given mass, velocity, and time.

5.3. Conservation of Momentum
- Describe the principle of conservation of momentum.
- Explain conservation of momentum with examples.

5.4. Elastic Collisions in One Dimension
- Describe an elastic collision of two objects in one dimension.
- Determine the final velocities in an elastic collision given masses and initial velocities.

5.5. Inelastic Collisions in One Dimension
- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to everyday situations.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

Introduction to Linear Momentum and Collisions

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.

5.1 Linear Momentum and Force

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

$$\mathbf{p} = m\mathbf{v}. \tag{5.1}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

$$\mathbf{p} = m\mathbf{v}. \tag{5.2}$$

Example 5.1 Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

$$p = mv \tag{5.3}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

$$p_{\text{player}} = (110\text{ kg})(8.00\text{ m/s}) = 880\text{ kg} \cdot \text{m/s} \tag{5.4}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

$$p_{\text{ball}} = (0.410\text{ kg})(25.0\text{ m/s}) = 10.3\text{ kg} \cdot \text{m/s} \tag{5.5}$$

The ratio of the player's momentum to that of the ball is

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9. \tag{5.6}$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}, \tag{5.7}$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change in time.

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes.

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} \tag{5.8}$$

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}} = m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta\mathbf{p}$ is given by

$$\Delta\mathbf{p} = \Delta(m\mathbf{v}). \tag{5.9}$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta\mathbf{v}. \tag{5.10}$$

So that for constant mass, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{m\Delta\mathbf{v}}{\Delta t}. \tag{5.11}$$

Because $\frac{\Delta\mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \tag{5.12}$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example 5.2 Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}. \tag{5.13}$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta p = m\Delta v = m(v_{\text{f}} - v_{\text{i}}). \tag{5.14}$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

$$\begin{aligned} \Delta p &= m(v_{\text{f}} - v_{\text{i}}) \\ &= (0.057\text{ kg})(58\text{ m/s} - 0\text{ m/s}) \\ &= 3.306\text{ kg}\cdot\text{m/s} \approx 3.3\text{ kg}\cdot\text{m/s} \end{aligned} \tag{5.15}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306\text{ kg}\cdot\text{m/s}}{5.0\times10^{-3}\text{ s}} \\ &= 661\text{ N} \approx 660\text{ N}, \end{aligned} \tag{5.16}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = ma$, but one additional step would be required compared with the strategy used in this example.

5.2 Impulse

The effect of a force on an object depends on how long it acts, as well as how great the force is. In previous examples, a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta\mathbf{p}$.

By rearranging the equation $\mathbf{F}_{\text{net}} = \dfrac{\Delta\mathbf{p}}{\Delta t}$ to be

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t, \tag{5.17}$$

we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name **impulse**. Impulse is the same as the change in momentum.

Impulse: Change in Momentum

Change in momentum equals the average net external force multiplied by the time this force acts.

$$\Delta\mathbf{p} = \mathbf{F}_{\text{net}}\Delta t \tag{5.18}$$

The quantity $\mathbf{F}_{\text{net}}\Delta t$ is given the name impulse.

There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt . *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. **Figure 5.2** shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2 . That area is equal to the area inside the rectangle bounded by F_{eff} , t_1 , and t_2 . Thus the impulses and their effects are the same for both the actual and effective forces.

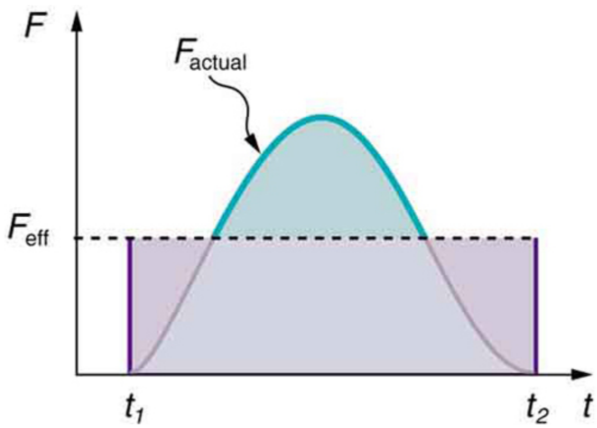


Figure 5.2 A graph of force versus time with time along the x -axis and force along the y -axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

Making Connections: Take-Home Investigation—Hand Movement and Impulse

Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Making Connections: Constant Force and Constant Acceleration

The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

5.3 Conservation of Momentum

In the previous sections in this chapter, changes in momentum for an object were considered in terms of impulse and force. But for certain conditions, the total momentum is conserved (stays constant). What are these conditions?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar (i.e, both objects are about the same mass) than the masses of a football player and Earth—for example, one car bumping into another, as shown in **Figure 5.3**. Both cars are coasting in the same direction when the lead car (labeled $m_2)$ is bumped by the trailing car (labeled $m_1).$ The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.

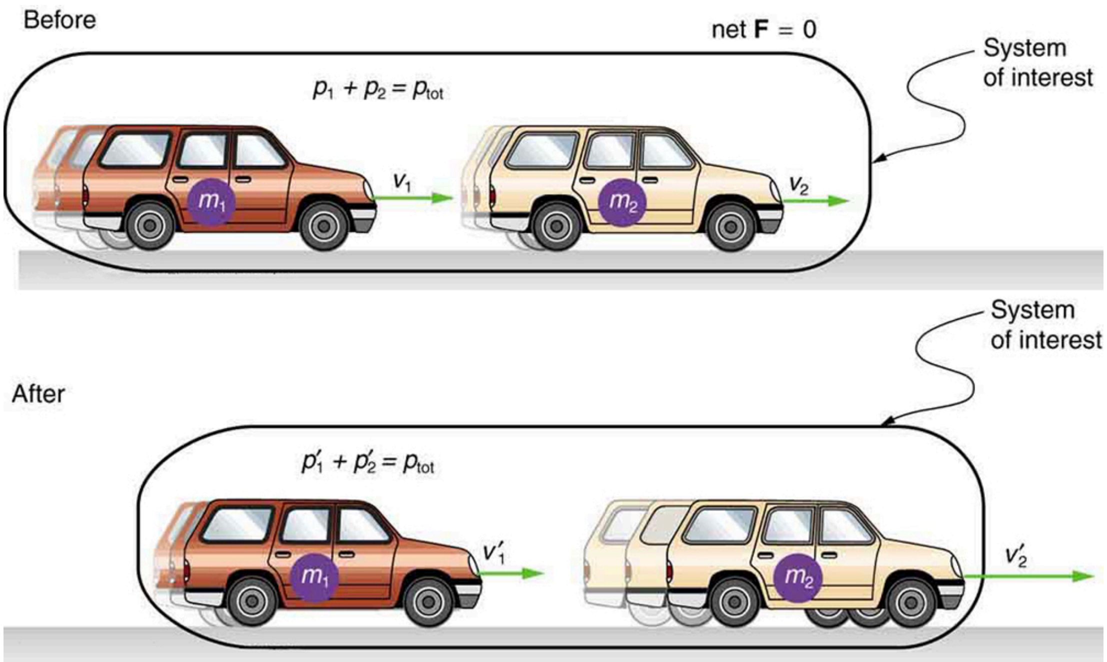


Figure 5.3 A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of v'_1 and the second speeds up to a velocity of v'_2. The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

$$\Delta p_1 = F_1 \Delta t, \tag{5.19}$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Similarly, the change in momentum of car 2 is

$$\Delta p_2 = F_2 \Delta t, \tag{5.20}$$

where F_2 is the force on car 2 due to car 1, and, intuitively, the duration of the collision Δt is the same for both cars. We know from Newton's third law that $F_2 = -F_1$, and so

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1. \tag{5.21}$$

Thus, the changes in momentum cancel out, and

$$\Delta p_1 + \Delta p_2 = 0. \tag{5.22}$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$p_1 + p_2 = \text{constant}. \tag{5.23}$$

Because of Newton's third law, when two objects in a system interact with each other, it does not change the total momentum of the system. In the absence of an external force (that is, force due to an object outside of the system, so that we can ignore the impulse due to the reaction force), the total momentum of the system is constant, or conserved. We call this the **conservation of momentum principle**.

We have noted that the motion in perpendicular directions—x, y, and z—are independent. Momentum conservation follows this paradigm, and components of total momentum along each direction are conserved separately. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See **Figure 5.4**.) However, if the momentum of the projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.

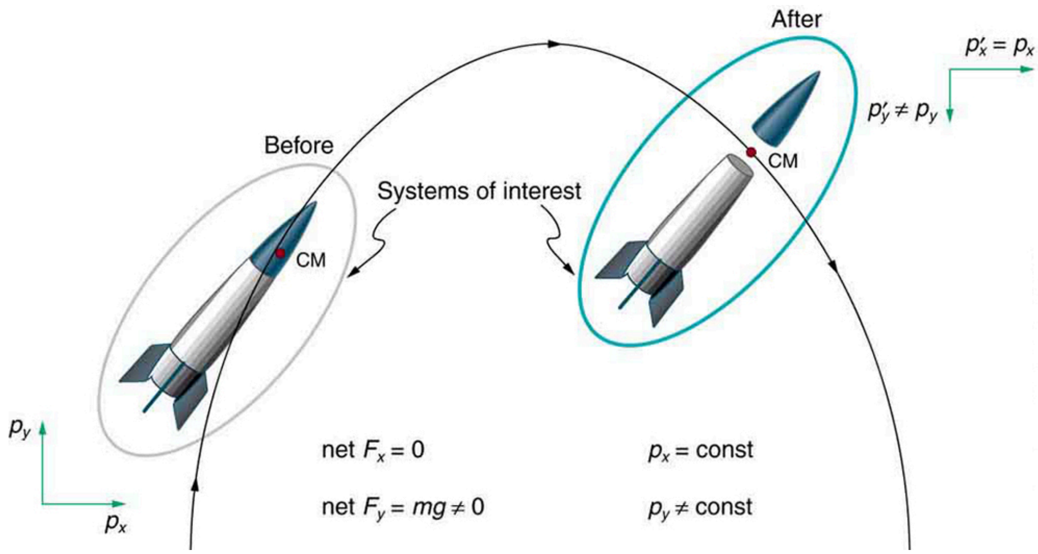


Figure 5.4 The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x-\text{net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y-\text{net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton's third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Making Connections: Conservation of Momentum and Collision

Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

5.4 Elastic Collisions in One Dimension

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves total kinetic energy, in addition to the total momentum. Figure 5.5 illustrates an elastic collision in which kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Elastic Collision

An **elastic collision** is one that conserves total kinetic energy.

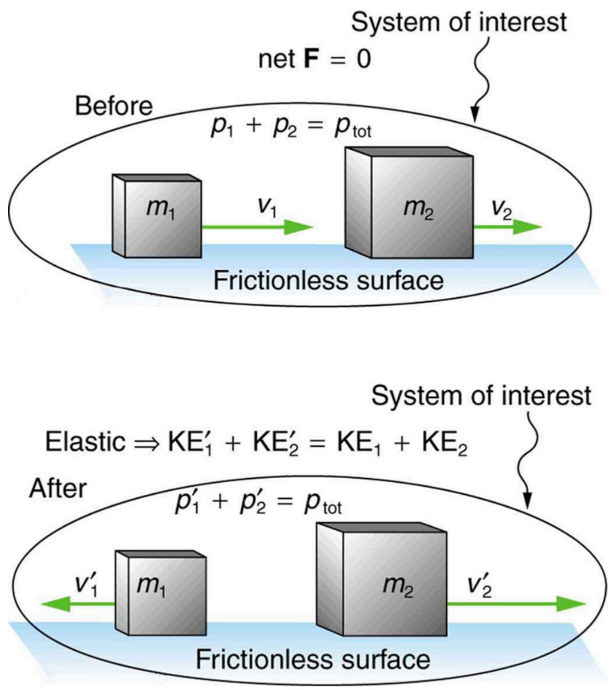


Figure 5.5 An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0) \tag{5.24}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0), \tag{5.25}$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves total kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

$$\frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 = \frac{1}{2}m_1 v'_1{}^2 + \frac{1}{2}m_2 v'_2{}^2 \quad \text{(two-object elastic collision)} \tag{5.26}$$

expresses the equation for conservation of total kinetic energy in a one-dimensional collision.

Example 5.3 Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

$$m_1 = 0.500\text{ kg},\ m_2 = 3.50\text{ kg},\ v_1 = 4.00\text{ m/s, and }\ v_2 = 0. \tag{5.27}$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in **Figure 5.5** where both objects are initially moving. We are asked to find two unknowns (the final velocities v'_1 and v'_2). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2 = 0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2 = 0$ and use conservation of momentum. Thus,

$$p_1 = p'_1 + p'_2 \tag{5.28}$$

or

$$m_1 v_1 = m_1 v'_1 + m_2 v'_2. \tag{5.29}$$

Using conservation of internal kinetic energy and that $v_2 = 0$,

$$\frac{1}{2} m_1 v_1{}^2 = \frac{1}{2} m_1 v'_1{}^2 + \frac{1}{2} m_2 v'_2{}^2. \tag{5.30}$$

Solving the first equation (momentum equation) for v'_2 , we obtain

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1). \tag{5.31}$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable v'_2 , leaving only v'_1 as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are

$$v'_1 = 4.00 \text{ m/s} \tag{5.32}$$

and

$$v'_1 = -3.00 \text{ m/s}. \tag{5.33}$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v'_1 = -3.00 \text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of v'_1 is used to find the velocity of the second object after the collision, we get

$$v'_2 = \frac{m_1}{m_2}(v_1 - v'_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}[4.00 - (-3.00)] \text{ m/s} \tag{5.34}$$

or

$$v'_2 = 1.00 \text{ m/s}. \tag{5.35}$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

5.5 Inelastic Collisions in One Dimension

We have seen that in an elastic collision, total kinetic energy is conserved. An **inelastic collision** is one in which the total kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add total kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some kinetic energy into thermal energy. Or it may convert stored energy into total kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Inelastic Collision

An inelastic collision is one in which the total kinetic energy changes (it is not conserved).

Figure 5.6 shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving total momentum. And the total kinetic energy is zero after this collision. A collision in which the objects stick together is called a **perfectly inelastic collision** because it reduces the total kinetic energy as much as possible, while conserving total momentum.

Perfectly Inelastic Collision

A collision in which the objects stick together is sometimes called “perfectly inelastic.”

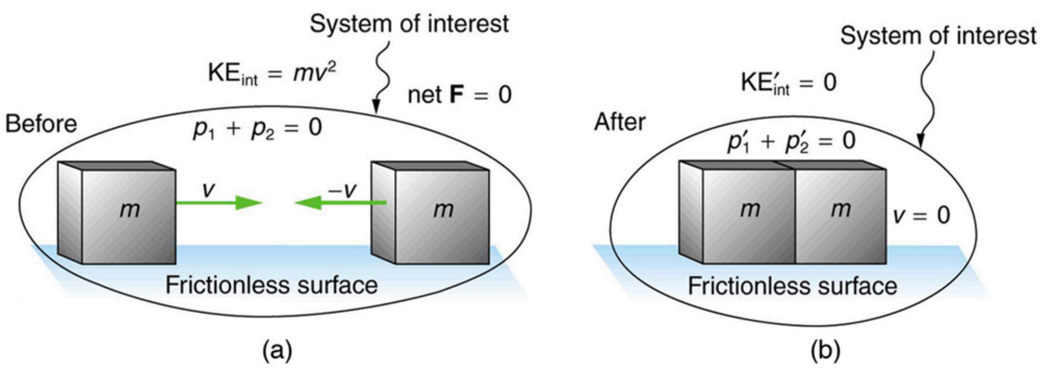


Figure 5.6 An inelastic one-dimensional two-object collision. Momentum is conserved, but total kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero.

Example 5.4 Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie

(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See **Figure 5.7**)

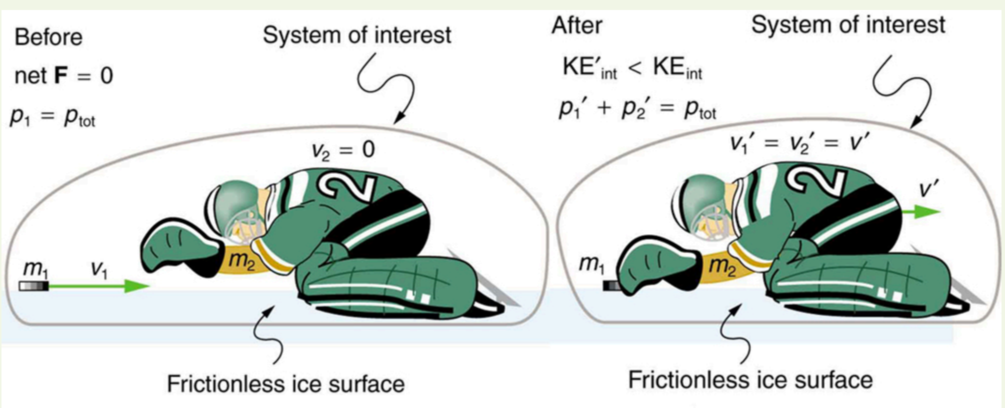


Figure 5.7 An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.

Solution for (a)

Momentum is conserved because the net external force on the puck-goalie system is zero.

Conservation of momentum is

$$p_1 + p_2 = p'_1 + p'_2 \tag{5.36}$$

or

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{5.37}$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v'_1 = v'_2 = v'$. Thus, the conservation of momentum equation simplifies to

$$m_1 v_1 = (m_1 + m_2)v'. \tag{5.38}$$

Solving for v' yields

$$v' = \frac{m_1}{m_1 + m_2} v_1. \tag{5.39}$$

Entering known values in this equation, we get

$$v' = \left(\frac{0.150 \text{ kg}}{0.150 \text{ kg} + 70.0 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48{\times}10^{-2} \text{ m/s}. \tag{5.40}$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the total kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned} \tag{5.41}$$

After the collision, the total kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2}(m + M)v^2 = \frac{1}{2}(70.15 \text{ kg})\left(7.48{\times}10^{-2} \text{ m/s}\right)^2 \\ &= 0.196 \text{ J}. \end{aligned} \tag{5.42}$$

The change in total kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 0.196 \text{ J} - 91.9 \text{ J} \\ &= -91.7 \text{ J} \end{aligned} \tag{5.43}$$

where the minus sign indicates that the energy was lost.

Discussion for (b)

Nearly all of the initial total kinetic energy is lost in this perfectly inelastic collision. KE_{int} is mostly converted to thermal energy and sound. Note that in this case, the total kinetic energy does not decrease all the way down to zero, because doing so would violate conservation of momentum, because initial total momentum is not zero. This is the most amount of energy that can be converted from the initial total kinetic energy without violating conservation of momentum.

Example 5.5 Calculating Final Velocity and Energy Release: Two Carts Collide

In the collision pictured in **???**, two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to total kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in **???**) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the total kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2. \tag{5.44}$$

The only unknown in this equation is v'_2. Solving for v'_2 and substituting known values into the previous equation yields

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350\text{ kg})(2.00\text{ m/s}) + (0.500\text{ kg})(-0.500\text{ m/s})}{0.500\text{ kg}} - \frac{(0.350\text{ kg})(-4.00\text{ m/s})}{0.500\text{ kg}} \\ &= 3.70\text{ m/s}. \end{aligned} \tag{5.45}$$

Solution for (b)

The total kinetic energy before the collision is

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2}(0.350\text{ kg})(2.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(-0.500\text{ m/s})^2 \\ &= 0.763\text{ J}. \end{aligned} \tag{5.46}$$

After the collision, the total kinetic energy is

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \\ &= \frac{1}{2}(0.350\text{ kg})(-4.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(3.70\text{ m/s})^2 \\ &= 6.22\text{ J}. \end{aligned} \tag{5.47}$$

The change in total kinetic energy is thus

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22\text{ J} - 0.763\text{ J} \\ &= 5.46\text{ J}. \end{aligned} \tag{5.48}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The total kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

Glossary

change in momentum: the difference between the final and initial momentum; the mass times the change in velocity

conservation of momentum principle: when the net external force is zero, the total momentum of the system is conserved or constant

elastic collision: a collision that also conserves total kinetic energy

impulse: the average net external force times the time it acts; equal to the change in momentum

inelastic collision: a collision in which total kinetic energy is not conserved

linear momentum: the product of mass and velocity

perfectly inelastic collision: a collision in which the colliding objects stick together

second law of motion: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Section Summary

5.1 Linear Momentum and Force

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be

$$\mathbf{p} = m\mathbf{v},$$

  where m is the mass of the system and $\mathbf{v}$ is its velocity.
- The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

  $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change time.

5.2 Impulse

- Impulse, or change in momentum, equals the average net external force multiplied by the time this force acts:

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

- Forces are usually not constant over a period of time.

5.3 Conservation of Momentum

- The conservation of momentum principle says that the total momentum is conserved, or constant, in the absence of a net external force, as a direct consequence of Newton's third law.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.

5.4 Elastic Collisions in One Dimension

- An elastic collision is one that conserves total kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

5.5 Inelastic Collisions in One Dimension

- An inelastic collision is one in which the total kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces total kinetic energy as much as possible while conserving total momentum.

Conceptual Questions

5.1 Linear Momentum and Force

1. An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

2. An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

3. Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

4. How can a small force impart the same momentum to an object as a large force?

5.2 Impulse

5. Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

6. While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

7. Professional Application

Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

5.3 Conservation of Momentum

8. Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

9. Under what circumstances is momentum conserved?

10. Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

11. Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

12. Professional Application

Explain in terms of momentum and Newton’s laws how a car’s air resistance is due in part to the fact that it pushes air in its direction of motion.

13. Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

14. Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

5.4 Elastic Collisions in One Dimension

15. What is an elastic collision?

5.5 Inelastic Collisions in One Dimension

16. What is an inelastic collision? What is a perfectly inelastic collision?

17. Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

18. A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

Problems & Exercises

5.1 Linear Momentum and Force

1. (a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s . (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of 600 m/s . (c) What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

2. (a) What is the mass of a large ship that has a momentum of $1.60\times10^{9} \text{ kg}\cdot\text{m/s}$, when the ship is moving at a speed of 48.0 km/h? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of 1200 m/s .

3. (a) At what speed would a $2.00\times10^{4}\text{-kg}$ airplane have to fly to have a momentum of $1.60\times10^{9} \text{ kg}\cdot\text{m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of 60.0 m/s ? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

4. (a) What is the momentum of a garbage truck that is $1.20\times10^{4} \text{ kg}$ and is moving at 10.0 m/s ? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

5. A runaway train car that has a mass of 15,000 kg travels at a speed of 5.4 m/s down a track. Compute the time required for a force of 1500 N to bring the car to rest.

6. The mass of Earth is $5.972\times10^{24} \text{ kg}$ and its orbital radius is an average of $1.496\times10^{11} \text{ m}$. Calculate its linear momentum.

5.2 Impulse

7. A bullet is accelerated down the barrel of a gun by hot gases produced in the combustion of gun powder. What is the average force exerted on a 0.0300-kg bullet to accelerate it to a speed of 600 m/s in a time of 2.00 ms (milliseconds)?

8. Professional Application

A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seat belt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.

9. A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.

10. Professional Application

A professional boxer hits his opponent with a 1000-N horizontal blow that lasts for 0.150 s. (a) Calculate the impulse imparted by this blow. (b) What is the opponent's final velocity, if his mass is 105 kg and he is motionless in midair when struck near his center of mass? (c) Calculate the recoil velocity of the opponent's 10.0-kg head if hit in this manner, assuming the head does not initially transfer significant momentum to the boxer's body. (d) Discuss the implications of your answers for parts (b) and (c).

11. Professional Application

Suppose a child drives a bumper car head on into the side rail, which exerts a force of 4000 N on the car for 0.200 s. (a) What impulse is imparted by this force? (b) Find the final velocity of the bumper car if its initial velocity was 2.80 m/s and the car plus driver have a mass of 200 kg. You may neglect friction between the car and floor.

12. Professional Application

One hazard of space travel is debris left by previous missions. There are several thousand objects orbiting Earth that are large enough to be detected by radar, but there are far greater numbers of very small objects, such as flakes of paint. Calculate the force exerted by a 0.100-mg chip of paint that strikes a spacecraft window at a relative speed of $4.00\times10^{3} \text{ m/s}$, given the collision lasts $6.00\times10^{-8} \text{ s}$.

13. Professional Application

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment. (a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm. (b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

14. Professional Application

Military rifles have a mechanism for reducing the recoil forces of the gun on the person firing it. An internal part recoils over a relatively large distance and is stopped by damping mechanisms in the gun. The larger distance reduces the average force needed to stop the internal part. (a) Calculate the recoil velocity of a 1.00-kg plunger that directly interacts with a 0.0200-kg bullet fired at 600 m/s from the gun. (b) If this part is stopped over a distance of 20.0 cm, what average force is exerted upon it by the gun? (c) Compare this to the force exerted on the gun if the bullet is accelerated to its velocity in 10.0 ms (milliseconds).

15. A cruise ship with a mass of $1.00\times10^{7} \text{ kg}$ strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)

16. Calculate the final speed of a 110-kg rugby player who is initially running at 8.00 m/s but collides head-on with a padded goalpost and experiences a backward force of $1.76\times10^{4} \text{ N}$ for $5.50\times10^{-2} \text{ s}$.

17. Water from a fire hose is directed horizontally against a wall at a rate of 50.0 kg/s and a speed of 42.0 m/s. Calculate the magnitude of the force exerted on the wall, assuming the water's horizontal momentum is reduced to zero.

18. A 0.450-kg hammer is moving horizontally at 7.00 m/s when it strikes a nail and comes to rest after driving the nail 1.00 cm into a board. (a) Calculate the duration of the impact. (b) What was the average force exerted on the nail?

19. Starting with the definitions of momentum and kinetic energy, derive an equation for the kinetic energy of a particle expressed as a function of its momentum.

20. A ball with an initial velocity of 10 m/s moves at an angle 60^{o} above the $+x$ -direction. The ball hits a vertical wall and bounces off so that it is moving 60^{o} above the $-x$ -direction with the same speed. What is the impulse delivered by the wall?

21. When serving a tennis ball, a player hits the ball when its velocity is zero (at the highest point of a vertical toss). The racquet exerts a force of 540 N on the ball for 5.00 ms, giving it a final velocity of 45.0 m/s. Using these data, find the mass of the ball.

22. A punter drops a ball from rest vertically 1 meter down onto his foot. The ball leaves the foot with a speed of 18 m/s at an angle 55^{o} above the horizontal. What is the impulse delivered by the foot (magnitude and direction)?

5.3 Conservation of Momentum

23. Professional Application

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of -0.120 m/s . (The minus indicates direction of motion.) What is their final velocity?

24. Suppose a clay model of a koala bear has a mass of 0.200 kg and slides on ice at a speed of 0.750 m/s. It runs into another clay model, which is initially motionless and has a mass of 0.350 kg. Both being soft clay, they naturally stick together. What is their final velocity?

25. Professional Application

Consider the following question: *A car moving at 10 m/s crashes into a tree and stops in 0.26 s. Calculate the force the seatbelt exerts on a passenger in the car to bring him to a halt. The mass of the passenger is 70 kg.* Would the answer to this question be different if the car with the 70-kg passenger had collided with a car that has a mass equal to and is traveling in the opposite direction and at the same speed? Explain your answer.

26. What is the velocity of a 900-kg car initially moving at 30.0 m/s, just after it hits a 150-kg deer initially running at 12.0 m/s in the same direction? Assume the deer remains on the car.

27. A 1.80-kg falcon catches a 0.650-kg dove from behind in midair. What is their velocity after impact if the falcon's velocity is initially 28.0 m/s and the dove's velocity is 7.00 m/s in the same direction?

5.4 Elastic Collisions in One Dimension

28. Two identical objects (such as billiard balls) have a one-dimensional collision in which one is initially motionless. After the collision, the moving object is stationary and the other moves with the same speed as the other originally had. Show that both momentum and kinetic energy are conserved.

29. Professional Application

Two manned satellites approach one another at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^{3}\text{ kg}$, and the second a mass of $7.50\times10^{3}\text{ kg}$. If the two satellites collide elastically rather than dock, what is their final relative velocity?

30. A 70.0-kg ice hockey goalie, originally at rest, catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. Suppose the goalie and the ice puck have an elastic collision and the puck is reflected back in the direction from which it came. What would their final velocities be in this case?

5.5 Inelastic Collisions in One Dimension

31. A 0.240-kg billiard ball that is moving at 3.00 m/s strikes the bumper of a pool table and bounces straight back at 2.40 m/s (80% of its original speed). The collision lasts 0.0150 s. (a) Calculate the average force exerted on the ball by the bumper. (b) How much kinetic energy in joules is lost during the collision? (c) What percent of the original energy is left?

32. During an ice show, a 60.0-kg skater leaps into the air and is caught by an initially stationary 75.0-kg skater. (a) What is their final velocity assuming negligible friction and that the 60.0-kg skater's original horizontal velocity is 4.00 m/s? (b) How much kinetic energy is lost?

33. Professional Application

Using mass and speed data from **Example 5.1** and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?

34. A battleship that is $6.00\times10^{7}\text{ kg}$ and is originally at rest fires a 1100-kg artillery shell horizontally with a velocity of 575 m/s. (a) If the shell is fired straight aft (toward the rear of the ship), there will be negligible friction opposing the ship's recoil. Calculate its recoil velocity. (b) Calculate the increase in total kinetic energy (that is, for the ship and the shell). This energy is less than the energy released by the gun powder—significant heat transfer occurs.

35. Professional Application

Two manned satellites approaching one another, at a relative speed of 0.250 m/s, intending to dock. The first has a mass of $4.00\times10^{3}\text{ kg}$, and the second a mass of $7.50\times10^{3}\text{ kg}$.

(a) Calculate the final velocity (after docking) by using the frame of reference in which the first satellite was originally at rest. (b) What is the loss of kinetic energy in this inelastic collision? (c) Repeat both parts by using the frame of reference in which the second satellite was originally at rest. Explain why the change in velocity is different in the two frames, whereas the change in kinetic energy is the same in both.

36. Professional Application

A 30,000-kg freight car is coasting at 0.850 m/s with negligible friction under a hopper that dumps 110,000 kg of scrap metal into it. (a) What is the final velocity of the loaded freight car? (b) How much kinetic energy is lost?

37. Professional Application

Space probes may be separated from their launchers by exploding bolts. (They bolt away from one another.) Suppose a 4800-kg satellite uses this method to separate from the 1500-kg remains of its launcher, and that 5000 J of kinetic energy is supplied to the two parts. What are their subsequent velocities using the frame of reference in which they were at rest before separation?

38. A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) Calculate the momentum of a 110-kg football player running at 8.00 m/s. Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s. Discuss its relationship to this problem.

39. Professional Application

One of the waste products of a nuclear reactor is plutonium-239 $\left({}^{239}\text{Pu}\right)$. This nucleus is radioactive and decays by splitting into a helium-4 nucleus and a uranium-235 nucleus $\left({}^{4}\text{He} + {}^{235}\text{U}\right)$, the latter of which is also radioactive and will itself decay some time later. The energy emitted in the plutonium decay is $8.40\times10^{-13}\text{ J}$ and is entirely converted to kinetic energy of the helium and uranium nuclei. The mass of the helium nucleus is $6.68\times10^{-27}\text{ kg}$, while that of the uranium is $3.92\times10^{-25}\text{ kg}$ (note that the ratio of the masses is 4 to 235). (a) Calculate the velocities of the two nuclei, assuming the plutonium nucleus is originally at rest. (b) How much kinetic energy does each nucleus carry away? Note that the data given here are accurate to three digits only.

40. Professional Application

The Moon's craters are remnants of meteorite collisions. Suppose a fairly large asteroid that has a mass of $5.00\times10^{12}\text{ kg}$ (about a kilometer across) strikes the Moon at a speed of 15.0 km/s. (a) At what speed does the Moon recoil after the perfectly inelastic collision (the mass of the Moon is $7.36\times10^{22}\text{ kg}$) ? (b) How much kinetic energy is lost in the collision? Such an event may have been observed by medieval English monks who reported observing a red glow and subsequent haze about the Moon. (c) In October 2009, NASA crashed a rocket into the Moon, and analyzed the plume produced by the impact. (Significant amounts of water were detected.) Answer part (a) and (b) for this real-life experiment. The mass of the rocket was 2000 kg and its speed upon impact was 9000 km/h. How does the plume produced alter these results?

41. Professional Application

Two football players collide head-on in midair while trying to catch a thrown football. The first player is 95.0 kg and has an initial velocity of 6.00 m/s, while the second player is 115 kg and has an initial velocity of –3.50 m/s. What is their velocity just after impact if they cling together?

42. What is the speed of a garbage truck that is $1.20\times10^{4}\text{ kg}$ and is initially moving at 25.0 m/s just after it hits and adheres to a trash can that is 80.0 kg and is initially at rest?

43. During a circus act, an elderly performer thrills the crowd by catching a cannon ball shot at him. The cannon ball has a mass of 10.0 kg and the horizontal component of its velocity is 8.00 m/s when the 65.0-kg performer catches it. If the performer is on nearly frictionless roller skates, what is his recoil velocity?

44. (a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?

6 OSCILLATIONS AND WAVES

Figure 6.1 There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

Chapter Outline

6.1. Period and Frequency in Oscillations
- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.

6.2. Simple Harmonic Motion: A Special Periodic Motion
- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

6.3. Forced Oscillations and Resonance
- Observe resonance of a paddle ball on a string.
- Observe amplitude of a damped harmonic oscillator.

6.4. Waves
- State the characteristics of a wave.
- Calculate the velocity of wave propagation.

6.5. Superposition and Interference
- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.

6.6. Sound
- Define sound and hearing.
- Describe sound as a longitudinal wave.

6.7. Speed of Sound, Frequency, and Wavelength
- Define pitch.
- Describe the relationship between the speed of sound, its frequency, and its wavelength.
- Describe the effects on the speed of sound as it travels through various media.
- Describe the effects of temperature on the speed of sound.

6.8. Doppler Effect and Sonic Booms
- Define Doppler effect, Doppler shift, and sonic boom.
- Describe the sounds produced by objects moving faster than the speed of sound.

Introduction to Oscillatory Motion and Waves

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined.

6.1 Period and Frequency in Oscillations

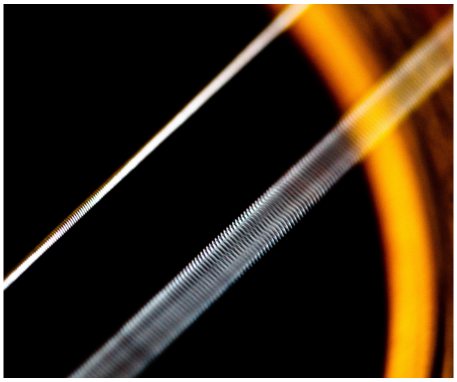


Figure 6.2 The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time.

The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time.

The relationship between frequency and period is

$$f = \frac{1}{T}. \tag{6.1}$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

$$1\text{ Hz} = 1\frac{\text{cycle}}{\text{sec}} \text{or } 1\text{ Hz} = \frac{1}{\text{s}} \tag{6.2}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example 6.1 Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute $0.400\ \mu\text{s}$ for T in $f = \frac{1}{T}$:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\text{ s}}. \tag{6.3}$$

Solve to find

$$f = 2.50 \times 10^{6}\text{ Hz}. \tag{6.4}$$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:
   The time for one complete oscillation is the period T:

$$f = \frac{1}{T}. \tag{6.5}$$

2. Solve for T:

$$T = \frac{1}{f}. \tag{6.6}$$

3. Substitute the given value for the frequency into the resulting expression:

$$T = \frac{1}{f} = \frac{1}{264\text{ Hz}} = \frac{1}{264\text{ cycles/s}} = 3.79 \times 10^{-3}\text{ s} = 3.79\text{ ms}. \tag{6.7}$$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Check your Understanding

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution
I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

6.2 Simple Harmonic Motion: A Special Periodic Motion

The oscillations of a system in which the net force can be described by Hooke’s law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke’s law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke’s law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in **Figure 6.3**. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?

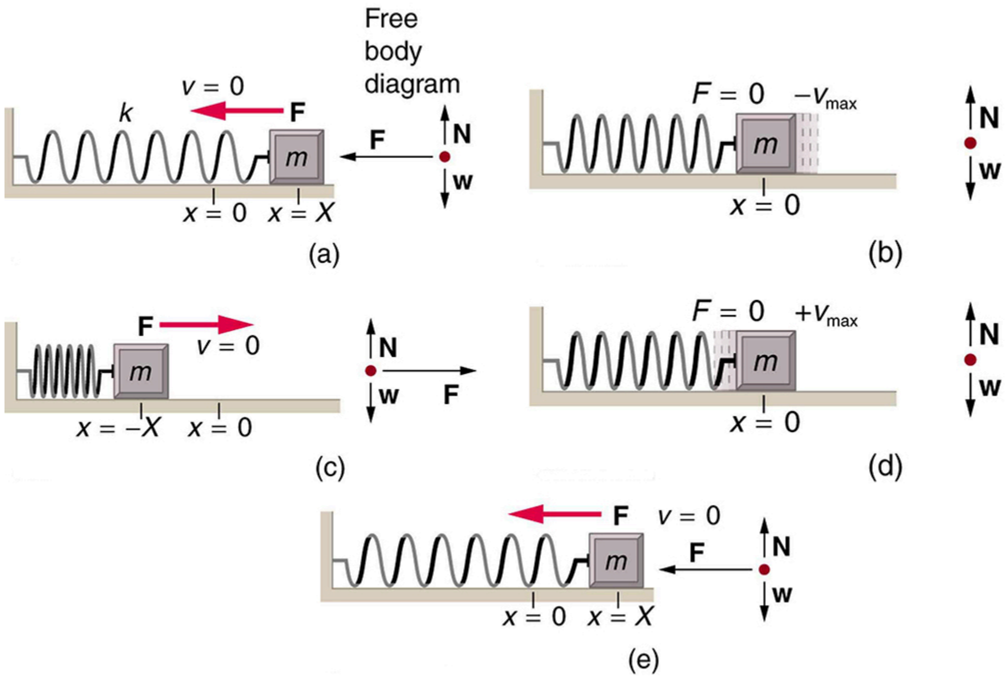


Figure 6.3 An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Period of Simple Harmonic Oscillator

The *period of a simple harmonic oscillator* is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{6.8}$$

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{6.9}$$

Note that neither T nor f has any dependence on amplitude.

Take-Home Experiment: Mass and Ruler Oscillations

Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example 6.2 Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See **Figure 6.4**). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is $6.53\times10^4\ \text{N/m}$.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the force constant are both given.

Solution

1. Enter the known values of *k* and *m*:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53\times10^4\ \text{N/m}}{900\ \text{kg}}}. \tag{6.10}$$

2. Calculate the frequency:

$$\frac{1}{2\pi}\sqrt{72.6\,/\,\text{s}^{-2}} = 1.3656\,/\,\text{s}^{-1} \approx 1.36\,/\,\text{s}^{-1} = 1.36\ \text{Hz}. \tag{6.11}$$

3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1\,/\,f$ and substitute the value just found for f:

$$T = \frac{1}{f} = \frac{1}{1.356\ \text{Hz}} = 0.738\ \text{s}. \tag{6.12}$$

Discussion

The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in **Figure 6.4**. Similarly, **Figure 6.5** shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.

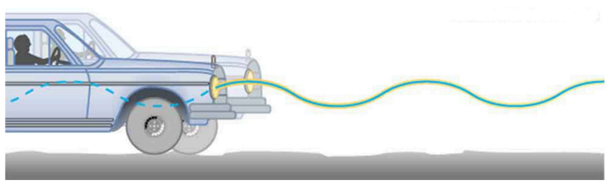


Figure 6.4 The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)

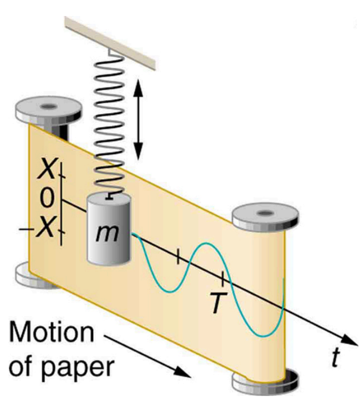


Figure 6.5 The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement as a function of time *t* in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke's law, is given by

$$x(t) = X \cos\frac{2\pi t}{T}, \tag{6.13}$$

where X is amplitude. At $t = 0$, the initial position is $x_0 = X$, and the displacement oscillates back and forth with a period T. (When $t = T$, we get $x = X$ again because $\cos 2\pi = 1$.). Furthermore, from this expression for x, the velocity v as a function of time is given by:

$$v(t) = -v_{\max} \sin\left(\frac{2\pi t}{T}\right), \tag{6.14}$$

where $v_{\max} = 2\pi X/T = X\sqrt{k/m}$. The object has zero velocity at maximum displacement—for example, $v = 0$ when $t = 0$, and at that time $x = X$. The minus sign in the first equation for $v(t)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x(t),\ v(t),\ t,$ and $a(t)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a = F/m = kx/m$. So, $a(t)$ is also a cosine function:

$$a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}. \tag{6.15}$$

Hence, $a(t)$ is directly proportional to and in the opposite direction to $x(t)$.

Figure 6.6 shows the simple harmonic motion of an object on a spring and presents graphs of $x(t), v(t),$ and $a(t)$ versus time.

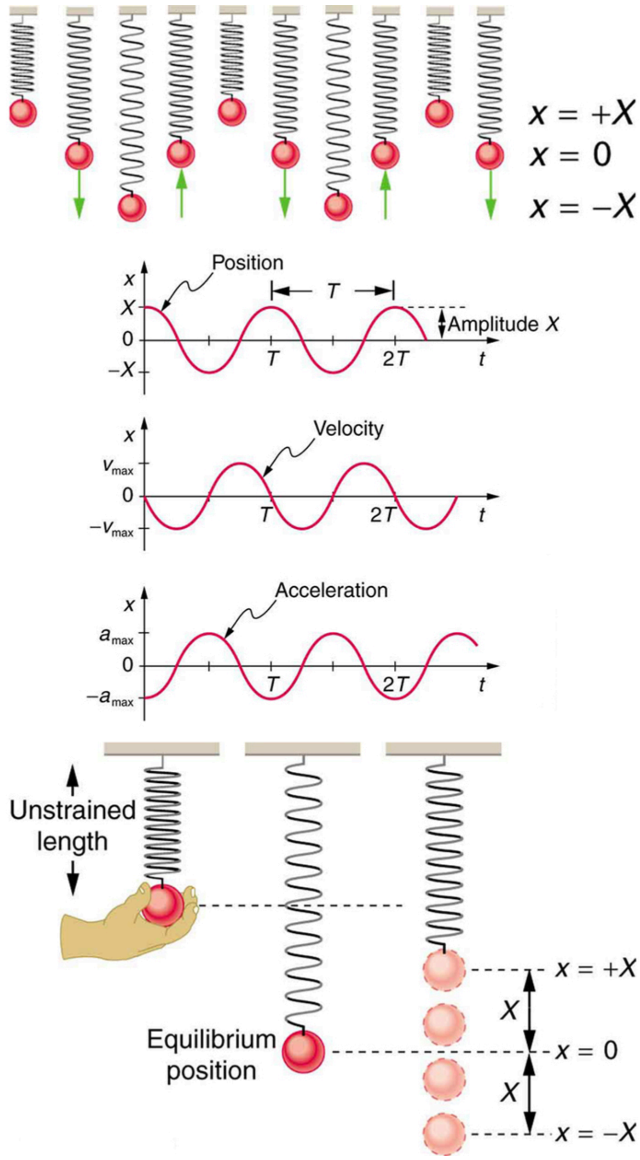


Figure 6.6 Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Check Your Understanding

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution
Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Check Your Understanding

A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located?

Solution

x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

PhET Explorations: Masses and Springs

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

PhET Interactive Simulation

Figure 6.7 Masses and Springs (http://legacy.cnx.org/content/m42242/1.7/mass-spring-lab_en.jar)

6.3 Forced Oscillations and Resonance

Figure 6.8 You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force.

Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in **Figure 6.9**. Imagine the finger in the figure is your finger. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

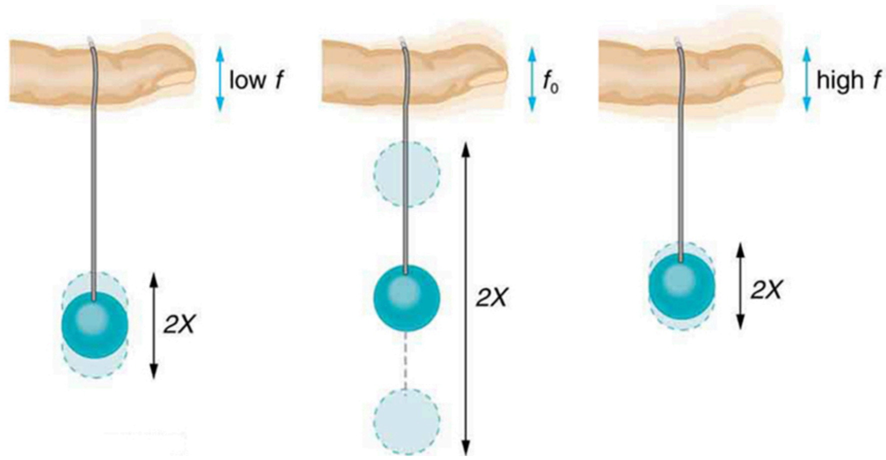


Figure 6.9 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

Figure 6.10 shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.

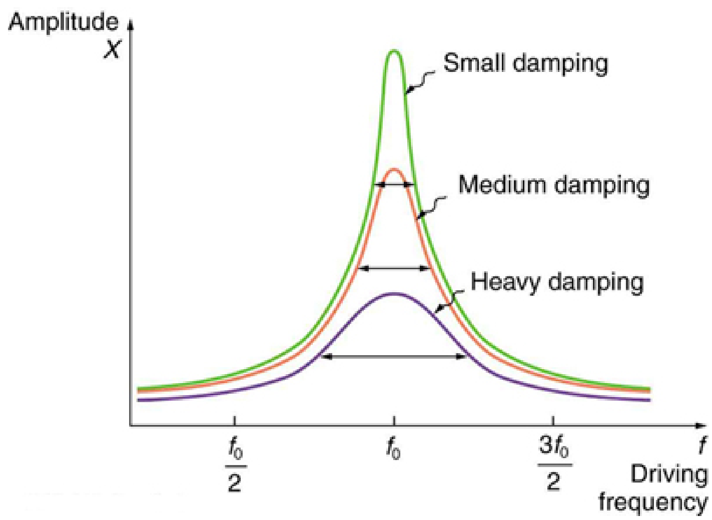


Figure 6.10 Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in **Figure 6.10** depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. **Figure 6.11** shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.

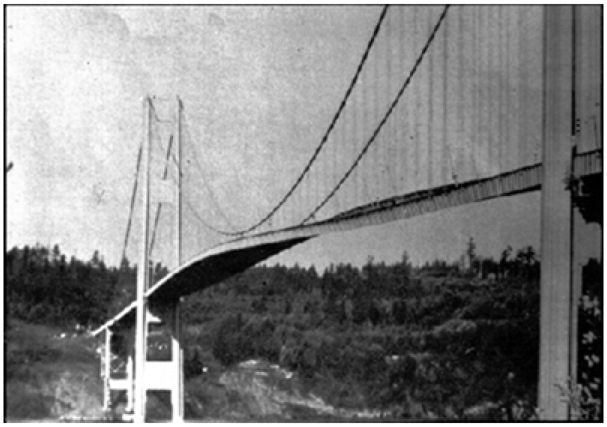


Figure 6.11 In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's *Studio 360*, via Flickr)

Check Your Understanding

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution
The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave. With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

6.4 Waves

Figure 6.12 Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in **Figure 6.13**. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_{w} to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed,* because the disturbance propagates from one location to another.

Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.

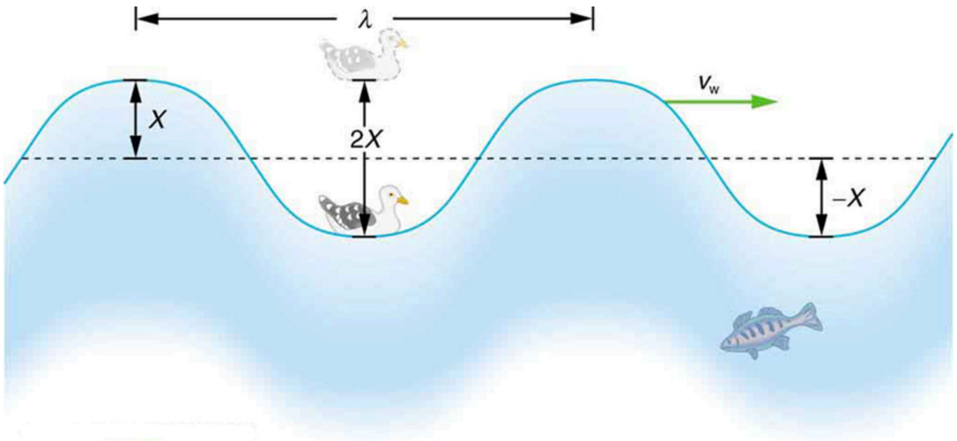


Figure 6.13 An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_w.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

$$v_\text{w} = \frac{\lambda}{T} \tag{6.16}$$

or

$$v_\text{w} = f\lambda. \tag{6.17}$$

This fundamental relationship holds for all types of waves. For water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light, for example.

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example 6.3 Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in **Figure 6.13** if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_w. The given information tells us that $\lambda = 10.0\text{ m}$ and $T = 5.00\text{ s}$. Therefore, we can use $v_\text{w} = \frac{\lambda}{T}$ to find the wave velocity.

Solution

1. Enter the known values into $v_\text{w} = \frac{\lambda}{T}$:

$$v_w = \frac{10.0\text{ m}}{5.00\text{ s}}. \tag{6.18}$$

2. Solve for v_w to find $v_w = 2.00\text{ m/s}$.

Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in **Figure 6.14** propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. **Figure 6.15** shows an example of a longitudinal wave. The size of the disturbance is its amplitude *X* and is completely independent of the speed of propagation v_w.

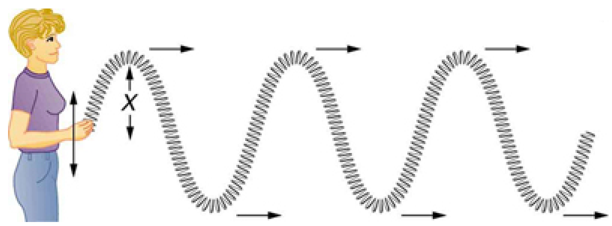


Figure 6.14 In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.

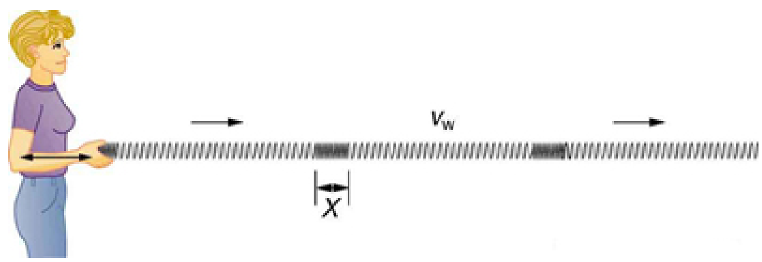


Figure 6.15 In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in **Figure 6.13** shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.

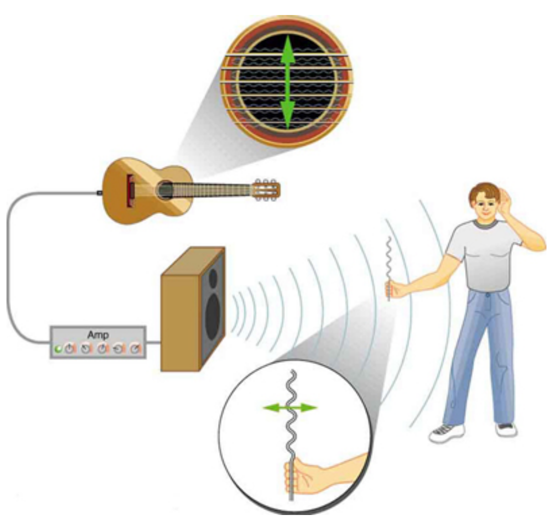


Figure 6.16 The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Check Your Understanding

Why is it important to differentiate between longitudinal and transverse waves?

Solution
In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

6.5 Superposition and Interference

Figure 6.17 These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

Most waves do not look very simple. They look more like the waves in **Figure 6.17** than like the simple water wave considered in "Waves". (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. **Figure 6.18** and **Figure 6.19** illustrate superposition in two special cases, both of which produce simple results.

Figure 6.18 shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

Figure 6.19 shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.

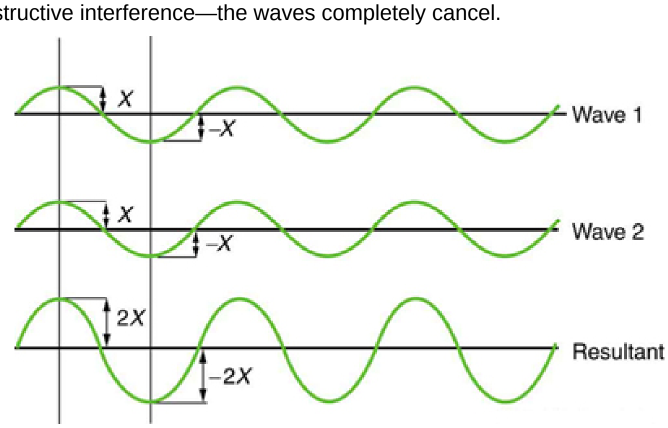


Figure 6.18 Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.

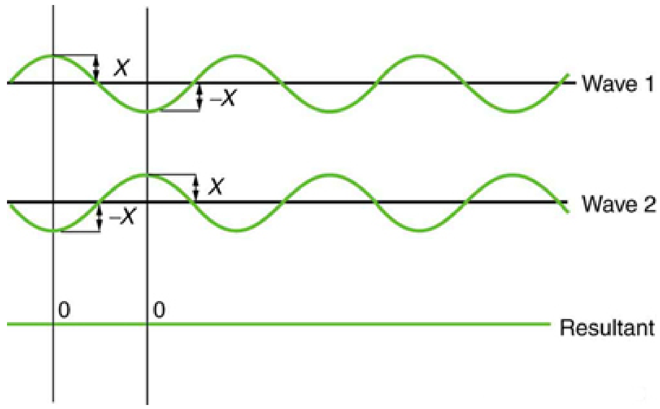


Figure 6.19 Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in **Figure 6.20**. Here again, the disturbances add and subtract, producing a more complicated looking wave.

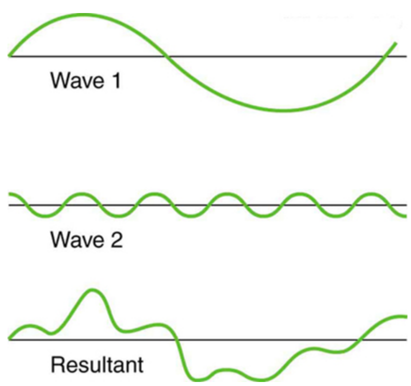


Figure 6.20 Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in **Figure 6.21** for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

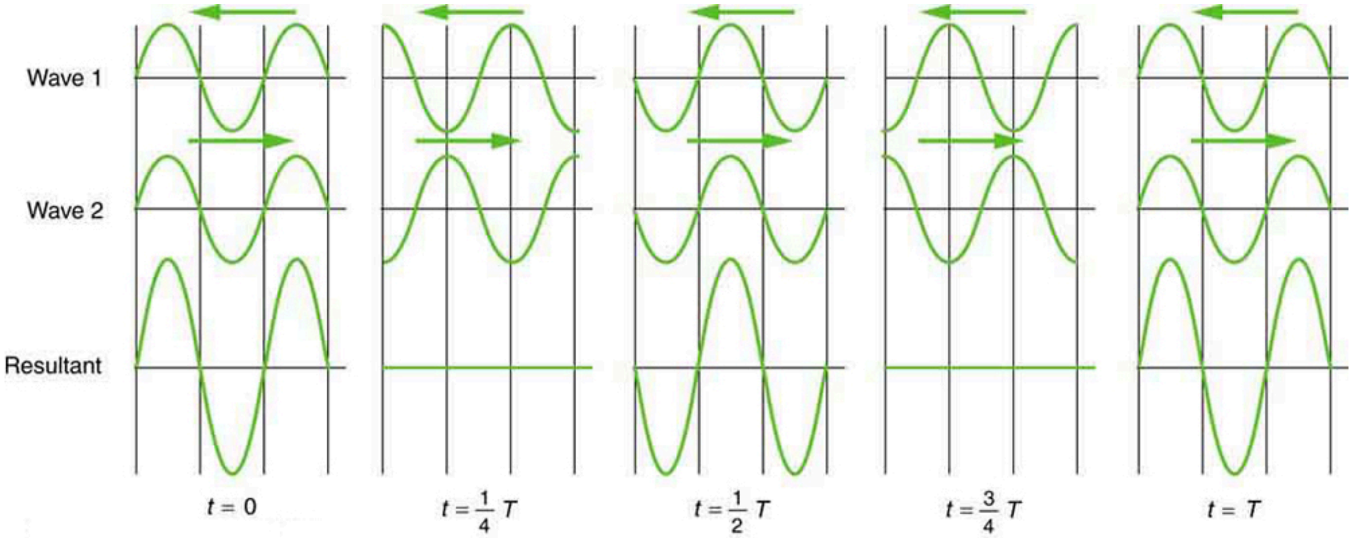


Figure 6.21 Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. **Figure 6.22** and **Figure 6.23** show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_\text{w} / \lambda_1 = v_\text{w} / 2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in **Figure 6.23**, the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_\text{w} / \lambda_2 = v_\text{w} / 2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.

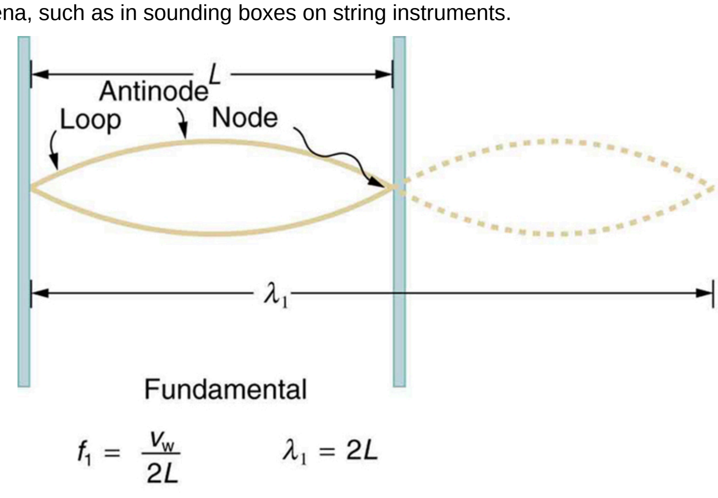


Figure 6.22 The figure shows a string oscillating at its fundamental frequency.

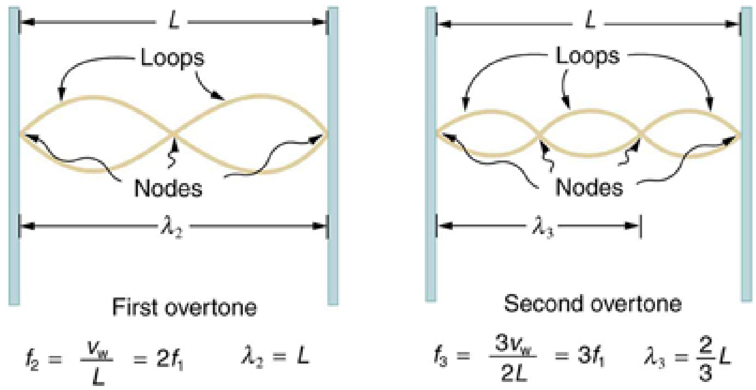


Figure 6.23 First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. **Figure 6.24** illustrates this graphically.

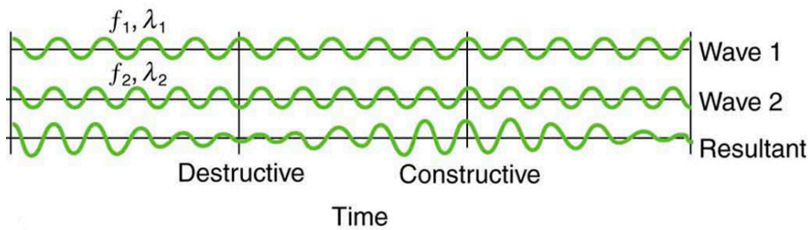


Figure 6.24 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. The result,

$$f_B = |f_1 - f_2| \tag{6.19}$$

is the beat frequency. The resultant wave has the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_B . This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz . Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Check Your Understanding

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution
The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium

with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Check Your Understanding

Define nodes and antinodes.

Solution
Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Check Your Understanding

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution
With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

6.6 Sound

Figure 6.25 This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. (credit: ||read||, Flickr)

Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. **Hearing** is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

The physical phenomenon of **sound** is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

A vibrating string produces a sound wave as illustrated in Figure 6.26, Figure 6.27, and Figure 6.28. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.) Figure 6.28 shows a graph of gauge pressure versus distance from the vibrating string.

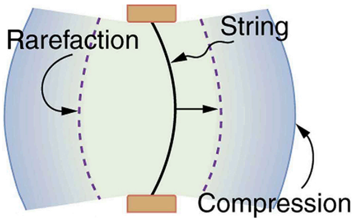


Figure 6.26 A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

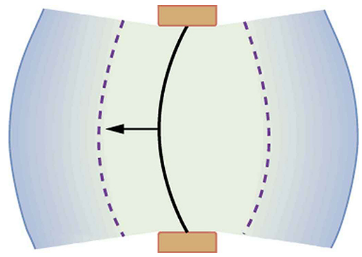


Figure 6.27 As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string.

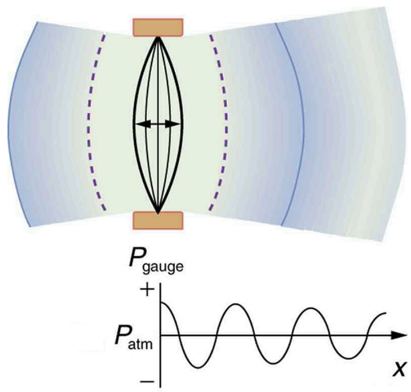


Figure 6.28 After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in **Figure 6.29**, and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.

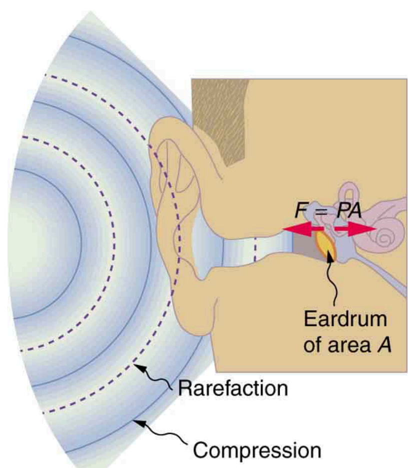


Figure 6.29 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person.

6.7 Speed of Sound, Frequency, and Wavelength

Figure 6.30 When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (credit: Dominic Alves, Flickr)

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower than light. You can also directly sense the frequency of a sound. Perception of frequency is called **pitch**. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v_{\mathrm{w}} = f\lambda, \tag{6.20}$$

where v_{w} is the speed of sound, f is its frequency, and λ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in Figure 6.31. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.

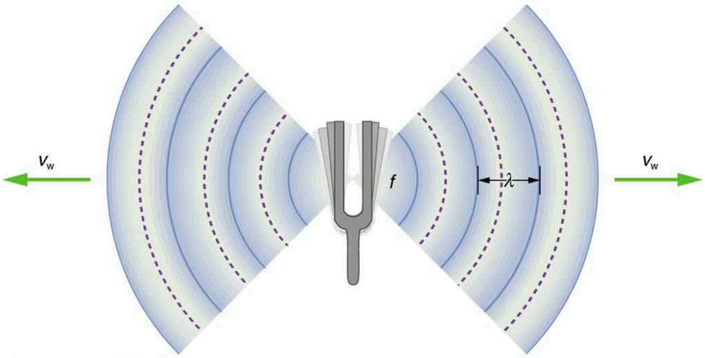


Figure 6.31 A sound wave emanates from a source vibrating at a frequency f, propagates at v_{w}, and has a wavelength λ.

Table 6.1 makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is directly proportional to the stiffness of the oscillating object. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Table 6.1 Speed of Sound in Various Media

Medium	v_W(m/s)
Gases at $0^{\circ}C$	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at $20^{\circ}C$	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Earthquakes, essentially sound waves in Earth's crust, are an interesting example of how the speed of sound depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake.

The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

$$v_{\mathrm{w}} = (331\ \mathrm{m/s})\sqrt{\frac{T}{273\ \mathrm{K}}}, \tag{6.21}$$

where the temperature (denoted as T) is in units of kelvin. While not negligible, this is not a strong dependence. At $0^{\circ}\mathrm{C}$, the speed of sound is 331 m/s, whereas at $20.0^{\circ}\mathrm{C}$ it is 343 m/s, less than a 4% increase. **Figure 6.32** shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.

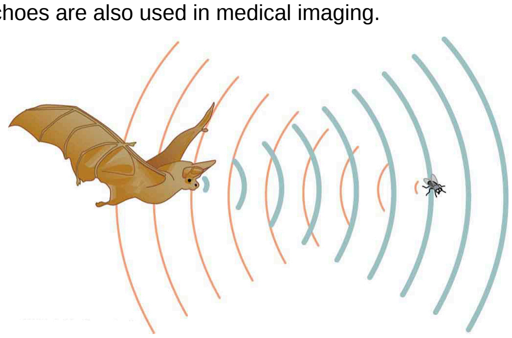


Figure 6.32 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20,000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed. Recall that

$$v_{\text{w}} = f\lambda. \tag{6.22}$$

In a given medium under fixed conditions, v_{w} is constant, so that there is a relationship between f and λ; the higher the frequency, the smaller the wavelength. See **Figure 6.33** and consider the following example.

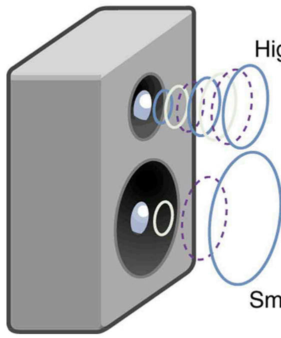


Figure 6.33 Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

Example 6.4 Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in 30.0°C air. (Assume that the frequency values are accurate to two significant figures.)

Strategy

To find wavelength from frequency, we can use $v_{\text{w}} = f\lambda$.

Solution

1. Identify knowns. The value for v_{w}, is given by

$$v_{\text{w}} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}. \tag{6.23}$$

2. Convert the temperature into kelvin and then enter the temperature into the equation

$$v_{\text{w}} = (331\text{ m/s})\sqrt{\frac{303\text{ K}}{273\text{ K}}} = 348.7\text{ m/s}. \tag{6.24}$$

3. Solve the relationship between speed and wavelength for λ:

$$\lambda = \frac{v_{\text{w}}}{f}. \tag{6.25}$$

4. Enter the speed and the minimum frequency to give the maximum wavelength:

$$\lambda_{\text{max}} = \frac{348.7\text{ m/s}}{20\text{ Hz}} = 17\text{ m}. \tag{6.26}$$

5. Enter the speed and the maximum frequency to give the minimum wavelength:

$$\lambda_{\text{min}} = \frac{348.7\text{ m/s}}{20{,}000\text{ Hz}} = 0.017\text{ m} = 1.7\text{ cm}. \tag{6.27}$$

Discussion

Because the product of f multiplied by λ equals a constant, the smaller f is, the larger λ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If v_{w} changes and f remains the same, then the wavelength λ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Making Connections: Take-Home Investigation—Voice as a Sound Wave

Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Check Your Understanding

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Solution
Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.

Check Your Understanding

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Solution
Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

6.8 Doppler Effect and Sonic Booms

The characteristic sound of a motorcycle buzzing by is an example of the **Doppler effect**. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. The actual change in frequency due to relative motion of source and observer is called a **Doppler shift**. The Doppler effect and Doppler shift are named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? **Figure 6.34**, **Figure 6.35**, and **Figure 6.36** compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in **Figure 6.34**. If the source is moving, as in **Figure 6.35**, then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in **Figure 6.35**), and longer in the opposite direction (on the left in **Figure 6.35**). Finally, if the observers move, as in **Figure 6.36**, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

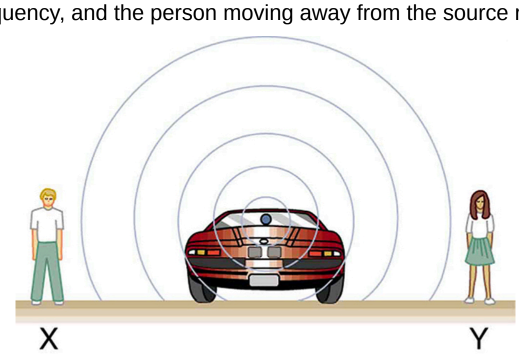


Figure 6.34 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

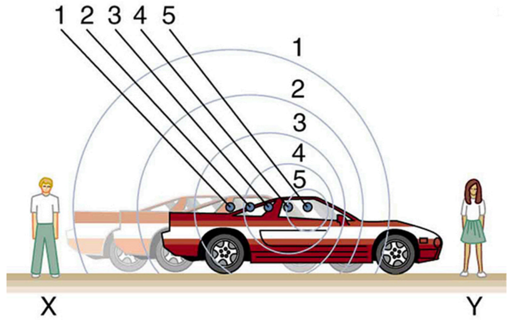


Figure 6.35 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

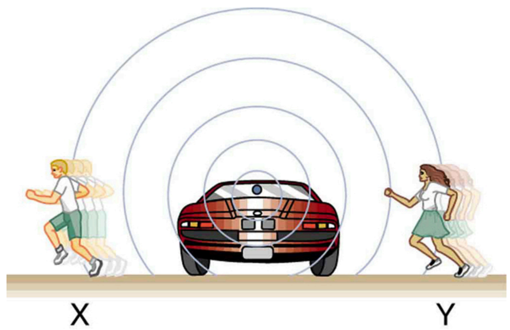


Figure 6.36 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v_\text{w} = f\lambda$, where v_w is the fixed speed of sound. The sound moves in a medium and has the same speed v_w in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in **Figure 6.35** receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in **Figure 6.36**. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

The Doppler Effect

The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_s. The greater the plane's speed v_s, the greater the Doppler shift and the greater the value observed for f_obs. Now, as v_s approaches the speed of sound, f_obs approaches infinity. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See **Figure 6.37**.)

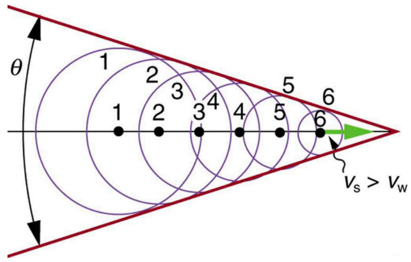


Figure 6.37 Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each. Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ.

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See Figure 6.38.) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in Figure 6.38. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

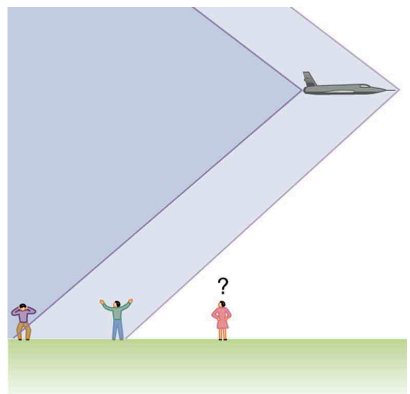


Figure 6.38 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A **bow wake**, such as the one in Figure 6.39, is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium. (In a vacuum, the maximum speed of light will be $c = 3.00 \times 10^8 \text{ m/s}$; in the medium of water, the speed of light is closer to $0.75c$. If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in Figure 6.40. Such a bow wake is called Cerenkov radiation and is commonly observed in particle physics.

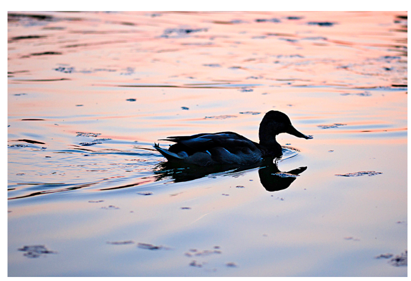


Figure 6.39 Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)

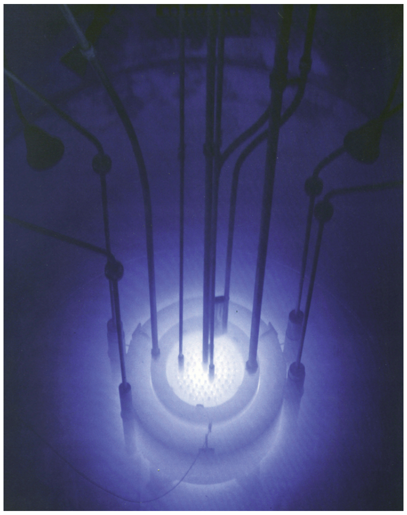


Figure 6.40 The blue glow in this research reactor pool is Cerenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such “Doppler Radar” can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Check Your Understanding

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Solution
Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound source and the observer are both in motion.

Check Your Understanding

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Solution
If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

Glossary

amplitude: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

antinode: the location of maximum amplitude in standing waves

beat frequency: the frequency of the amplitude fluctuations of a wave

bow wake: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

constructive interference: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

Doppler effect: an alteration in the observed frequency of a sound due to motion of either the source or the observer

Doppler shift: the actual change in frequency due to relative motion of source and observer

frequency: number of events per unit of time

fundamental frequency: the lowest frequency of a periodic waveform

hearing: the perception of sound

longitudinal wave: a wave in which the disturbance is parallel to the direction of propagation

natural frequency: the frequency at which a system would oscillate if there were no driving and no damping forces

nodes: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

oscillate: moving back and forth regularly between two points

overtones: multiples of the fundamental frequency of a sound

period: time it takes to complete one oscillation

periodic motion: motion that repeats itself at regular time intervals

pitch: the perception of the frequency of a sound

resonance: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate: a system being driven at its natural frequency

simple harmonic motion: the oscillatory motion in a system where the net force can be described by Hooke’s law

simple harmonic oscillator: a device that implements Hooke’s law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

sonic boom: a constructive interference of sound created by an object moving faster than sound

sound: a disturbance of matter that is transmitted from its source outward

superposition: the phenomenon that occurs when two or more waves arrive at the same point

transverse wave: a wave in which the disturbance is perpendicular to the direction of propagation

wave: a disturbance that moves from its source and carries energy

wave velocity: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength: the distance between adjacent identical parts of a wave

Section Summary

6.1 Period and Frequency in Oscillations

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T .
- The number of oscillations per unit time is the frequency f .
- These quantities are related by

$$f = \frac{1}{T}.$$

6.2 Simple Harmonic Motion: A Special Periodic Motion

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke’s law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X . The period T and frequency f of a simple harmonic oscillator are given by

  $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.

- Displacement in simple harmonic motion as a function of time is given by $x(t) = X\cos\frac{2\pi t}{T}$.
- The velocity is given by $v(t) = -v_{\text{max}}\sin\frac{2\pi t}{T}$, where $v_{\text{max}} = \sqrt{k/m}X$.
- The acceleration is found to be $a(t) = -\frac{kX}{m}\cos\frac{2\pi t}{T}$.

6.3 Forced Oscillations and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

6.4 Waves

- A wave is a disturbance that moves from the point of creation with a wave velocity v_w.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_\text{w} = \frac{\lambda}{T}$ or $v_\text{w} = f\lambda$.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

6.5 Superposition and Interference

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by

$$f_\text{B} = |f_1 - f_2|.$$

6.6 Sound

- Sound is a disturbance of matter that is transmitted from its source outward.
- Sound is one type of wave.
- Hearing is the perception of sound.

6.7 Speed of Sound, Frequency, and Wavelength

The relationship of the speed of sound v_w, its frequency f, and its wavelength λ is given by

$$v_\text{w} = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature T by

$$v_\text{w} = (331\text{ m/s})\sqrt{\frac{T}{273\text{ K}}}.$$

v_w is the same for all frequencies and wavelengths.

6.8 Doppler Effect and Sonic Booms

- The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer.
- The actual change in frequency is called the Doppler shift.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A sonic boom is a type of bow wake created when any wave source moves faster than the wave propagation speed.

Conceptual Questions

6.2 Simple Harmonic Motion: A Special Periodic Motion

1. What conditions must be met to produce simple harmonic motion?

2. (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?

(b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

3. Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

4. Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.

5. As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

6. Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

6.3 Forced Oscillations and Resonance

7. Why are soldiers in general ordered to “route step” (walk out of step) across a bridge?

6.4 Waves

8. Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

9. What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

6.5 Superposition and Interference

10. Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

6.7 Speed of Sound, Frequency, and Wavelength

11. How do sound vibrations of atoms differ from thermal motion?

12. When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

6.8 Doppler Effect and Sonic Booms

13. Is the Doppler shift real or just a sensory illusion?

14. Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.

15. When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

Problems & Exercises

6.1 Period and Frequency in Oscillations

1. What is the period of 60.0 Hz electrical power?

2. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

3. Find the frequency of a tuning fork that takes $2.50\times10^{-3} \text{ s}$ to complete one oscillation.

4. A stroboscope is set to flash every $8.00\times10^{-5} \text{ s}$. What is the frequency of the flashes?

5. A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

6. Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

6.2 Simple Harmonic Motion: A Special Periodic Motion

7. A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?

8. If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?

9. A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

10. By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

11. Suppose you attach the object with mass m to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of $2.00\ mg$ on the object at its lowest point. (b) If the spring has a force constant of 10.0 N/m and a 0.25-kg-mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity.

12. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 kg and the period of her motion is 0.800 s. The next diver is a male whose period of simple harmonic oscillation is 1.05 s. What is his mass if the mass of the board is negligible?

13. Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of 4.00 Hz. The board has an effective mass of 10.0 kg. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board?

14.

Figure 6.41 This child's toy relies on springs to keep infants entertained. (credit: By Humboldthead, Flickr)

The device pictured in **Figure 6.41** entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring constant.

(a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its spring constant?

(b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

15. A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in **Figure 6.42**.

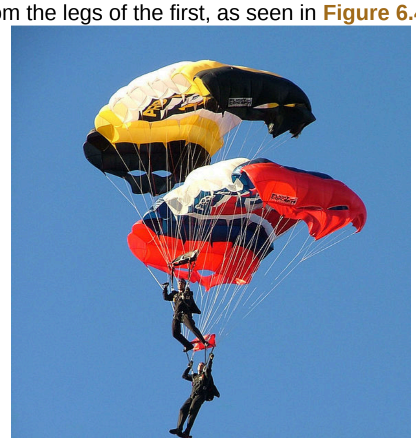


Figure 6.42 The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil)

6.3 Forced Oscillations and Resonance

16. How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

17. If a car has a suspension system with a force constant of $5.00\times10^{4} \text{ N/m}$, how much energy must the car's shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

18. (a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

19. Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_{\text{s}} = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_{\text{k}} = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

20. Engineering Application: A suspension bridge oscillates with an effective force constant of $1.00\times10^{8}\text{ N/m}$. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart $1.00\times10^{4}\text{ J}$ of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

6.4 Waves

21. Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

22. Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

23. Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

24. How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

25. Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

26. What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

27. What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

28. Radio waves transmitted through space at $3.00\times10^{8}\text{ m/s}$ by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

29. Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

30. (a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. **Figure 6.43**) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)

Figure 6.43 A seismograph as described in above problem.(credit: Oleg Alexandrov)

6.5 Superposition and Interference

31. A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

32. The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

33. Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

34. Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

35. A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

36. Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

6.7 Speed of Sound, Frequency, and Wavelength

37. When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?

38. What frequency sound has a 0.10-m wavelength when the speed of sound is 340 m/s?

39. Calculate the speed of sound on a day when a 1500 Hz frequency has a wavelength of 0.221 m.

40. (a) What is the speed of sound in a medium where a 100-kHz frequency produces a 5.96-cm wavelength? (b) Which substance in **Table 6.1** is this likely to be?

41. Show that the speed of sound in 20.0°C air is 343 m/s, as claimed in the text.

42. Air temperature in the Sahara Desert can reach 56.0°C (about 134°F). What is the speed of sound in air at that temperature?

43. Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is 20.0°C.

44. A sonar echo returns to a submarine 1.20 s after being emitted. What is the distance to the object creating the echo? (Assume that the submarine is in the ocean, not in fresh water.)

45. (a) If a submarine's sonar can measure echo times with a precision of 0.0100 s, what is the smallest difference in distances it can detect? (Assume that the submarine is in the ocean, not in fresh water.)

(b) Discuss the limits this time resolution imposes on the ability of the sonar system to detect the size and shape of the object creating the echo.

46. A physicist at a fireworks display times the lag between seeing an explosion and hearing its sound, and finds it to be 0.400 s. (a) How far away is the explosion if air temperature is 24.0°C and if you neglect the time taken for light to reach the physicist? (b) Calculate the distance to the explosion taking the speed of light into account. Note that this distance is negligibly greater.

47. Suppose a bat uses sound echoes to locate its insect prey, 3.00 m away. (See **Figure 6.32**.) (a) Calculate the echo times for temperatures of 5.00°C and 35.0°C. (b) What percent uncertainty does this cause for the bat in locating the insect? (c) Discuss the significance of this uncertainty and whether it could cause difficulties for the bat. (In practice, the bat continues to use sound as it closes in, eliminating most of any difficulties imposed by this and other effects, such as motion of the prey.)

7 ROTATION

Figure 7.1 The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. (credit: Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Chapter Outline

7.1. Dynamics of Rotational Motion: Rotational Inertia
- Understand the relationship between force, mass and acceleration.
- Study the turning effect of force.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

7.2. Rotational Kinetic Energy
- Guess the equation for rotational kinetic energy by analogy.
- Calculate rotational kinetic energy.

7.3. Angular Momentum and Its Conservation
- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

7.4. Gyroscopic Effects: Vector Aspects of Angular Momentum
- Describe the right-hand rule to find the direction of angular velocity, momentum, and torque.
- Explain the gyroscopic effect.
- Study how Earth acts like a gigantic gyroscope.

Introduction to Rotational Motion and Angular Momentum

Why do tornadoes spin at all? And why do tornados spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in Figure 7.2. The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.

Figure 7.2 This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

7.1 Dynamics of Rotational Motion: Rotational Inertia

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in Figure 7.3. In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton's second law of motion. There are, in fact, precise rotational analogs to both force and mass.

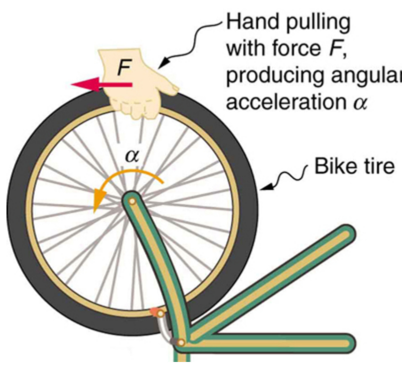


Figure 7.3 Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in Figure 7.4. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that $F = ma$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = ma$, yielding

$$F = mr\alpha. \tag{7.1}$$

Recall that **torque** is the turning effectiveness of a force. In this case, because $\mathbf{F}$ is perpendicular to r, torque is simply $\tau = Fr$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

$$rF = mr^2\alpha \tag{7.2}$$

or

$$\tau = mr^2\alpha. \tag{7.3}$$

This last equation is the rotational analog of Newton's second law ($F = ma$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.

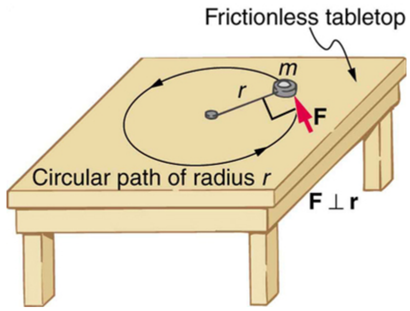


Figure 7.4 An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r, causing it to accelerate about the pivot point. The force is kept perpendicular to r.

Making Connections: Rotational Motion Dynamics

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in **Figure 7.4**, we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the **moment of inertia** I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \sum mr^2$. Here I is analogous to m in translational motion. Because of the distance r, the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis. A hoop's moment of inertia around its axis is therefore MR^2, where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) In all other cases, we must consult **Figure 7.5** (note that the table is piece of artwork that has shapes as well as formulae) for formulas for I that have been derived from integration over the continuous body. Note that I has units of mass multiplied by distance squared ($\mathrm{kg \cdot m^2}$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is

$$\text{net } \tau = I\alpha \tag{7.4}$$

or

$$\alpha = \frac{\text{net } \tau}{I}, \tag{7.5}$$

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha,\ \alpha = \frac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases; but the moment of inertia is much larger when the children are at the edge.

Take-Home Experiment

Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle's moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

Problem-Solving Strategy for Rotational Dynamics

1. *Examine the situation to determine that torque and mass are involved in the rotation*. Draw a careful sketch of the situation.
2. *Determine the system of interest*.
3. *Draw a free body diagram*. That is, draw and label all external forces acting on the system of interest.
4. *Apply* $\text{net } \tau = I\alpha,\ \alpha = \frac{\text{net } \tau}{I}$ *, the rotational equivalent of Newton's second law, to solve the problem*. Care must be taken to use the correct moment of inertia and to consider the torque about the point of rotation.
5. *As always, check the solution to see if it is reasonable*.

Making Connections

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton's second law of motion for rotation.

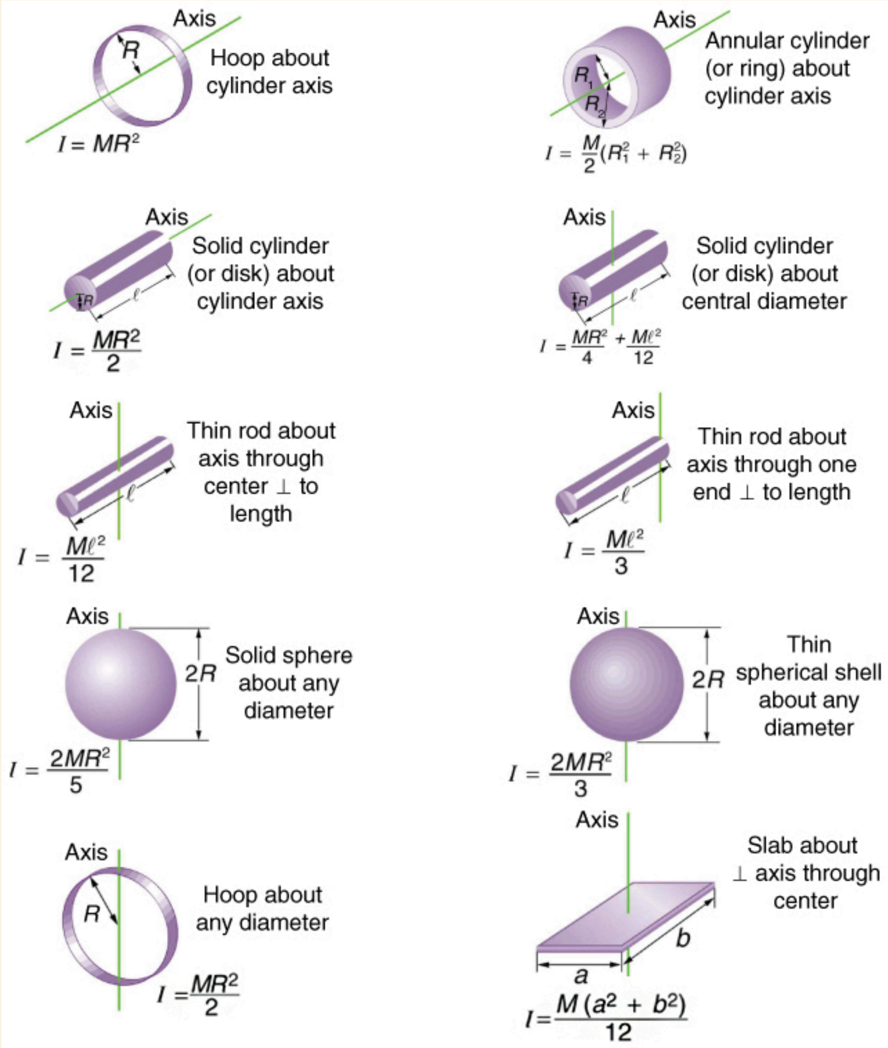


Figure 7.5 Some rotational inertias.

Example 7.1 Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in **Figure 7.6**. He exerts a force of 250 N at the edge of the

50.0-kg merry-go-round, which has a 1.50 m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible retarding friction.

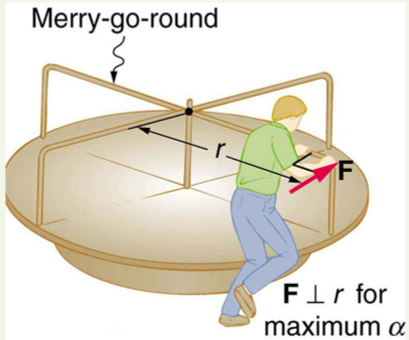


Figure 7.6 A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Strategy

Angular acceleration is given directly by the expression $\alpha = \frac{\text{net } \tau}{I}$:

$$\alpha = \frac{\tau}{I}. \tag{7.6}$$

To solve for α , we must first calculate the torque τ (which is the same in both cases) and moment of inertia I (which is greater in the second case). To find the torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50\text{ m})(250\text{ N}) = 375\text{ N}\cdot\text{m}. \tag{7.7}$$

Solution for (a)

The moment of inertia of a solid disk about this axis is given in **Figure 7.5** to be

$$\frac{1}{2}MR^2, \tag{7.8}$$

where $M = 50.0\text{ kg}$ and $R = 1.50\text{ m}$, so that

$$I = (0.500)(50.0\text{ kg})(1.50\text{ m})^2 = 56.25\text{ kg}\cdot\text{m}^2. \tag{7.9}$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{56.25\text{ kg}\cdot\text{m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}. \tag{7.10}$$

Solution for (b)

We expect the angular acceleration for the system to be less in this part, because the moment of inertia is greater when the child is on the merry-go-round. To find the total moment of inertia I , we first find the child's moment of inertia I_c by considering the child to be equivalent to a point mass at a distance of 1.25 m from the axis. Then,

$$I_\text{c} = MR^2 = (18.0\text{ kg})(1.25\text{ m})^2 = 28.13\text{ kg}\cdot\text{m}^2. \tag{7.11}$$

The total moment of inertia is the sum of moments of inertia of the merry-go-round and the child (about the same axis). To justify this sum to yourself, examine the definition of I :

$$I = 28.13\text{ kg}\cdot\text{m}^2 + 56.25\text{ kg}\cdot\text{m}^2 = 84.38\text{ kg}\cdot\text{m}^2. \tag{7.12}$$

Substituting known values into the equation for α gives

$$\alpha = \frac{\tau}{I} = \frac{375\text{ N}\cdot\text{m}}{84.38\text{ kg}\cdot\text{m}^2} = 4.44\frac{\text{rad}}{\text{s}^2}. \tag{7.13}$$

Discussion

The angular acceleration is less when the child is on the merry-go-round than when the merry-go-round is empty, as expected. The angular accelerations found are quite large, partly due to the fact that friction was considered to be negligible. If, for example, the father kept pushing perpendicularly for 2.00 s, he would give the merry-go-round an angular velocity of

13.3 rad/s when it is empty but only 8.89 rad/s when the child is on it. In terms of revolutions per second, these angular velocities are 2.12 rev/s and 1.41 rev/s, respectively. The father would end up running at about 50 km/h in the first case. Summer Olympics, here he comes! Confirmation of these numbers is left as an exercise for the reader.

Check Your Understanding

Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Solution
No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

7.2 Rotational Kinetic Energy

In this module, we will learn about work and energy associated with rotational motion. **Figure 7.7** shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable **rotational kinetic energy**.

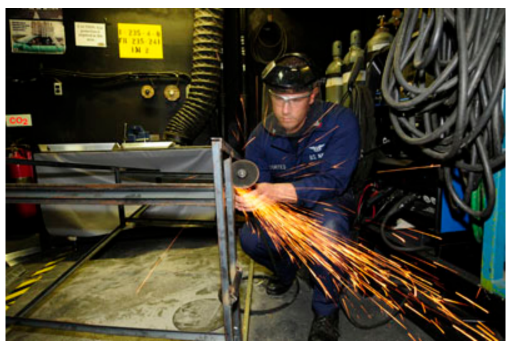


Figure 7.7 The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

Instead of deriving the formula for rotational kinetic energy by calculating the work done by torque on a rotating object, we can simply guess the correct formula by analogy. We know the kinetic energy in linear, or translational motion, $\mathrm{KE} = \frac{1}{2}mv^2$. We can find the rotational version of kinetic energy by replacing mass m with rotational version of mass, rotational inertia I, and by replacing speed v with rotational speed ω. Rotational speed is measured in units of radians per second. With these replacements, we get the formula for rotational kinetic energy,

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2. \tag{7.14}$$

Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in **Figure 7.8**.

Figure 7.8 Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into $\mathrm{KE}_{\mathrm{rot}}$. It can also convert translational kinetic energy, when the bus stops, into $\mathrm{KE}_{\mathrm{rot}}$. The flywheel's energy can then be used to accelerate, to go up another hill, or to keep the bus from going against friction.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Example 7.2 Calculating Helicopter Energies

A typical small rescue helicopter, similar to the one in **Figure 7.9**, has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?

Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

$$\mathrm{KE_{rot}} = \frac{1}{2}I\omega^2. \tag{7.15}$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find $\mathrm{KE_{rot}}$. The angular velocity ω is

$$\omega = \frac{300\ \text{rev}}{1.00\ \text{min}} \cdot \frac{2\pi\ \text{rad}}{1\ \text{rev}} \cdot \frac{1.00\ \text{min}}{60.0\ \text{s}} = 31.4\frac{\text{rad}}{\text{s}}. \tag{7.16}$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, $I_{\text{rod}} = \frac{1}{3}M\ell^2$. The total I is four times this moment of inertia, because there are four blades. Thus,

$$I = 4\frac{M\ell^2}{3} = 4\times\frac{(50.0\ \text{kg})(4.00\ \text{m})^2}{3} = 1067\ \text{kg}\cdot\text{m}^2. \tag{7.17}$$

Entering ω and I into the expression for rotational kinetic energy gives

$$\begin{aligned}\mathrm{KE_{rot}} &= 0.5(1067\ \text{kg}\cdot\text{m}^2)(31.4\ \text{rad/s})^2 \\ &= 5.26\times10^5\ \text{J}\end{aligned} \tag{7.18}$$

Solution for (b)

Entering the given values of mass and velocity into formula for translational kinetic energy, we obtain

$$\mathrm{KE_{trans}} = \frac{1}{2}mv^2 = (0.5)(1000\ \text{kg})(20.0\ \text{m/s})^2 = 2.00\times10^5\ \text{J}. \tag{7.19}$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

$$\frac{2.00\times10^5\ \text{J}}{5.26\times10^5\ \text{J}} = 0.380. \tag{7.20}$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

$$\mathrm{KE_{rot}} = \mathrm{PE_{grav}} \tag{7.21}$$

or

$$\frac{1}{2}I\omega^2 = mgh. \tag{7.22}$$

We now solve for h and substitute known values into the resulting equation

$$h = \frac{\frac{1}{2}I\omega^2}{mg} = \frac{5.26\times10^5\ \text{J}}{(1000\ \text{kg})(9.80\ \text{m/s}^2)} = 53.7\ \text{m}. \tag{7.23}$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.

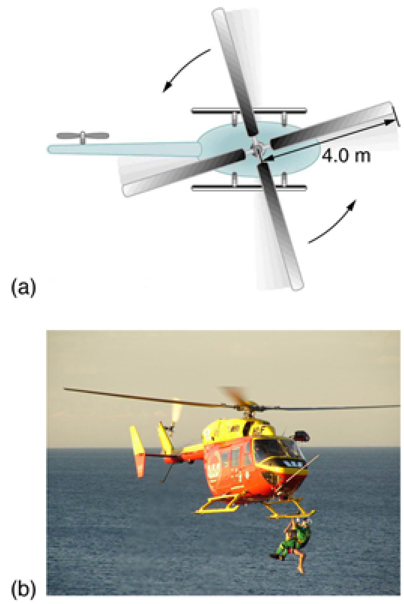


Figure 7.9 The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy PE_{grav}, which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of KE_{trans} or KE_{rot}, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in **Figure 7.10**.

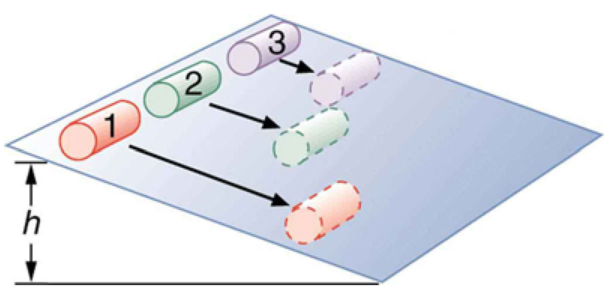


Figure 7.10 Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

$$\mathrm{PE_i} = \mathrm{KE_f}. \tag{7.24}$$

More specifically,

$$\mathrm{PE_{grav}} = \mathrm{KE_{trans}} + \mathrm{KE_{rot}} \tag{7.25}$$

or

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{7.26}$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

Take-Home Experiment

Locate several cans each containing different types of food. First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. You could also do this experiment by collecting several empty cylindrical containers of the same size and filling them with different materials such as wet or dry sand.

Example 7.3 Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with v as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2. \tag{7.27}$$

Before we can solve for v , we must look up an expression for I for a solid disk: $I_{\text{disk}} = \frac{1}{2}M\ell^2$. Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right). \tag{7.28}$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2. \tag{7.29}$$

Solving algebraically, the equation for the final velocity v gives

$$v = \left(\frac{4gh}{3}\right)^{1/2}. \tag{7.30}$$

Substituting known values into the resulting expression yields

$$v = \left[\frac{4\left(9.80 \text{ m/s}^2\right)(2.00 \text{ m})}{3}\right]^{1/2} = 5.11 \text{ m/s}. \tag{7.31}$$

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}gh\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}mv^2 = mgh$ and $v = (2gh)^{1/2}$, which is 22% greater than $(4gh/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

Check Your Understanding

Analogy of Rotational and Translational Kinetic Energy

Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Solution

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

7.3 Angular Momentum and Its Conservation

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define **angular momentum** L as

$$L = I\omega. \tag{7.32}$$

This equation is an analog to the definition of linear momentum as $p = mv$. Units for linear momentum are $\text{kg} \cdot \text{m/s}$ while units for angular momentum are $\text{kg} \cdot \text{m}^2\text{/s}$. As we would expect, an object that has a large moment of inertia I, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

Making Connections

Angular momentum is completely analogous to linear momentum, first presented in **Uniform Circular Motion and Gravitation (https://legacy.cnx.org/content/m42140/latest/)**. It has the same implications in terms of carrying rotation forward, and it is conserved when the net external torque is zero. Angular momentum, like linear momentum, is also a property of the atoms and subatomic particles.

Example 7.4 Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to **Figure 7.5**, the formula for the moment of inertia of a sphere is

$$I = \frac{2MR^2}{5} \tag{7.33}$$

so that

$$L = I\omega = \frac{2MR^2\omega}{5}. \tag{7.34}$$

Earth's mass M is $5.979\times10^{24}\ \text{kg}$ and its radius R is $6.376\times10^6\ \text{m}$. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must covert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

$$\begin{aligned} L &= 0.4\left(5.979\times10^{24}\ \text{kg}\right)\left(6.376\times10^{6}\ \text{m}\right)^2\left(\frac{1\ \text{rev}}{\text{d}}\right) \\ &= 9.72\times10^{37}\ \text{kg}\cdot\text{m}^2\cdot\text{rev/d}. \end{aligned} \tag{7.35}$$

Substituting 2π rad for 1 rev and 8.64×10^4 s for 1 day gives

$$\begin{aligned} L &= \left(9.72\times10^{37}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{2\pi\ \text{rad/rev}}{8.64\times10^4\ \text{s/d}}\right)(1\ \text{rev/d}) \\ &= 7.07\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}. \end{aligned} \tag{7.36}$$

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L. The relationship between torque and angular momentum is

$$\text{net}\ \tau = \frac{\Delta L}{\Delta t}. \tag{7.37}$$

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p / \Delta t$. The equation $\text{net}\ \tau = \frac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.

Example 7.5 Calculating the Torque Putting Angular Momentum Into a Lazy Susan

Figure 7.11 shows a Lazy Susan food tray being rotated by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?

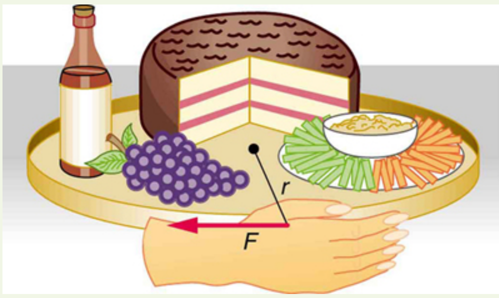


Figure 7.11 A partygoer exerts a torque on a lazy Susan to make it rotate. The equation $\text{net}\ \tau = \frac{\Delta L}{\Delta t}$ gives the relationship between torque and the angular momentum produced.

Strategy

We can find the angular momentum by solving $\text{net}\ \tau = \frac{\Delta L}{\Delta t}$ for ΔL, and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving $\text{net}\ \tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

$$\Delta L = (\text{net}\ \tau)\Delta \text{t}. \tag{7.38}$$

Because the force is perpendicular to r, we see that $\text{net}\ \tau = rF$, so that

$$\begin{aligned} L &= \text{rF}\Delta t = (0.260\ \text{m})(2.50\ \text{N})(0.150\ \text{s}) \\ &= 9.75\times10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}. \end{aligned} \tag{7.39}$$

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

$$L = I\omega. \tag{7.40}$$

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}. \tag{7.41}$$

And substituting known values into the preceding equation yields

$$\omega = \frac{9.75\times10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}}{(0.500)(4.00\ \text{kg})(0.260\ \text{m})} = 0.721\ \text{rad/s}. \tag{7.42}$$

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

Example 7.6 Calculating the Torque in a Kick

The person whose leg is shown in Figure 7.12 kicks his leg by exerting a 2000-N force with his upper leg muscle. The effective perpendicular lever arm is 2.20 cm. Given the moment of inertia of the lower leg is $1.25\ \text{kg}\cdot\text{m}^2$, (a) find the angular acceleration of the leg. (b) Neglecting the gravitational force, what is the rotational kinetic energy of the leg after it has rotated through 57.3° (1.00 rad)?

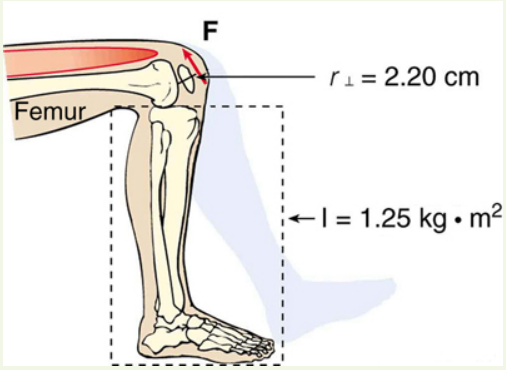


Figure 7.12 The muscle in the upper leg gives the lower leg an angular acceleration and imparts rotational kinetic energy to it by exerting a torque about the knee. $\mathbf{F}$ is a vector that is perpendicular to r. This example examines the situation.

Strategy

The angular acceleration can be found using the rotational analog to Newton's second law, or $\alpha = \text{net}\ \tau / I$. The moment of inertia I is given and the torque can be found easily from the given force and perpendicular lever arm. Once the angular acceleration α is known, the final angular velocity and rotational kinetic energy can be calculated.

Solution to (a)

From the rotational analog to Newton's second law, the angular acceleration α is

$$\alpha = \frac{\text{net}\ \tau}{I}. \tag{7.43}$$

Because the force and the perpendicular lever arm are given and the leg is vertical so that its weight does not create a torque, the net torque is thus

$$\begin{aligned} \text{net}\ \tau &= r_\perp\ F \\ &= (0.0220\ \text{m})(2000\ \text{N}) \\ &= 44.0\ \text{N}\cdot\text{m}. \end{aligned} \tag{7.44}$$

Substituting this value for the torque and the given value for the moment of inertia into the expression for α gives

$$\alpha = \frac{44.0\ \text{N} \cdot \text{m}}{1.25\ \text{kg} \cdot \text{m}^2} = 35.2\ \text{rad/s}^2. \quad (7.45)$$

Solution to (b)

The final angular velocity can be calculated from the kinematic expression

$$\omega^2 = \omega_0{}^2 + 2\alpha\theta \quad (7.46)$$

or

$$\omega^2 = 2\alpha\theta \quad (7.47)$$

because the initial angular velocity is zero. The kinetic energy of rotation is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2 \quad (7.48)$$

so it is most convenient to use the value of ω^2 just found and the given value for the moment of inertia. The kinetic energy is then

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5\left(1.25\ \text{kg} \cdot \text{m}^2\right)\left(70.4\ \text{rad}^2/\text{s}^2\right) \\ &= 44.0\ \text{J} \end{aligned} \quad (7.49)$$

Discussion

These values are reasonable for a person kicking his leg starting from the position shown. The weight of the leg can be neglected in part (a) because it exerts no torque when the center of gravity of the lower leg is directly beneath the pivot in the knee. In part (b), the force exerted by the upper leg is so large that its torque is much greater than that created by the weight of the lower leg as it rotates. The rotational kinetic energy given to the lower leg is enough that it could give a ball a significant velocity by transferring some of this energy in a kick.

Making Connections: Conservation Laws

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net } \tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

$$\text{net}\tau = 0 \quad (7.50)$$

implying that

$$\frac{\Delta L}{\Delta t} = 0. \quad (7.51)$$

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

$$L = \text{constant } (\text{net } \tau = 0) \quad (7.52)$$

or

$$L = L'(\text{net}\tau = 0). \quad (7.53)$$

These expressions are the **law of conservation of angular momentum**. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in **Figure 7.13**, in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

$$L = L'. \tag{7.54}$$

Expressing this equation in terms of the moment of inertia,

$$I\omega = I'\omega', \tag{7.55}$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity ω' must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.

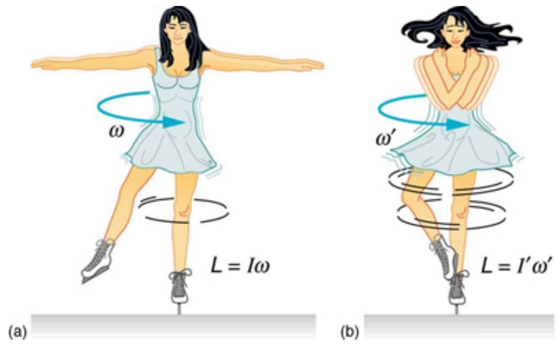


Figure 7.13 (a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

Example 7.7 Calculating the Angular Momentum of a Spinning Skater

Suppose an ice skater, such as the one in **Figure 7.13**, is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of $2.34 \text{ kg} \cdot \text{m}^2$ with her arms extended and of $0.363 \text{ kg} \cdot \text{m}^2$ with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy

In the first part of the problem, we are looking for the skater's angular velocity ω' after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{7.56}$$

Solution for (a)

Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I'\omega'$ is applicable. Thus,

$$L = L' \tag{7.57}$$

or

$$I\omega = I'\omega' \tag{7.58}$$

Solving for ω' and substituting known values into the resulting equation gives

$$\begin{aligned} \omega' &= \frac{I}{I'}\omega = \left(\frac{2.34 \text{ kg} \cdot \text{m}^2}{0.363 \text{ kg} \cdot \text{m}^2}\right)(0.800 \text{ rev/s}) \\ &= 5.16 \text{ rev/s}. \end{aligned} \tag{7.59}$$

Solution for (b)

Rotational kinetic energy is given by

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2. \tag{7.60}$$

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

$$\begin{aligned} KE_{rot} &= (0.5)\left(2.34\ \text{kg}\cdot\text{m}^2\right)\left((0.800\ \text{rev/s})(2\pi\ \text{rad/rev})\right)^2 \\ &= 29.6\ \text{J}. \end{aligned} \tag{7.61}$$

The final rotational kinetic energy is

$$KE_{rot}{}' = \frac{1}{2}I'\omega'^2. \tag{7.62}$$

Substituting known values into this equation gives

$$\begin{aligned} KE_{rot}{}' &= (0.5)\left(0.363\ \text{kg}\cdot\text{m}^2\right)\left[(5.16\ \text{rev/s})(2\pi\ \text{rad/rev})\right]^2 \\ &= 191\ \text{J}. \end{aligned} \tag{7.63}$$

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See **Figure 7.14**.)

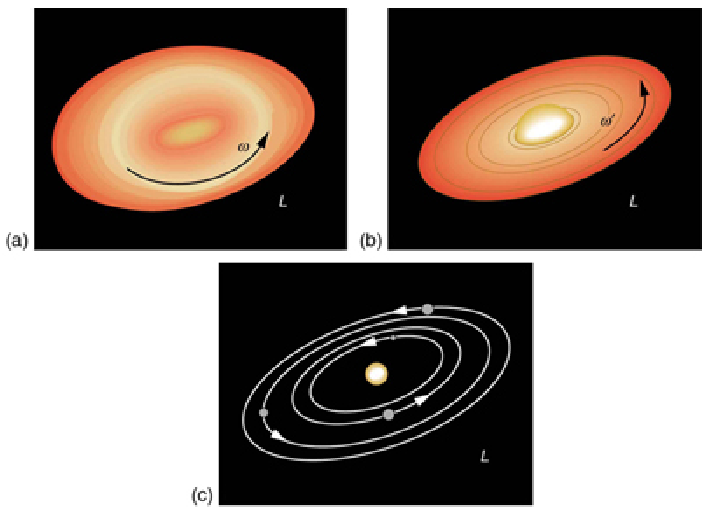


Figure 7.14 The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

Check Your Undestanding

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Solution

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

7.4 Gyroscopic Effects: Vector Aspects of Angular Momentum

Angular momentum is a vector and, therefore, *has direction as well as magnitude*. Torque affects both the direction and the magnitude of angular momentum. What is the direction of the angular momentum of a rotating object like the disk in **Figure 7.15**? The figure shows the **right-hand rule** used to find the direction of both angular momentum and angular velocity. Both $\mathbf{L}$ and $\boldsymbol{\omega}$ are vectors—each has direction and magnitude. Both can be represented by arrows. The right-hand rule defines both to be perpendicular to the plane of rotation in the direction shown. Because angular momentum is related to angular velocity by $\mathbf{L} = I\boldsymbol{\omega}$, the direction of $\mathbf{L}$ is the same as the direction of $\boldsymbol{\omega}$. Notice in the figure that both point along the axis of rotation.

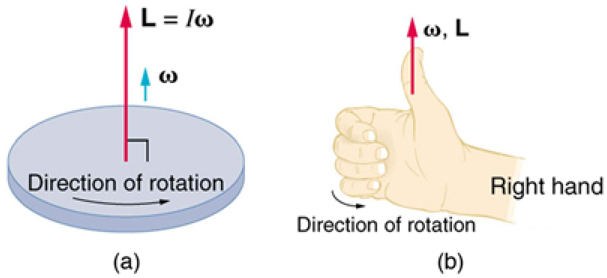


Figure 7.15 Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity $\boldsymbol{\omega}$ size and angular momentum $\mathbf{L}$ are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation as shown.

Now, recall that torque changes angular momentum as expressed by

$$\text{net } \boldsymbol{\tau} = \frac{\Delta \mathbf{L}}{\Delta t}. \tag{7.64}$$

This equation means that the direction of $\Delta\mathbf{L}$ is the same as the direction of the torque $\boldsymbol{\tau}$ that creates it. This result is illustrated in **Figure 7.16**, which shows the direction of torque and the angular momentum it creates.

Let us now consider a bicycle wheel with a couple of handles attached to it, as shown in **Figure 7.17**. (This device is popular in demonstrations among physicists, because it does unexpected things.) With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it—but what happens is quite different. The forces exerted create a torque that is horizontal toward the person, as shown in **Figure 7.17**(a). This torque creates a change in angular momentum $\mathbf{L}$ in the same direction, perpendicular to the original angular momentum $\mathbf{L}$, thus changing the direction of $\mathbf{L}$ but not the magnitude of $\mathbf{L}$. **Figure 7.17** shows how $\Delta\mathbf{L}$ and $\mathbf{L}$ add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved *perpendicular to the forces exerted on it*, instead of in the expected direction.

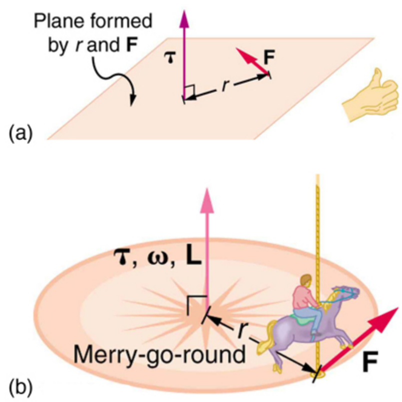


Figure 7.16 In figure (a), the torque is perpendicular to the plane formed by r and $\mathbf{F}$ and is the direction your right thumb would point to if you curled your fingers in the direction of $\mathbf{F}$. Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

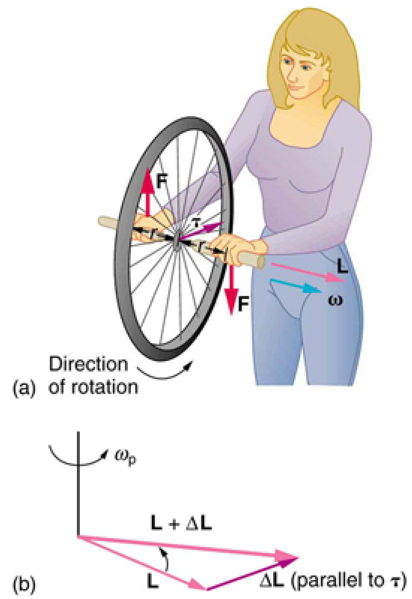


Figure 7.17 In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum $\Delta\mathbf{L}$ in exactly the same direction. Figure (b) shows a vector diagram depicting how $\Delta\mathbf{L}$ and $\mathbf{L}$ add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

This same logic explains the behavior of gyroscopes. **Figure 7.18** shows the two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the torque is changed, but not its magnitude. The gyroscope *precesses* around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is *not* spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates around a horizontal axis, falling over just as we would expect.

Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star. But Earth is slowly precessing (once in about 26,000 years) due to the torque of the Sun and the Moon on its nonspherical shape.

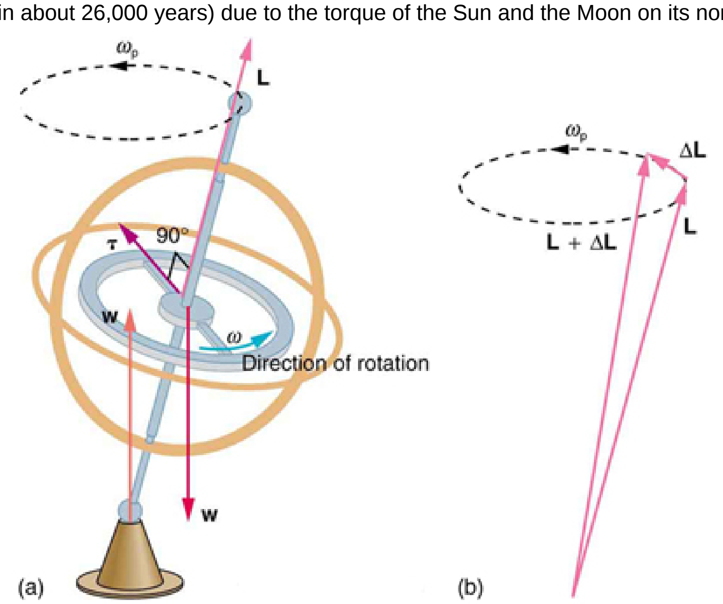


Figure 7.18 As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum $\Delta\mathbf{L}$ that is also horizontal. In figure (b), $\Delta\mathbf{L}$ and $\mathbf{L}$ add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Check Your Understanding

Rotational kinetic energy is associated with angular momentum? Does that mean that rotational kinetic energy is a vector?

Solution
No, energy is always a scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Glossary

angular momentum: the product of moment of inertia and angular velocity

law of conservation of angular momentum: angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system

moment of inertia: mass times the square of perpendicular distance from the rotation axis; for a point mass, it is $I = mr^2$ and, because any object can be built up from a collection of point masses, this relationship is the basis for all other moments of inertia

right-hand rule: direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation

rotational inertia: resistance to change of rotation. The more rotational inertia an object has, the harder it is to rotate

rotational kinetic energy: the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

torque: the turning effectiveness of a force

Section Summary

7.1 Dynamics of Rotational Motion: Rotational Inertia

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- If we exert a force F on a point mass m that is at a distance r from a pivot point and because the force is perpendicular to r, an acceleration $a = F/m$ is obtained in the direction of F. We can rearrange this equation such that
$$F = ma,$$
and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F=ma$, yielding
$$F=mr\alpha$$
- Torque is the turning effectiveness of a force. In this case, because F is perpendicular to r, torque is simply $\tau = rF$. If we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,
$$rF = mr^2\alpha$$
or
$$\tau = mr^2\alpha.$$
- The moment of inertia I of an object is the sum of MR^2 for all the point masses of which it is composed. That is,
$$I = \sum mr^2.$$
- The general relationship among torque, moment of inertia, and angular acceleration is
$$\tau = I\alpha$$
or
$$\alpha = \frac{\text{net } \tau}{I}.$$

7.2 Rotational Kinetic Energy

- The rotational kinetic energy KE_{rot} for an object with a moment of inertia I and an angular velocity ω is given by
$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$
- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.

7.3 Angular Momentum and Its Conservation

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of linear momentum as $p = mv$. The relationship between torque and angular momentum is $\text{net } \tau = \frac{\Delta L}{\Delta t}$.
- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

7.4 Gyroscopic Effects: Vector Aspects of Angular Momentum

- Torque is perpendicular to the plane formed by r and $\mathbf{F}$ and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of $\mathbf{F}$. The direction of the torque is thus the same as that of the angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta \mathbf{L}$), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.

Conceptual Questions

7.1 Dynamics of Rotational Motion: Rotational Inertia

1. The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)

2. Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?

3. Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.

4. While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to achieve greater accelerations than would an identical reduction in the mass of the bicycle's frame?

Figure 7.19 The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? (credit: Jesús Rodriguez)

5. A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

7.2 Rotational Kinetic Energy

6. Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user's hand.

7. What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.

8. The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?

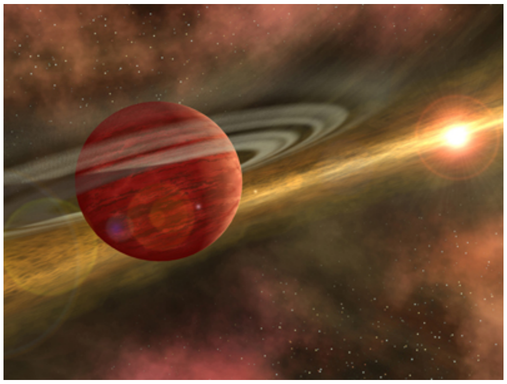


Figure 7.20 An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. (credit: NASA)

7.3 Angular Momentum and Its Conservation

9. When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine's rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?

10. Suppose a child walks from the outer edge of a rotating merry-go round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.

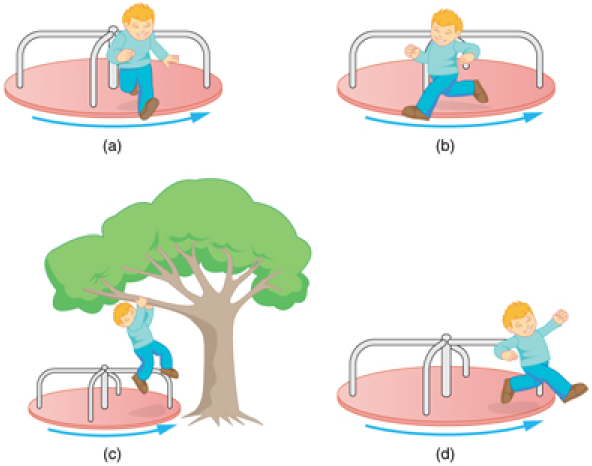


Figure 7.21 A child may jump off a merry-go-round in a variety of directions.

11. Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to **Figure 7.21**).

12. Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton's third law why the helicopter body rotates in the opposite direction to the blades.

13. Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.

14. Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?

15. When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.

16. Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?

17. Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.

18. An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.

19. Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain the effect on their angular momenta.

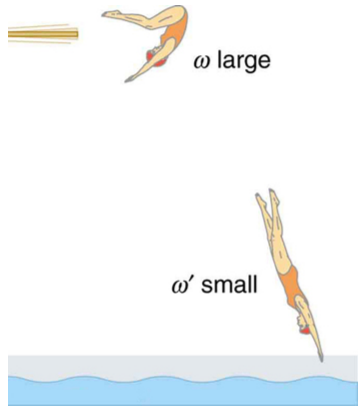


Figure 7.22 The diver spins rapidly when curled up and slows when she extends her limbs before entering the water.

20. Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.

21. In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?

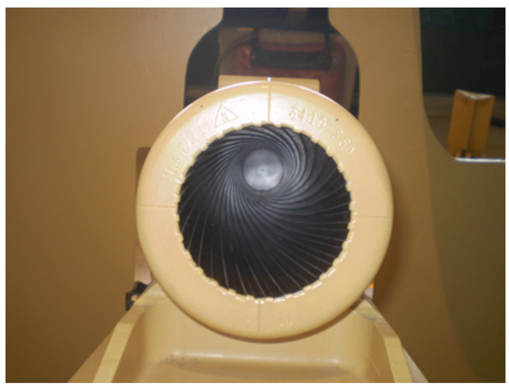


Figure 7.23 The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). (credit: Elsie esq., Flickr)

7.4 Gyroscopic Effects: Vector Aspects of Angular Momentum

22. While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

23. Gyroscopes used in guidance systems to indicate directions in space must have an angular momentum that does not change in direction. Yet they are often subjected to large forces and accelerations. How can the direction of their angular momentum be constant when they are accelerated?

Problems & Exercises

7.1 Dynamics of Rotational Motion: Rotational Inertia

1. This problem considers additional aspects of example **Calculating the Effect of Mass Distribution on a Merry-Go-Round**. (a) How long does it take the father to give the merry-go-round an angular velocity of 1.50 rad/s? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m, how long would it take him to stop them?

2. Calculate the moment of inertia of a skater given the following information. (a) The 60.0-kg skater is approximated as a cylinder that has a 0.110-m radius. (b) The skater with arms extended is approximately a cylinder that is 52.5 kg, has a 0.110-m radius, and has two 0.900-m-long arms which are 3.75 kg each and extend straight out from the cylinder like rods rotated about their ends.

3. The triceps muscle in the back of the upper arm extends the forearm. This muscle in a professional boxer exerts a force of $2.00\times10^{3}\ \mathrm{N}$ with an effective perpendicular lever arm of 3.00 cm, producing an angular acceleration of the forearm of $120\ \mathrm{rad/s}^2$. What is the moment of inertia of the boxer's forearm?

4. A soccer player extends her lower leg in a kicking motion by exerting a force with the muscle above the knee in the front of her leg. She produces an angular acceleration of $30.00\ \mathrm{rad/s}^2$ and her lower leg has a moment of inertia of $0.750\ \mathrm{kg\cdot m}^2$. What is the force exerted by the muscle if its effective perpendicular lever arm is 1.90 cm?

5. Suppose you exert a force of 180 N tangential to a 0.280-m-radius 75.0-kg grindstone (a solid disk).

(a)What torque is exerted? (b) What is the angular acceleration assuming negligible opposing friction? (c) What is the angular acceleration if there is an opposing frictional force of 20.0 N exerted 1.50 cm from the axis?

6. Consider the 12.0 kg motorcycle wheel shown in **Figure 7.24**. Assume it to be approximately an annular ring with an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. (a) If the drive chain exerts a force of 2200 N at a radius of 5.00 cm, what is the angular acceleration of the wheel? (b) What is the tangential acceleration of a point on the outer edge of the tire? (c) How long, starting from rest, does it take to reach an angular velocity of 80.0 rad/s?

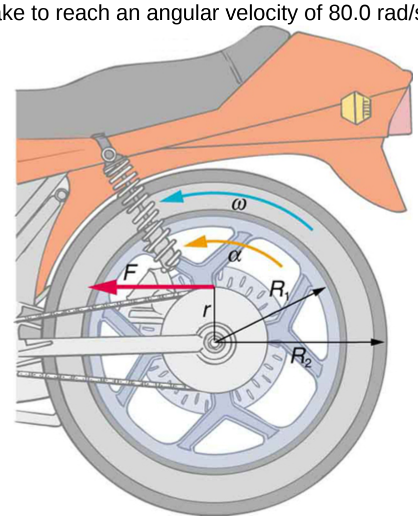


Figure 7.24 A motorcycle wheel has a moment of inertia approximately that of an annular ring.

7. Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of $4.00\times10^{7}\ \mathrm{N}$ (a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in **Problem-Solving Strategy for Rotational Dynamics**.

8. An automobile engine can produce 200 N · m of torque. Calculate the angular acceleration produced if 95.0% of this torque is applied to the drive shaft, axle, and rear wheels of a car, given the following information. The car is suspended so that the wheels can turn freely. Each wheel acts like a 15.0 kg disk that has a 0.180 m radius. The walls of each tire act like a 2.00-kg annular ring that has inside radius of 0.180 m and outside radius of 0.320 m. The tread of each tire acts like a 10.0-kg hoop of radius 0.330 m. The 14.0-kg axle acts like a rod that has a 2.00-cm radius. The 30.0-kg drive shaft acts like a rod that has a 3.20-cm radius.

9. Starting with the formula for the moment of inertia of a rod rotated around an axis through one end perpendicular to its length $\left(I = M\ell^{2}/3\right)$, prove that the moment of inertia of a rod rotated about an axis through its center perpendicular to its length is $I = M\ell^{2}/12$. You will find the graphics in **Figure 7.5** useful in visualizing these rotations.

10. Unreasonable Results

A gymnast doing a forward flip lands on the mat and exerts a 500-N · m torque to slow and then reverse her angular velocity. Her initial angular velocity is 10.0 rad/s, and her moment of inertia is $0.050\ \text{kg} \cdot \text{m}^2$. (a) What time is required for her to exactly reverse her spin? (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

11. Unreasonable Results

An advertisement claims that an 800-kg car is aided by its 20.0-kg flywheel, which can accelerate the car from rest to a speed of 30.0 m/s. The flywheel is a disk with a 0.150-m radius. (a) Calculate the angular velocity the flywheel must have if 95.0% of its rotational energy is used to get the car up to speed. (b) What is unreasonable about the result? (c) Which premise is unreasonable or which premises are inconsistent?

7.2 Rotational Kinetic Energy

12. What is the final velocity of a hoop that rolls without slipping down a 5.00-m-high hill, starting from rest?

13. (a) Calculate the rotational kinetic energy of Earth on its axis. (b) What is the rotational kinetic energy of Earth in its orbit around the Sun?

14. A baseball pitcher throws the ball in a motion where there is rotation of the forearm about the elbow joint as well as other movements. If the linear velocity of the ball relative to the elbow joint is 20.0 m/s at a distance of 0.480 m from the joint and the moment of inertia of the forearm is $0.500\ \text{kg} \cdot \text{m}^2$, what is the rotational kinetic energy of the forearm?

15. While punting a football, a kicker rotates his leg about the hip joint. The moment of inertia of the leg is $3.75\ \text{kg} \cdot \text{m}^2$ and its rotational kinetic energy is 175 J. (a) What is the angular velocity of the leg? (b) What is the velocity of tip of the punter's shoe if it is 1.05 m from the hip joint? (c) Explain how the football can be given a velocity greater than the tip of the shoe (necessary for a decent kick distance).

16. A bus contains a 1500 kg flywheel (a disk that has a 0.600 m radius) and has a total mass of 10,000 kg. (a) Calculate the angular velocity the flywheel must have to contain enough energy to take the bus from rest to a speed of 20.0 m/s, assuming 90.0% of the rotational kinetic energy can be transformed into translational energy. (b) How high a hill can the bus climb with this stored energy and still have a speed of 3.00 m/s at the top of the hill?

17. A ball with an initial velocity of 8.00 m/s rolls up a hill without slipping. Treating the ball as a spherical shell, calculate the vertical height it reaches. (b) Repeat the calculation for the same ball if it slides up the hill without rolling.

18. Consider two cylinders that start down identical inclines from rest except that one is frictionless. Thus one cylinder rolls without slipping, while the other slides frictionlessly without rolling. They both travel a short distance at the bottom and then start up another incline. (a) Show that they both reach the same height on the other incline, and that this height is equal to their original height. (b) Find the ratio of the time the rolling cylinder takes to reach the height on the second incline to the time the sliding cylinder takes to reach the height on the second incline. (c) Explain why the time for the rolling motion is greater than that for the sliding motion.

19. What is the moment of inertia of an object that rolls without slipping down a 2.00-m-high incline starting from rest, and has a final velocity of 6.00 m/s? Express the moment of inertia as a multiple of MR^2, where M is the mass of the object and R is its radius.

20. In softball, the pitcher throws with the arm fully extended (straight at the elbow). In a fast pitch the ball leaves the hand with a speed of 139 km/h. (a) Find the rotational kinetic energy of the pitcher's arm given its moment of inertia is $0.720\ \text{kg} \cdot \text{m}^2$ and the ball leaves the hand at a distance of 0.600 m from the pivot at the shoulder. (b) What force did the muscles exert to cause the arm to rotate if their effective perpendicular lever arm is 4.00 cm and the ball is 0.156 kg?

21. Construct Your Own Problem

Consider the work done by a spinning skater pulling her arms in to increase her rate of spin. Construct a problem in which you calculate the work done with a "force multiplied by distance" calculation and compare it to the skater's increase in kinetic energy.

7.3 Angular Momentum and Its Conservation

22. (a) Calculate the angular momentum of the Earth in its orbit around the Sun.

(b) Compare this angular momentum with the angular momentum of Earth on its axis.

23. (a) What is the angular momentum of the Moon in its orbit around Earth?

(b) How does this angular momentum compare with the angular momentum of the Moon on its axis? Remember that the Moon keeps one side toward Earth at all times.

(c) Discuss whether the values found in parts (a) and (b) seem consistent with the fact that tidal effects with Earth have caused the Moon to rotate with one side always facing Earth.

24. Suppose you start an antique car by exerting a force of 300 N on its crank for 0.250 s. What angular momentum is given to the engine if the handle of the crank is 0.300 m from the pivot and the force is exerted to create maximum torque the entire time?

25. A playground merry-go-round has a mass of 120 kg and a radius of 1.80 m and it is rotating with an angular velocity of 0.500 rev/s. What is its angular velocity after a 22.0-kg child gets onto it by grabbing its outer edge? The child is initially at rest.

26. Three children are riding on the edge of a merry-go-round that is 100 kg, has a 1.60-m radius, and is spinning at 20.0 rpm. The children have masses of 22.0, 28.0, and 33.0 kg. If the child who has a mass of 28.0 kg moves to the center of the merry-go-round, what is the new angular velocity in rpm?

27. (a) Calculate the angular momentum of an ice skater spinning at 6.00 rev/s given his moment of inertia is $0.400 \text{ kg} \cdot \text{m}^2$. (b) He reduces his rate of spin (his angular velocity) by extending his arms and increasing his moment of inertia. Find the value of his moment of inertia if his angular velocity decreases to 1.25 rev/s. (c) Suppose instead he keeps his arms in and allows friction of the ice to slow him to 3.00 rev/s. What average torque was exerted if this takes 15.0 s?

28. Consider the Earth-Moon system. Construct a problem in which you calculate the total angular momentum of the system including the spins of the Earth and the Moon on their axes and the orbital angular momentum of the Earth-Moon system in its nearly monthly rotation. Calculate what happens to the Moon's orbital radius if the Earth's rotation decreases due to tidal drag. Among the things to be considered are the amount by which the Earth's rotation slows and the fact that the Moon will continue to have one side always facing the Earth.

7.4 Gyroscopic Effects: Vector Aspects of Angular Momentum

29. Integrated Concepts

The axis of Earth makes a 23.5° angle with a direction perpendicular to the plane of Earth's orbit. As shown in **Figure 7.25**, this axis precesses, making one complete rotation in 25,780 y.

(a) Calculate the change in angular momentum in half this time.

(b) What is the average torque producing this change in angular momentum?

(c) If this torque were created by a single force (it is not) acting at the most effective point on the equator, what would its magnitude be?

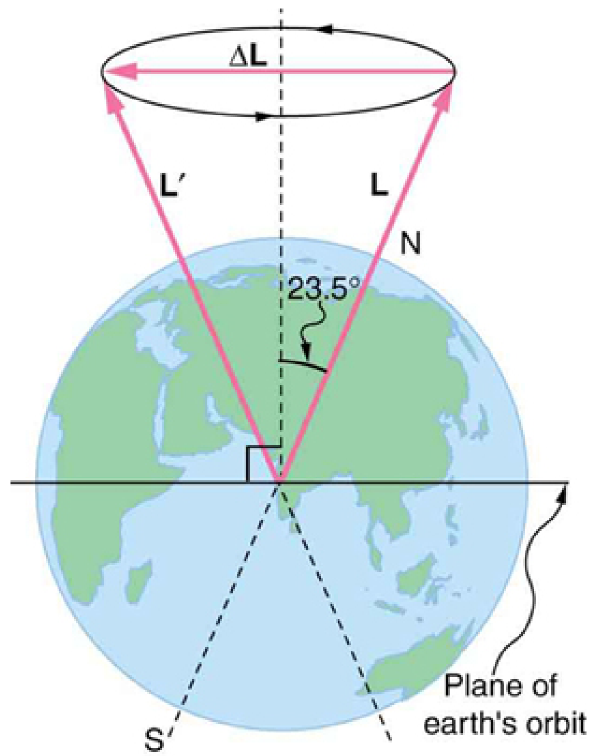


Figure 7.25 The Earth's axis slowly precesses, always making an angle of 23.5° with the direction perpendicular to the plane of Earth's orbit. The change in angular momentum for the two shown positions is quite large, although the magnitude $\mathbf{L}$ is unchanged.

8 FLUIDS

Figure 8.1 The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

Chapter Outline

8.1. What Is a Fluid?
- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

8.2. Density
- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

8.3. Pressure
- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

8.4. Archimedes' Principle
- Define buoyant force.
- State Archimedes' principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes' principle.

8.5. Flow Rate and Its Relation to Velocity
- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

8.6. Bernoulli's Equation
- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

Introduction to Fluids

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of fluids and some of the laws that govern their behavior are the topics of this chapter.

Also by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—fluid dynamics—allows us to answer these and many other questions.

8.1 What Is a Fluid?

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See **Figure 8.2**.) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.

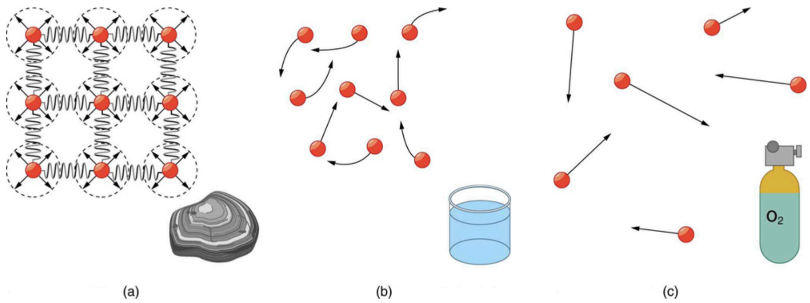


Figure 8.2 (a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

Connections: Submicroscopic Explanation of Solids and Liquids

Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

8.2 Density

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See **Figure 8.3**.)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}, \tag{8.1}$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Density

Density is mass per unit volume.

$$\rho = \frac{m}{V}, \tag{8.2}$$

where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3, representative values are given in **Table 8.1**. The metric system was originally devised so that water would have a density of $1\ \text{g/cm}^3$, equivalent to $10^3\ \text{kg/m}^3$. Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Table 8.1 Densities of Various Substances

Substance	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	Substance	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	Substance	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	1.29×10^{-3}
Brass	8.44	Blood	1.05	Carbon dioxide	1.98×10^{-3}
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	1.25×10^{-3}
Gold	19.32	Mercury	13.6	Hydrogen	0.090×10^{-3}
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	0.18×10^{-3}
Lead	11.3	Petrol	0.68	Methane	0.72×10^{-3}
Polystyrene	0.10	Glycerin	1.26	Nitrogen	1.25×10^{-3}
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	1.98×10^{-3}
Uranium	18.70			Oxygen	1.43×10^{-3}
Concrete	2.30–3.0			Steam $(100^\circ\ \text{C})$	0.60×10^{-3}
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth’s crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				

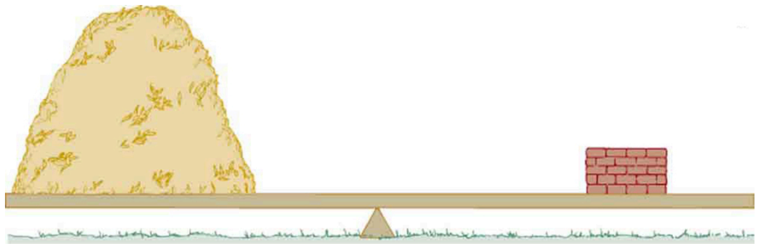


Figure 8.3 A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining **Table 8.1**, the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Example 8.1 Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of $50.0\ \text{km}^2$ and an average depth of 40.0 m. What mass of water is held behind the dam? (See **Figure 8.4** for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in **Table 8.1**. Then the mass m can be found from the definition of density

$$\rho = \frac{m}{V}. \tag{8.3}$$

Solution

Solving equation $\rho = m/V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h:

$$\begin{aligned} V &= Ah = \left(50.0\ \text{km}^2\right)(40.0\ \text{m}) \\ &= \left[\left(50.0\ \text{km}^2\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)^2\right](40.0\ \text{m}) = 2.00\times10^9\ \text{m}^3 \end{aligned} \tag{8.4}$$

The density of water ρ from **Table 8.1** is $1.000\times10^3\ \text{kg/m}^3$. Substituting V and ρ into the expression for mass gives

$$\begin{aligned} m &= \left(1.00\times10^3\ \text{kg/m}^3\right)\left(2.00\times10^9\ \text{m}^3\right) \\ &= 2.00\times10^{12}\ \text{kg}. \end{aligned} \tag{8.5}$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $mg = 1.96\times10^{13}\ \text{N}$, where g is the acceleration due to the Earth's gravity (about $9.80\ \text{m/s}^2$). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.

Figure 8.4 Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

8.3 Pressure

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

$$P = \frac{F}{A} \tag{8.6}$$

where F is a force applied to an area A that is perpendicular to the force.

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

$$P = \frac{F}{A}. \tag{8.7}$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in **Figure 8.5**. The SI unit for pressure is the *pascal*, where

$$1\ \text{Pa} = 1\ \text{N/m}^2. \tag{8.8}$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

$$100\ \text{mb} = 1\times10^5\ \text{Pa}\ . \tag{8.9}$$

Pounds per square inch $\left(\text{lb/in}^2\ \text{or psi}\right)$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.

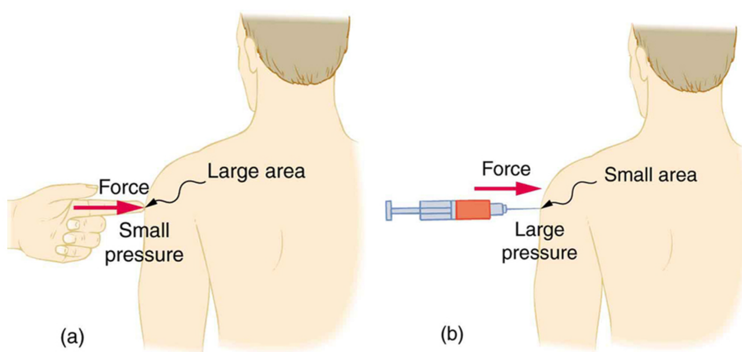


Figure 8.5 (a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

Example 8.2 Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads $6.90\times10^6 \text{ Pa}$. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P=\frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

$$F = PA. \tag{8.10}$$

Here, the pressure P is given, as is the area of the end of the cylinder A, given by $A=\pi r^2$. Thus,

$$\begin{aligned} F &= \left(6.90\times10^6 \text{ N/m}^2\right)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22\times10^5 \text{ N}. \end{aligned} \tag{8.11}$$

Discussion

Wow! No wonder the tank must be strong. Since we found $F = PA$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in **Figure 8.6**, for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See **Figure 8.7**.)

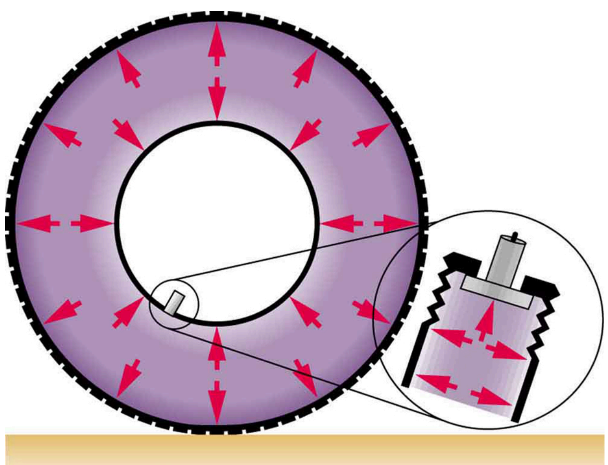


Figure 8.6 Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.

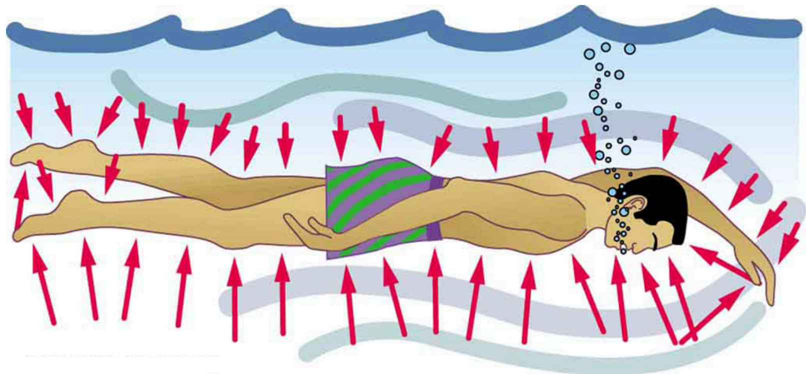
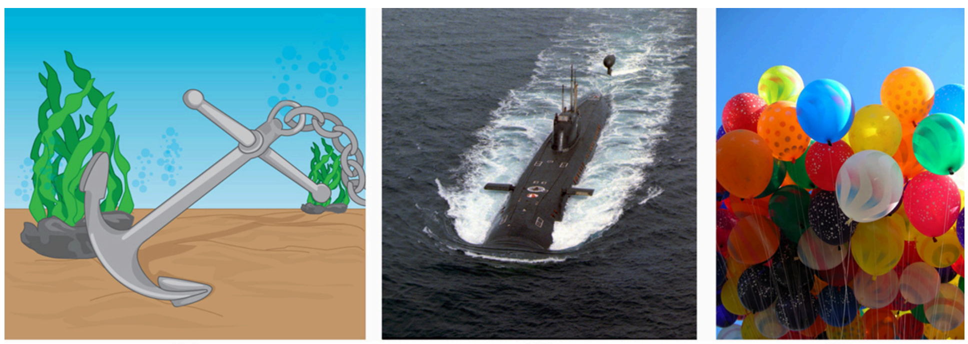


Figure 8.7 Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth, giving a net upward or buoyant force that is balanced by the weight of the swimmer.

8.4 Archimedes' Principle

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See **Figure 8.8**.)



(a) (b) (c)

Figure 8.8 (a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See **Figure 8.9**.) If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

Buoyant Force

The buoyant force is the net upward force on any object in any fluid.

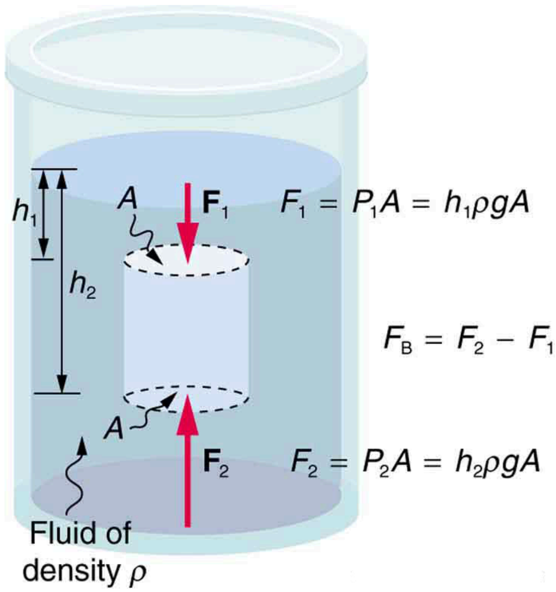


Figure 8.9 Pressure due to the weight of a fluid increases with depth since $P = h\rho g$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant force $\mathbf{F}_\mathrm{B}$. (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in **Figure 8.10**.

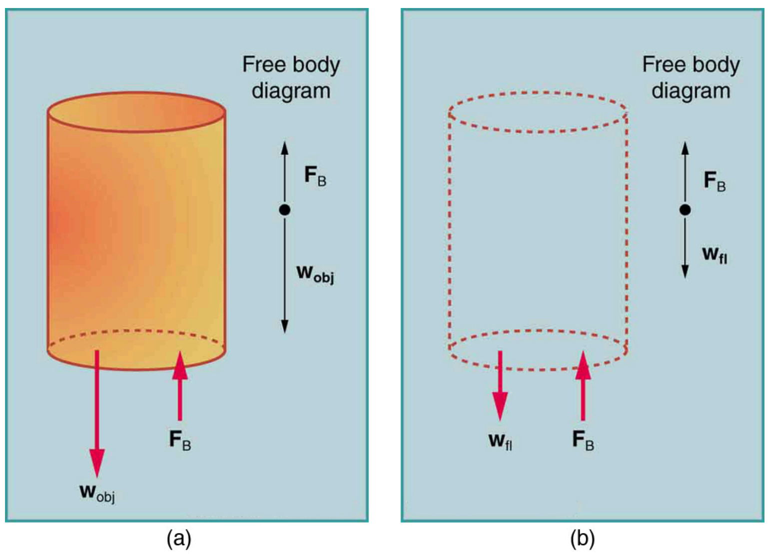


Figure 8.10 (a) An object submerged in a fluid experiences a buoyant force F_B . If F_B is greater than the weight of the object, the object will rise. If F_B is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight w_fl . Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_\mathrm{B} = w_\mathrm{fl}$,a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_fl . This weight is supported by the surrounding fluid, and so the buoyant force must equal w_fl , the weight of the fluid displaced by the object. It is a tribute to the genius of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_\mathrm{B} = w_\mathrm{fl}, \tag{8.12}$$

where F_B is the buoyant force and w_fl is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

$$F_B = w_{fl}, \tag{8.13}$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm … High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

Example 8.3 Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons $(1.00\times10^7 \text{ kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00\times10^5 \text{ m}^3$ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given earlier (see "Density"). We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

$$V_{st} = \frac{m_{st}}{\rho_{st}} = \frac{1.00\times10^7 \text{ kg}}{7.8\times10^3 \text{ kg/m}^3} = 1.28\times10^3 \text{ m}^3. \tag{8.14}$$

Because the steel is completely submerged, this is also the volume of water displaced, V_w. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

$$\begin{aligned} m_w &= \rho_w V_w = (1.000\times10^3 \text{ kg/m}^3)(1.28\times10^3 \text{ m}^3) \\ &= 1.28\times10^6 \text{ kg}. \end{aligned} \tag{8.15}$$

By Archimedes' principle, the weight of water displaced is $m_w g$, so the buoyant force is

$$\begin{aligned} F_B &= w_w = m_w g = \left(1.28\times10^6 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) \\ &= 1.3\times10^7 \text{ N}. \end{aligned} \tag{8.16}$$

The steel's weight is $m_w g = 9.80\times10^7 \text{ N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

$$\begin{aligned} m_w &= \rho_w V_w = \left(1.000\times10^3\ \text{kg/m}^3\right)\left(1.00\times10^5\ \text{m}^3\right) \\ &= 1.00\times10^8\ \text{kg}. \end{aligned} \tag{8.17}$$

The maximum buoyant force is the weight of this much water, or

$$\begin{aligned} F_B &= w_w = m_w g = \left(1.00\times10^8\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) \\ &= 9.80\times10^8\ \text{N}. \end{aligned} \tag{8.18}$$

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Making Connections: Take-Home Investigation

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this “boat,” what shape of the boat would allow it to hold the most “cargo” when placed in water? Test your prediction.

Density and Archimedes’ Principle

Density plays a crucial role in Archimedes’ principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object’s density is related to that of the fluid. In **Figure 8.11**, for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

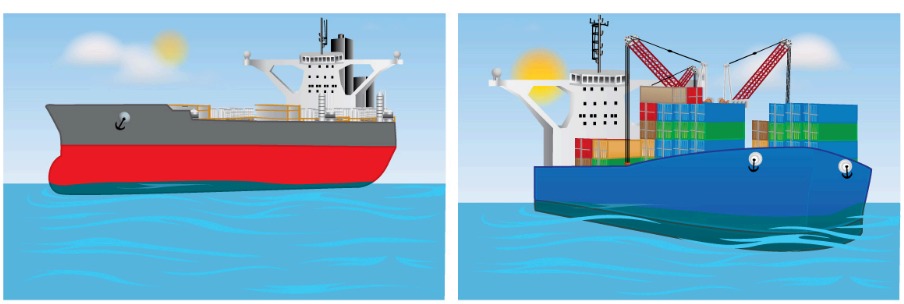
$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}. \tag{8.19}$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl} . Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}} / \rho_{\text{fl}}}{m_{\text{obj}} / \bar{\rho}_{\text{obj}}}, \tag{8.20}$$

where $\bar{\rho}_{\text{obj}}$ is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}. \tag{8.21}$$



(a) (b)

Figure 8.11 An unloaded ship (a) floats higher in the water than a loaded ship (b).

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

$$\text{specific gravity} = \frac{\bar{\rho}}{\rho_{\text{w}}}, \tag{8.22}$$

where $\bar{\rho}$ is the average density of the object or substance and ρ_{w} is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ. If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in **Figure 8.12**.

Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).

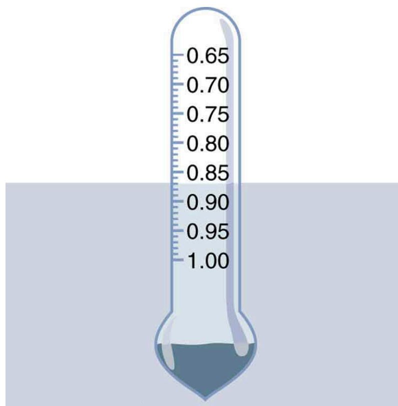


Figure 8.12 This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

Example 8.4 Calculating Average Density: Floating Woman

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}} \tag{8.23}$$

for the density of the object. This yields

$$\bar{\rho}_{\text{obj}} = \bar{\rho}_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}. \tag{8.24}$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

Entering the known values into the expression for her density, we obtain

$$\bar{\rho}_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}. \tag{8.25}$$

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See **Figure 8.13**.)

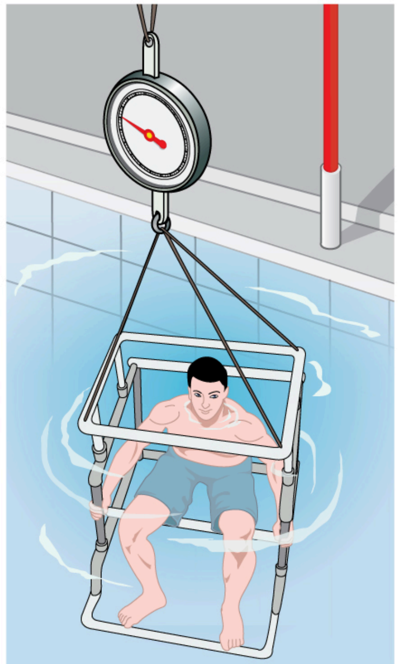


Figure 8.13 Subject in a "fat tank," where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air, and pinch tests of strategic fatty areas are used to calculate his percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a "lava lamp," to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in **Figure 8.14**.

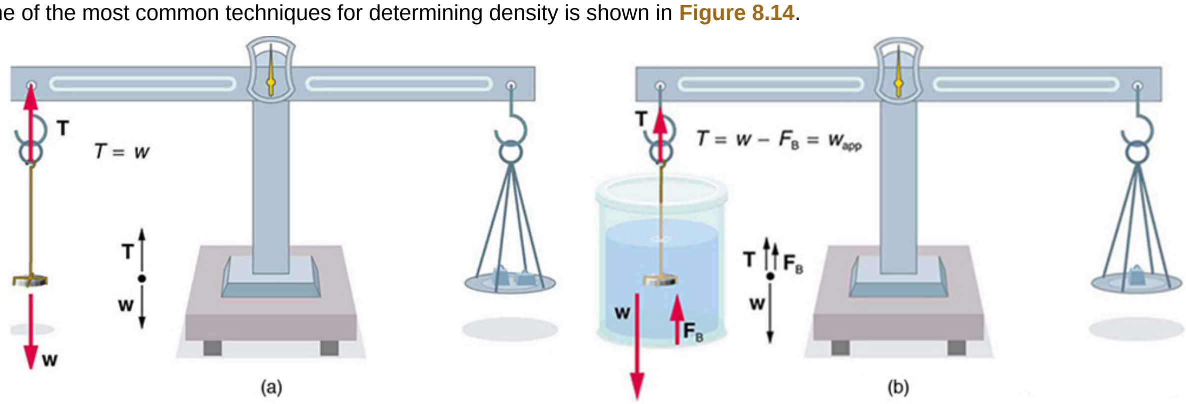


Figure 8.14 (a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

$$\text{apparent weight loss} = \text{weight of fluid displaced} \tag{8.26}$$

or

$$\text{apparent mass loss} = \text{mass of fluid displaced.} \tag{8.27}$$

The next example illustrates the use of this technique.

Example 8.5 Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in Figure 8.14, its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000 g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V.

Solution

The volume of water is $V_\text{w} = \frac{m_\text{w}}{\rho_\text{w}}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_\text{w} = 8.630 \text{ g} - 7.800 \text{ g} = 0.830 \text{ g}$. Thus the volume of water is $V_\text{w} = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830 \text{ cm}^3$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

$$\rho_\text{c} = \frac{m_\text{c}}{V_\text{c}} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 \text{ g/cm}^3. \tag{8.28}$$

Discussion

You see that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

8.5 Flow Rate and Its Relation to Velocity

Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in Figure 8.15. In symbols, this can be written as

$$Q = \frac{V}{t}, \tag{8.29}$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s, but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters (10^{-3} m^3 or 10^3 cm^3). In this text we shall use whatever metric units are most convenient for a given situation.

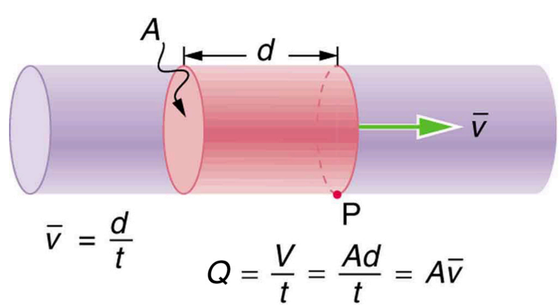


Figure 8.15 Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t. The volume of the cylinder is Ad and the average velocity is $\bar{v} = d/t$ so that the flow rate is $Q = Ad/t = A\bar{v}$.

Example 8.6 Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V / t$ for volume gives

$$V = Qt. \tag{8.30}$$

Substituting known values yields

$$\begin{aligned} V &= \left(\frac{5.00\ \text{L}}{1\ \text{min}}\right)(75\ \text{y})\left(\frac{1\ \text{m}^3}{10^3\ \text{L}}\right)\left(5.26\times10^5\frac{\text{min}}{\text{y}}\right) \\ &= 2.0\times10^5\ \text{m}^3. \end{aligned} \tag{8.31}$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity $\bar{v}$ is

$$Q = A\bar{v}, \tag{8.32}$$

where A is the cross-sectional area and $\bar{v}$ is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. Figure 8.15 illustrates how this relationship is obtained. The shaded cylinder has a volume

$$V = Ad, \tag{8.33}$$

which flows past the point P in a time t. Dividing both sides of this relationship by t gives

$$\frac{V}{t} = \frac{Ad}{t}. \tag{8.34}$$

We note that $Q = V / t$ and the average speed is $\bar{v} = d / t$. Thus the equation becomes $Q = A\bar{v}$.

Figure 8.16 shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1\bar{v}_1 &= A_2\bar{v}_2 \end{aligned}\right\}. \tag{8.35}$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.

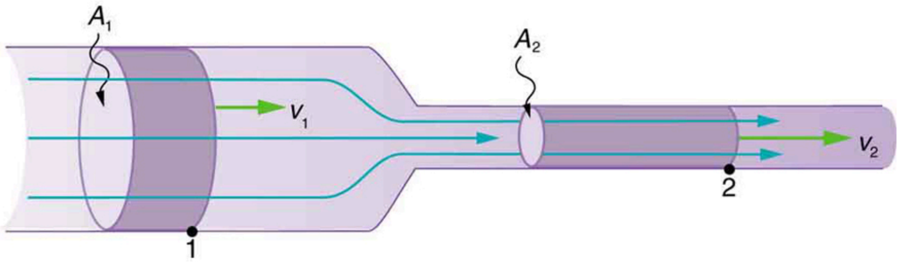


Figure 8.16 When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example 8.7 Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = A\bar{v}$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

$$\bar{v}_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}. \tag{8.36}$$

Substituting known values and making appropriate unit conversions yields

$$\bar{v}_1 = \frac{(0.500\text{ L/s})(10^{-3}\text{ m}^3/\text{L})}{\pi(9.00\times10^{-3}\text{ m})^2} = 1.96\text{ m/s}. \tag{8.37}$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle $\bar{v}_2$, but we will use the equation of continuity to give a somewhat different insight. Using the equation which states

$$A_1\bar{v}_1 = A_2\bar{v}_2, \tag{8.38}$$

solving for $\bar{v}_2$ and substituting πr^2 for the cross-sectional area yields

$$\bar{v}_2 = \frac{A_1}{A_2}\bar{v}_1 = \frac{\pi r_1^2}{\pi r_2^2}\bar{v}_1 = \frac{r_1{}^2}{r_2{}^2}\bar{v}_1. \tag{8.39}$$

Substituting known values,

$$\bar{v}_2 = \frac{(0.900\text{ cm})^2}{(0.250\text{ cm})^2}1.96\text{ m/s} = 25.5\text{ m/s}. \tag{8.40}$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

$$n_1 A_1 \bar{v}_1 = n_2 A_2 \bar{v}_2, \tag{8.41}$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example 8.8 Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is $8.0\ \mu\text{m}$, calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = A\bar{v}$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = A\bar{v}$ or $\bar{v} = \dfrac{Q}{\pi r^2}$ for a cylindrical vessel.

Substituting the known values (converted to units of meters and seconds) gives

$$\bar{v} = \frac{(5.0\ \text{L/min})\left(10^{-3}\ \text{m}^3/\text{L}\right)(1\ \text{min}/60\ \text{s})}{\pi(0.010\ \text{m})^2} = 0.27\ \text{m/s}. \tag{8.42}$$

Solution for (b)

Using $n_1 A_1 \bar{v}_1 = n_2 A_2 \bar{v}_1$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \dfrac{n_1 A_1 \bar{v}_1}{A_2 \bar{v}_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

$$n_2 = \frac{(1)(\pi)\left(10\times10^{-3}\ \text{m}\right)^2(0.27\ \text{m/s})}{(\pi)\left(4.0\times10^{-6}\ \text{m}\right)^2\left(0.33\times10^{-3}\ \text{m/s}\right)} = 5.0\times10^9\ \text{capillaries}. \tag{8.43}$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3, or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

8.6 Bernoulli's Equation

When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \tag{8.44}$$

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See **Figure 8.17**.) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.

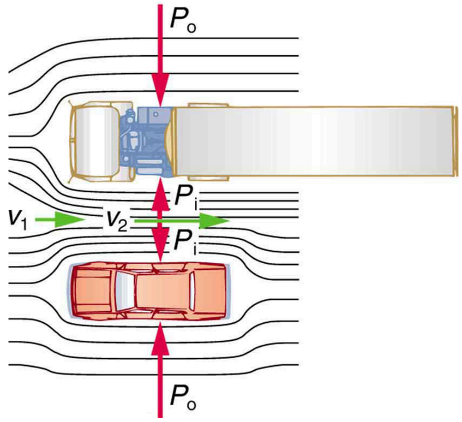


Figure 8.17 An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_i is less than P_o). Greater pressure on the outside pushes the car and truck together.

> **Making Connections: Take-Home Investigation with a Sheet of Paper**
>
> Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by **Bernoulli's equation**, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}, \tag{8.45}$$

where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2. \tag{8.46}$$

Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ . In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting $\rho = m / V$ into it and gathering terms:

$$\frac{1}{2}\rho v^2 = \frac{\frac{1}{2}mv^2}{V} = \frac{\text{KE}}{V}. \tag{8.47}$$

So $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find

$$\rho g h = \frac{mgh}{V} = \frac{\text{PE}_\text{g}}{V}, \tag{8.48}$$

so $\rho g h$ is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since $P = F / A$, its units are N/m^2 . If we multiply these by m/m, we obtain $\text{N} \cdot \text{m/m}^3 = \text{J/m}^3$, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

> **Making Connections: Conservation of Energy**
>
> Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results

in changes in the fluid's KE and $\mathrm{PE_g}$ per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$. Bernoulli's equation in that case is

$$P_1 + \rho g h_1 = P_2 + \rho g h_2. \tag{8.49}$$

We can further simplify the equation by taking $h_2 = 0$ (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

$$P_2 = P_1 + \rho g h_1. \tag{8.50}$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h_1 , and consequently, P_2 is greater than P_1 by an amount $\rho g h_1$. In the very simplest case, P_1 is zero at the top of the fluid, and we get the familiar relationship $P = \rho g h$. (Recall that $P = \rho g h$ and $\Delta \mathrm{PE_g} = mgh.$) Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is $\rho g h$. Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth

Another important situation is one in which the fluid moves but its depth is constant—that is, $h_1 = h_2$. Under that condition, Bernoulli's equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2. \tag{8.51}$$

Situations in which fluid flows at a constant depth are so important that this equation is often called **Bernoulli's principle**. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v_2 is greater than v_1 in the equation, then P_2 must be less than P_1 for the equality to hold.

Example 8.9 Calculating Pressure: Pressure Drops as a Fluid Speeds Up

In **Example 8.7**, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01\times10^5\ \mathrm{N/m^2}$ (atmospheric, as it must be) and assuming level, frictionless flow.

Strategy

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P_1 .

Solution

Solving Bernoulli's principle for P_1 yields

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2). \tag{8.52}$$

Substituting known values,

$$\begin{aligned} P_1 &= 1.01\times10^5\ \mathrm{N/m^2} \\ &\quad + \frac{1}{2}(10^3\ \mathrm{kg/m^3})\left[(25.5\ \mathrm{m/s})^2 - (1.96\ \mathrm{m/s})^2\right] \\ &= 4.24\times10^5\ \mathrm{N/m^2}. \end{aligned} \tag{8.53}$$

Discussion

This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in **Figure 8.18**.

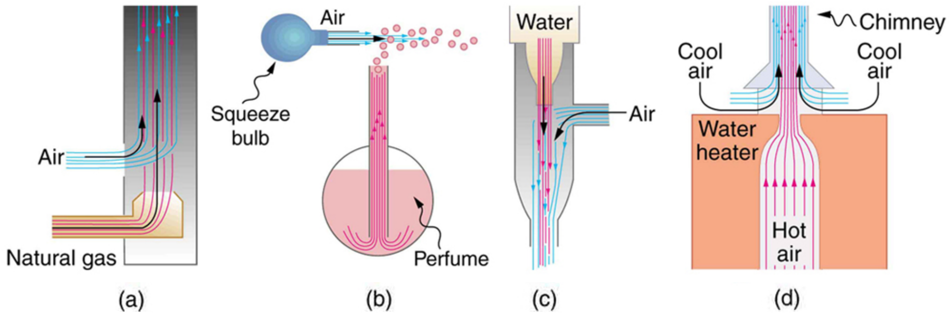


Figure 8.18 Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

Wings and Sails

The airplane wing is a beautiful example of Bernoulli's principle in action. **Figure 8.19**(a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See **Figure 8.19**(b).) The pressure on the front side of the sail, P_{front}, is lower than the pressure on the back of the sail, P_{back}. This results in a forward force and even allows you to sail into the wind.

> **Making Connections: Take-Home Investigation with Two Strips of Paper**
>
> For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

Velocity measurement

Figure 8.20 shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in **Figure 8.20**(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ($v_1 = 0$) in front of it, while fluid passing the other tube has velocity v_2. This means that Bernoulli's principle as stated in $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ becomes

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2. \tag{8.54}$$

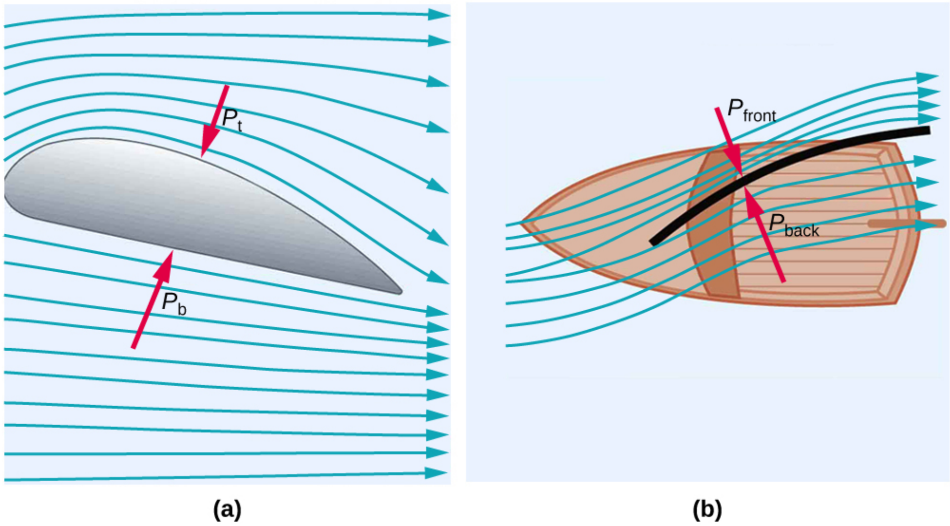


Figure 8.19 (a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Thus pressure P_2 over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, and so the fluid in the manometer rises by h on the side connected to the second opening, where

$$h \propto \frac{1}{2}\rho v_2^2. \tag{8.55}$$

(Recall that the symbol $\propto$ means "proportional to.") Solving for v_2, we see that

$$v_2 \propto \sqrt{h}. \tag{8.56}$$

Figure 8.20(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.

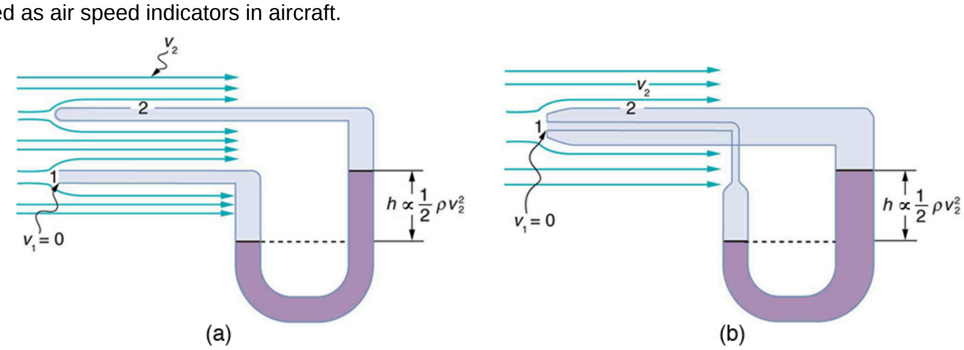


Figure 8.20 Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, and so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

Glossary

Archimedes' principle: the buoyant force on an object equals the weight of the fluid it displaces

Bernoulli's equation: the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2pv^2 + pgh =$ constant , through the fluid

Bernoulli's principle: Bernoulli's equation applied at constant depth: $P_1 + 1/2pv_1^2 = P_2 + 1/2pv_2^2$

buoyant force: the net upward force on any object in any fluid

density: the mass per unit volume of a substance or object

flow rate: abbreviated *Q*, it is the volume *V* that flows past a particular point during a time *t*, or *Q* = *V/t*

fluids: liquids and gases; a fluid is a state of matter that yields to shearing forces

liter: a unit of volume, equal to 10^{-3} m^3

pressure: the force per unit area perpendicular to the force, over which the force acts

specific gravity: the ratio of the density of an object to a fluid (usually water)

Section Summary

8.1 What Is a Fluid?

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

8.2 Density

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as

$$\rho = \frac{m}{V}.$$

- The SI unit of density is kg/m^3 .

8.3 Pressure

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as

$$P = \frac{F}{A}.$$

- The SI unit of pressure is pascal and $1\ \text{Pa} = 1\ \text{N/m}^2$.

8.4 Archimedes' Principle

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

8.5 Flow Rate and Its Relation to Velocity

- Flow rate Q is defined to be the volume V flowing past a point in time t , or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3 .
- Another common unit is the liter (L), which is $10^{-3}\ \text{m}^3$.
- Flow rate and velocity are related by $Q = A\bar{v}$ where A is the cross-sectional area of the flow and $\bar{v}$ is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1\bar{v}_1 &= A_2\bar{v}_2 \\ n_1A_1\bar{v}_1 &= n_2A_2\bar{v}_2 \end{aligned}\right\}.$$

8.6 Bernoulli's Equation

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

Conceptual Questions

8.1 What Is a Fluid?

1. What physical characteristic distinguishes a fluid from a solid?

2. Which of the following substances are fluids at room temperature: air, mercury, water, glass?

3. Why are gases easier to compress than liquids and solids?

4. How do gases differ from liquids?

8.2 Density

5. Approximately how does the density of air vary with altitude?

6. Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

7. Figure 8.21 shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

Figure 8.21

8.3 Pressure

8. How is pressure related to the sharpness of a knife and its ability to cut?

9. Why does a dull hypodermic needle hurt more than a sharp one?

10. The outward force on one end of an air tank was calculated in Example 8.2. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

11. Why is force exerted by static fluids always perpendicular to a surface?

12. In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

13. How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

14. Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

15. How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

8.4 Archimedes' Principle

16. More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

17. Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

18. Will the same ship float higher in salt water than in freshwater? Explain your answer.

19. Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

8.5 Flow Rate and Its Relation to Velocity

20. What is the difference between flow rate and fluid velocity? How are they related?

21. Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

22. Identify some substances that are incompressible and some that are not.

8.6 Bernoulli's Equation

23. You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

24. Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.

25. Look back to **Figure 8.17**. Answer the following two questions. Why is P_{o} less than atmospheric? Why is P_{o} greater than P_{i}?

26. Give an example of entrainment not mentioned in the text.

27. Many entrainment devices have a constriction, called a Venturi, such as shown in **Figure 8.22**. How does this bolster entrainment?

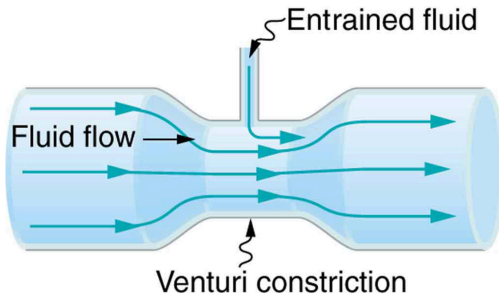


Figure 8.22 A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators.

28. Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.

29. Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

30. Why is it preferable for airplanes to take off into the wind rather than with the wind?

31. Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.

32. Why does a sailboat need a keel?

33. It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

34. Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

35. A perfume bottle or atomizer sprays a fluid that is in the bottle. (**Figure 8.23**.) How does the fluid rise up in the vertical tube in the bottle?

Figure 8.23 Atomizer: perfume bottle with tube to carry perfume up through the bottle. (credit: Antonia Foy, Flickr)

36. If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

Problems & Exercises

8.2 Density

1. Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

2. Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?

3. (a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.

4. A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 cm^3 of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

5. Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

6. (a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

7. A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

8. A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?

9. What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

10. There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3 kg/m^3. The nucleus of an atom has a radius about 10^{-5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8 \text{ m}$)?

8.3 Pressure

11. As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50 cm^2 and the woman's mass is 55.0 kg. Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

12. The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2?

13. Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{ N/m}^2$? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

8.4 Archimedes' Principle

14. What fraction of ice is submerged when it floats in freshwater, given the density of water at 0°C is very close to 1000 kg/m^3?

15. Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

16. Find the density of a fluid in which a hydrometer having a density of 0.750 g/mL floats with 92.0% of its volume submerged.

17. If your body has a density of 995 kg/m^3, what fraction of you will be submerged when floating gently in: (a) Freshwater? (b) Salt water, which has a density of 1027 kg/m^3?

18. Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

19. A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

20. Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron? (c) Calculate the fluid's density and identify it.

21. In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

22. Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is 1015 kg/m^3?

23. (a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.

24. (a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?

25. A certain man has a mass of 80 kg and a density of $955\ \text{kg/m}^3$ (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

26. A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

27. What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?

28. Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

29. A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?

30. Referring to **Figure 8.10**, prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A. Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

31. (a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

8.5 Flow Rate and Its Relation to Velocity

32. What is the average flow rate in cm^3/s of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

33. The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to cm^3/s. (b) What is this rate in m^3/s?

34. Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.

35. Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

36. The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see **Figure 8.24**). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?

Figure 8.24 The Huka Falls in Taupo, New Zealand, demonstrate flow rate. (credit: RaviGogna, Flickr)

37. A major artery with a cross-sectional area of $1.00\ \text{cm}^2$ branches into 18 smaller arteries, each with an average cross-sectional area of $0.400\ \text{cm}^2$. By what factor is the average velocity of the blood reduced when it passes into these branches?

38. (a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is $10.0\ \text{cm}^2$, what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0\ \mu\text{m}$?

39. The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8\ \mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

40. (a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000\ \text{m}^3/\text{s}$, into it?

41. The flow rate of blood through a $2.00\times10^{-6}\text{-m}$ -radius capillary is $3.80\times10^{-9}\ \text{cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0\ \text{cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)

42. (a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

43. The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

44. Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm. (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s. What is the nozzle's inside diameter?

45. Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

46. Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

47. Unreasonable Results

A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches $100{,}000\ \text{m}^3/\text{s}$. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

8.6 Bernoulli's Equation

48. Verify that pressure has units of energy per unit volume.

49. Suppose you have a wind speed gauge like the pitot tube shown in **Example 8.7**(b). By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer?

50. If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?

51. Calculate the maximum height to which water could be squirted with the hose in **Example 8.7** example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.

52. Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of $220\ \text{m}^2$? Typical air density in Boulder is $1.14\ \text{kg/m}^3$, and the corresponding atmospheric pressure is $8.89\times10^4\ \text{N/m}^2$. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

53. (a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be $1.29\ \text{kg/m}^3$. (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.

54. (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

55. (a) Using Bernoulli's equation, show that the measured fluid speed v for a pitot tube, like the one in **Figure 8.20**(b), is given by $v = \left(\dfrac{2\rho' gh}{\rho}\right)^{1/2}$,

where h is the height of the manometer fluid, ρ' is the density of the manometer fluid, ρ is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h , as stated in the text.) (b) Calculate v for moving air if a mercury manometer's h is 0.200 m.

III UNIT 3: CLASSICAL PHYSICS - THERMODYNAMICS, ELECTRICITY AND MAGNETISM, AND LIGHT

9 THERMAL PHYSICS

Figure 9.1 The welder’s gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O’Brien/U.S. Navy)

Chapter Outline

9.1. Temperature
- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

9.2. The Ideal Gas Law
- State and explain the ideal gas law using Boltzmann's constant
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules in a given volume.

9.3. Heat
- Define heat as transfer of energy.

9.4. Heat Transfer Methods
- Discuss the three different methods of heat transfer.

9.5. Temperature Change and Heat Capacity
- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

9.6. Phase Change and Latent Heat
- Describe phase changes.
- Explain the relationship between phase changes and heat transfer.

9.7. The First Law of Thermodynamics
- Define the first law of thermodynamics.
- Describe how conservation of energy relates to the first law of thermodynamics.
- Identify instances of the first law of thermodynamics working in everyday situations.

9.8. The First Law of Thermodynamics and Heat Engine Processes
- Describe the processes of a simple heat engine.
- Explain the differences among the simple thermodynamic processes—isobaric, isochoric, isothermal, and adiabatic.
- Explain the relationship between work done by a gas and change in its volume.

9.9. Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency
- State the expressions of the second law of thermodynamics.
- Calculate the efficiency of a coal-fired electricity plant, using second law characteristics.
- Describe and define the Otto cycle.

9.10. Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated
- Identify a Carnot cycle.
- Calculate maximum theoretical efficiency of a nuclear reactor.
- Explain how dissipative processes affect the ideal Carnot engine.

9.11. Applications of Thermodynamics: Heat Pumps and Refrigerators
- Describe the use of heat engines in heat pumps and refrigerators.
- Demonstrate how a heat pump works to warm an interior space.
- Explain the differences between heat pumps and refrigerators.
- Calculate a heat pump's coefficient of performance.

9.12. Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy
- Define entropy and calculate the increase of entropy in a system with reversible and irreversible processes.
- Calculate the increasing disorder of a system.

9.13. Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation
- Identify probabilities in entropy.
- Analyze statistical probabilities in entropic systems.

Introduction to Thermal Physics

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.

Most notably, heat transfer can be used to do work. It can also be converted to any other form of energy. A car engine, for example, burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance, converting its energy into a variety of other forms—into the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the heat transfer produced from burning fuel in the engine does not do work on the gas-the engine is quite inefficient.

Basic physical laws govern how heat transfer for doing work takes place and place insurmountable limits onto its efficiency. This chapter will explore these laws as well as some applications and concepts associated with them. These topics are part of thermodynamics—the study of heat transfer and its relationship to doing work.

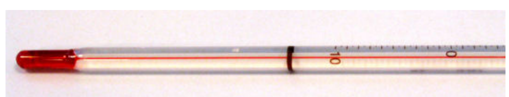


Figure 9.2 In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)

9.1 Temperature

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

Misconception Alert: Human Perception vs. Reality

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does. This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip (**Figure 9.3**). Other properties used to measure temperature include electrical resistance and color, as shown in **Figure 9.4**, and the emission of infrared radiation, as shown in **Figure 9.5**.

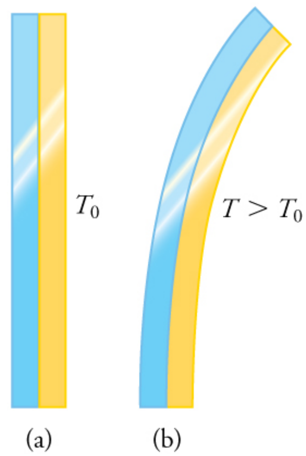


Figure 9.3 The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.

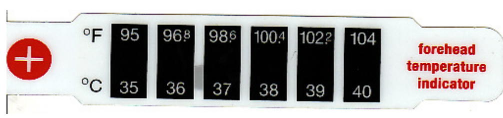


Figure 9.4 Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95°F , all six squares are black. When the plastic thermometer is exposed to temperature that increases to 95°F , the first liquid crystal square changes color. When the temperature increases above 96.8°F the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

Figure 9.5 Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced. Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at $0°C$ and the boiling point at $100°C$. Its unit is the **degree Celsius** $(°C)$. On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at $32°F$ and the boiling point is at $212°F$. The unit of temperature on this scale is the **degree Fahrenheit** $(°F)$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180 / 100 = 9 / 5.$

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

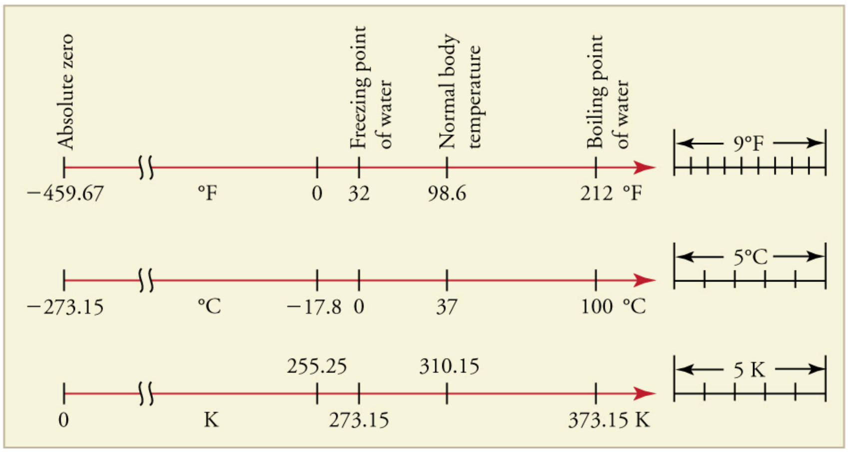


Figure 9.6 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in **Figure 9.6**.

Temperature Ranges in the Universe

Figure 9.8 shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from $24°C$ to $44°C$ $(75°F$ to $111°F$). The average normal body temperature is usually given as $37.0°C$ ($98.6°F$), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see **Figure 9.7**).

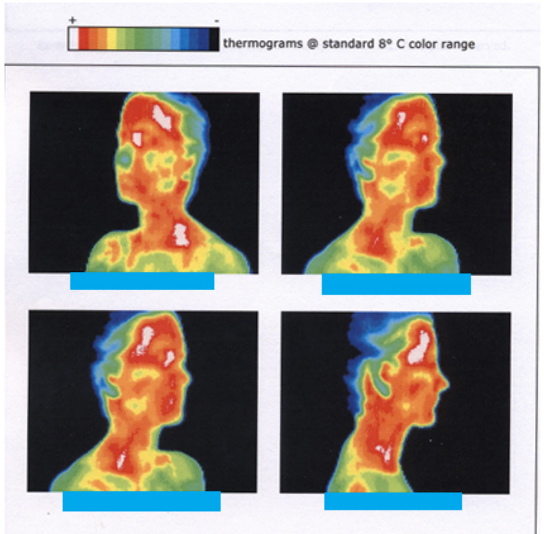


Figure 9.7 This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: $4.5\times10^{-10}\ \text{K}$ at the Massachusetts Institute of Technology (USA), and $1.0\times10^{-10}\ \text{K}$ at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (-89°C), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

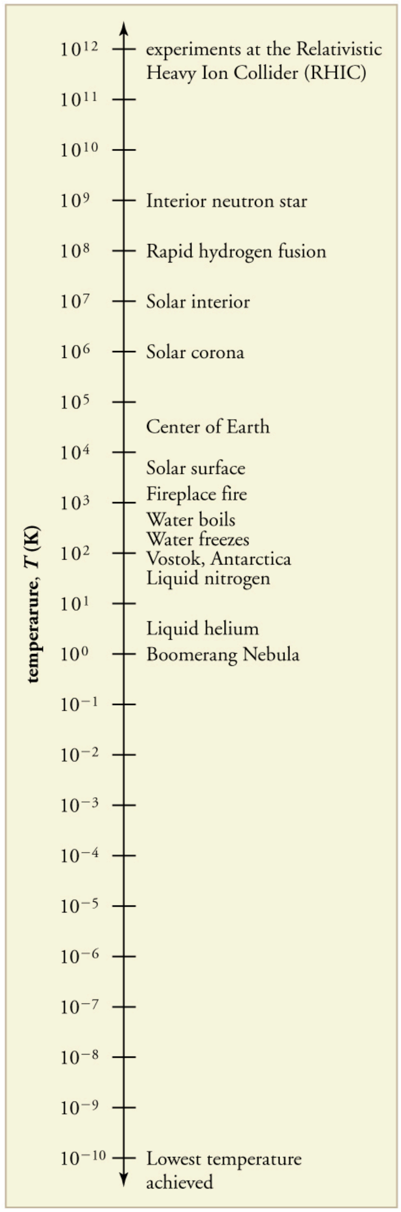


Figure 9.8 Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. **Figure 9.9** shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, -273.15°C. This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is -273.15°C or 0 K.

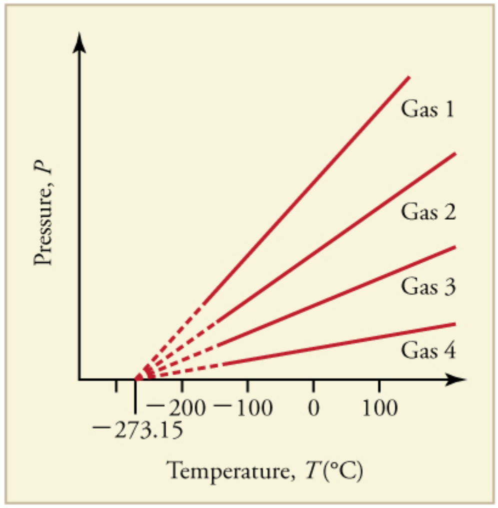


Figure 9.9 Graph of pressure versus temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws. An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.

Check Your Understanding

Does the temperature of a body depend on its size?

Solution
No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

9.2 The Ideal Gas Law

Figure 9.10 The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2, and oxygen, O_2, are composed of two or more atoms. We will primarily use the term "molecule" in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. Gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β. This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in **Figure 9.11**. Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.

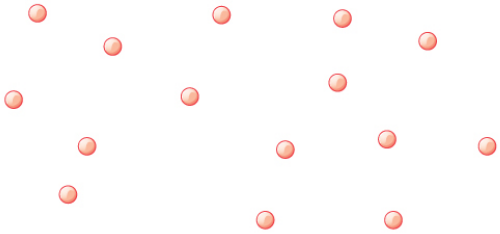


Figure 9.11 Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See **Figure 9.12**.)

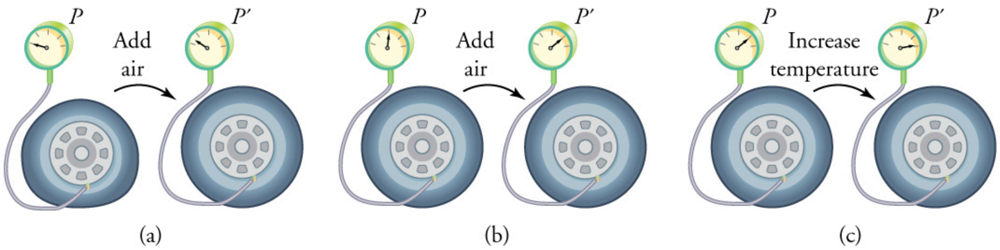


Figure 9.12 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

Ideal Gas Law

The **ideal gas law** states that

$$PV = NkT, \tag{9.1}$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

$$k = 1.38 \times 10^{-23}\ \text{J/K}. \tag{9.2}$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V. The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $PV = NkT$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example 9.1 Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of $7.00 \times 10^5\ \text{Pa}$ (a gauge pressure of just under $90.0\ \text{lb/in}^2$) at a temperature of 18.0°C. What is the pressure after its temperature has risen to 35.0°C? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5\ \text{Pa}$, the initial temperature $T_0 = 18.0^\circ\text{C}$, and the final temperature $T_\text{f} = 35.0^\circ\text{C}$. We must find the final pressure P_f. How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0 V_0 = NkT_0$ and $P_\text{f} V_\text{f} = NkT_\text{f}$. If we divide $P_\text{f} V_\text{f}$ by $P_0 V_0$ we can come up with an equation that allows us to solve for P_f.

$$\frac{P_\text{f} V_\text{f}}{P_0 V_0} = \frac{N_\text{f} k T_\text{f}}{N_0 k T_0} \tag{9.3}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0, and k, which is a constant. Therefore,

$$\frac{P_\mathrm{f}}{P_0} = \frac{T_\mathrm{f}}{T_0}. \tag{9.4}$$

We can then rearrange this to solve for P_f:

$$P_\mathrm{f} = P_0 \frac{T_\mathrm{f}}{T_0}, \tag{9.5}$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.

$$\begin{aligned} T_0 &= (18.0 + 273)\mathrm{K} = 291\ \mathrm{K} \\ T_\mathrm{f} &= (35.0 + 273)\mathrm{K} = 308\ \mathrm{K} \end{aligned} \tag{9.6}$$

2. Substitute the known values into the equation.

$$P_\mathrm{f} = P_0 \frac{T_\mathrm{f}}{T_0} = 7.00\times 10^5\ \mathrm{Pa}\left(\frac{308\ \mathrm{K}}{291\ \mathrm{K}}\right) = 7.41\times 10^5\ \mathrm{Pa} \tag{9.7}$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example 9.2 Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be $0^\mathrm{o}\mathrm{C}$ and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $PV = NkT$, to find N.

Solution

1. Identify the knowns.

$$\begin{aligned} T &= 0^\mathrm{o}\mathrm{C} = 273\ \mathrm{K} \\ P &= 1.01\times 10^5\ \mathrm{Pa} \\ V &= 1.00\ \mathrm{m}^3 \\ k &= 1.38\times 10^{-23}\ \mathrm{J/K} \end{aligned} \tag{9.8}$$

2. Identify the unknown: number of molecules, N.

3. Rearrange the ideal gas law to solve for N.

$$\begin{aligned} PV &= NkT \\ N &= \frac{PV}{kT} \end{aligned} \tag{9.9}$$

4. Substitute the known values into the equation and solve for N.

$$N = \frac{PV}{kT} = \frac{\left(1.01\times 10^5\ \mathrm{Pa}\right)\left(1.00\ \mathrm{m}^3\right)}{\left(1.38\times 10^{-23}\ \mathrm{J/K}\right)(273\ \mathrm{K})} = 2.68\times 10^{25}\ \mathrm{molecules} \tag{9.10}$$

Discussion

This number is undeniably large, considering that a gas is mostly empty space. N is huge, even in small volumes. For example, $1\ \text{cm}^3$ of a gas at STP has 2.68×10^{19} molecules in it. Once again, note that N is the same for all types or mixtures of gases.

9.3 Heat

We have defined work as force times distance and learned that work done on an object changes its kinetic energy. We have also seen that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

Heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00°C —specifically, between 14.5°C and 15.5°C, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00°C. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called "big calorie") are actually kilocalories ($1\ \text{kilocalorie} = 1000\ \text{calories}$), a fact not easily determined from package labeling.

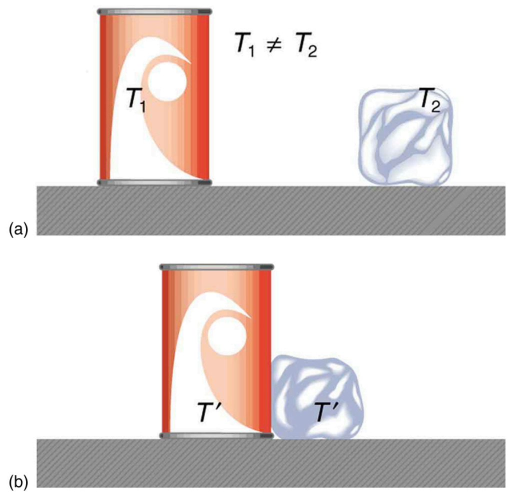


Figure 9.13 In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature T', achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

$$1.000\ \text{kcal} = 4186\ \text{J}. \tag{9.11}$$

We consider this equation as the conversion between two different units of energy.

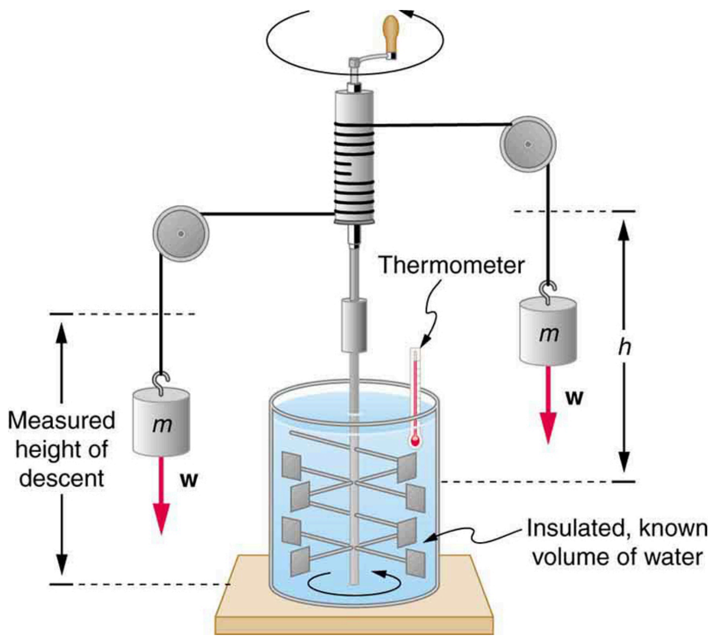


Figure 9.14 Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule's most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain "heat content" or "work content". We use the phrase "heat transfer" to emphasize its nature.

Check Your Understanding

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Solution
Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

9.4 Heat Transfer Methods

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.

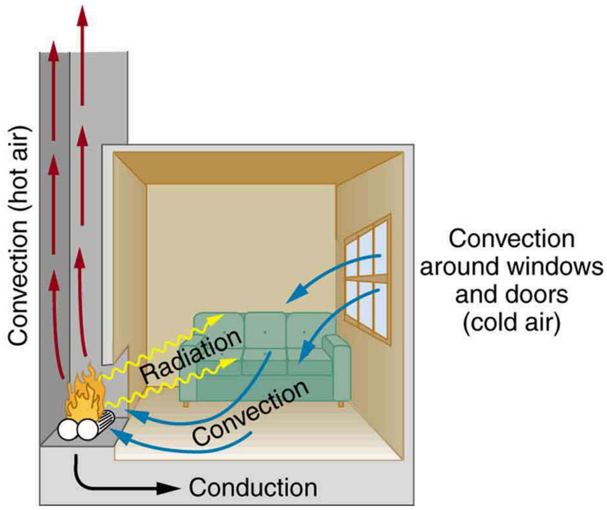


Figure 9.15 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

Figure 9.15 illustrates these three methods of heat transfer occurring in one system. While we skip detailed discussion of methods of heat transfer, the one thing all three methods-conduction, convection, and radiation-share is they are driven by temperature difference.

Check Your Understanding

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Solution

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista “steams” cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

9.5 Temperature Change and Heat Capacity

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.

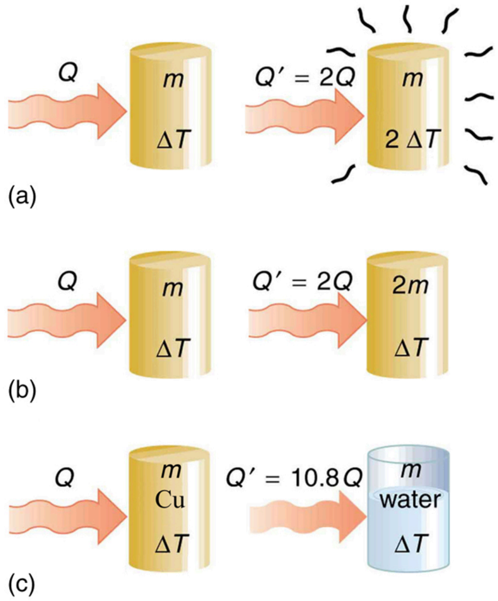


Figure 9.16 The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Heat Transfer and Temperature Change

The quantitative relationship between heat transfer and temperature change contains all three factors:

$$Q = mc\Delta T, \tag{9.12}$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00ºC. The specific heat c is a property of the substance; its SI unit is $\text{J/(kg} \cdot \text{K)}$ or $\text{J/(kg}\cdot\text{ºC)}.$ Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\text{kcal/(kg}\cdot\text{ºC)}.$

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. **Table 9.1** lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Example 9.3 Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0ºC to 80.0ºC. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in **Table 9.1**.

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:

$$\Delta T = T_{\text{f}} - T_{\text{i}} = 60.0\text{ºC}. \tag{9.13}$$

2. Calculate the mass of water. Because the density of water is $1000 \ \text{kg/m}^3$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_{\text{w}} = 0.250 \text{ kg}$.

3. Calculate the heat transferred to the water. Use the specific heat of water in **Table 9.1**:

$$Q_{\text{w}} = m_{\text{w}} c_{\text{w}} \Delta T = (0.250 \text{ kg})(4186 \text{ J/kgºC})(60.0\text{ºC}) = 62.8 \text{ kJ}. \tag{9.14}$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in **Table 9.1**:

$$Q_{\text{Al}} = m_{\text{Al}} c_{\text{Al}} \Delta T = (0.500 \text{ kg})(900 \text{ J/kgºC})(60.0\text{ºC}) = 27.0 \times 10^4 \text{ J} = 27.0 \text{ kJ}. \tag{9.15}$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_{\text{Total}} = Q_{\text{W}} + Q_{\text{Al}} = 62.8 \text{ kJ} + 27.0 \ \text{ kJ} = 89.8 \text{ kJ}. \tag{9.16}$$

Thus, the amount of heat going into heating the pan is

$$\frac{27.0 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 30.1\%, \tag{9.17}$$

and the amount going into heating the water is

$$\frac{62.8 \text{ kJ}}{89.8 \text{ kJ}} \times 100\% = 69.9\%. \tag{9.18}$$

Discussion

In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

Figure 9.17 The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

Example 9.4 Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of $800\ \text{J/kg}\cdot\text{ºC}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill

$$Mgh = (10{,}000\ \text{kg})\left(9.80\ \text{m/s}^2\right)(75.0\ \text{m}) = 7.35\times10^6\ \text{J}. \tag{9.19}$$

2. Calculate the temperature from the heat transferred using $Q{=}Mgh$ and

$$\Delta T = \frac{Q}{mc}, \tag{9.20}$$

where m is the mass of the brake material. Insert the values $m = 100\ \text{kg}$ and $c = 800\ \text{J/kg}\cdot\text{ºC}$ to find

$$\Delta T = \frac{\left(7.35\times10^6\ \text{J}\right)}{(100\ \text{kg})(800\ \text{J/kgºC})} = 92\text{ºC}. \tag{9.21}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, so this technique is not practical. However, the same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Table 9.1 Specific Heats[1] of Various Substances

Substances	Specific heat (*c*)	
Solids	J/kg·ºC	kcal/kg·ºC[2]
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [3]		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Note that **Example 9.4** is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

Example 9.5 Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of $20.0^\circ C$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150^\circ C$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $\mid Q_{\text{hot}} \mid = Q_{\text{cold}}$.

1. The values for solids and liquids are at constant volume and at $25^\circ C$, except as noted.
2. These values are identical in units of $\text{cal/g}\cdot{}^\circ C$.
3. c_v at constant volume and at $20.0^\circ C$, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_p at a constant pressure of 1.00 atm.

Solution

1. Use the equation for heat transfer $Q = mc\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\circ}\text{C}). \tag{9.22}$$

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\circ}\text{C}). \tag{9.23}$$

3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -Q_{\text{hot}}, \\ m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\circ}\text{C}) &= -m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\circ}\text{C}.) \end{aligned} \tag{9.24}$$

4. This an equation for the unknown final temperature, T_{f}

5. Bring all terms involving T_{f} on the left hand side and all other terms on the right hand side. Solve for T_{f} ,

$$T_{\text{f}} = \frac{m_{\text{Al}}c_{\text{Al}}(150^{\circ}\text{C}) + m_{\text{W}}c_{\text{W}}(20.0^{\circ}\text{C})}{m_{\text{Al}}c_{\text{Al}} + m_{\text{W}}c_{\text{W}}}, \tag{9.25}$$

and insert the numerical values:

$$\begin{aligned} T_{\text{f}} &= \frac{(0.500\text{ kg})(900\text{ J/kg}^{\circ}\text{C})(150^{\circ}\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^{\circ}\text{C})(20.0^{\circ}\text{C})}{(0.500\text{ kg})(900\text{ J/kg}^{\circ}\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^{\circ}\text{C})} \\ &= \frac{88430\text{ J}}{1496.5\text{ J/}^{\circ}\text{C}} \\ &= 59.1^{\circ}\text{C}. \end{aligned} \tag{9.26}$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to 20.0°C than 150°C ? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Take-Home Experiment: Temperature Change of Land and Water

What heats faster, land or water?

To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

Check Your Understanding

If 25 kJ is necessary to raise the temperature of a block from 25°C to 30°C , how much heat is necessary to heat the block from 45°C to 50°C ?

Solution

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

9.6 Phase Change and Latent Heat

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.

Figure 9.18 Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at 0°C stays at 0°C until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together **Figure 9.19**.

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

$$Q = mL_\text{f} \text{ (melting/freezing)}, \tag{9.27}$$

$$Q = mL_\text{v} \text{ (vaporization/condensation)}, \tag{9.28}$$

where the latent heat of fusion, L_f, and latent heat of vaporization, L_v, are material constants that are determined experimentally. See (**Table 9.2**).

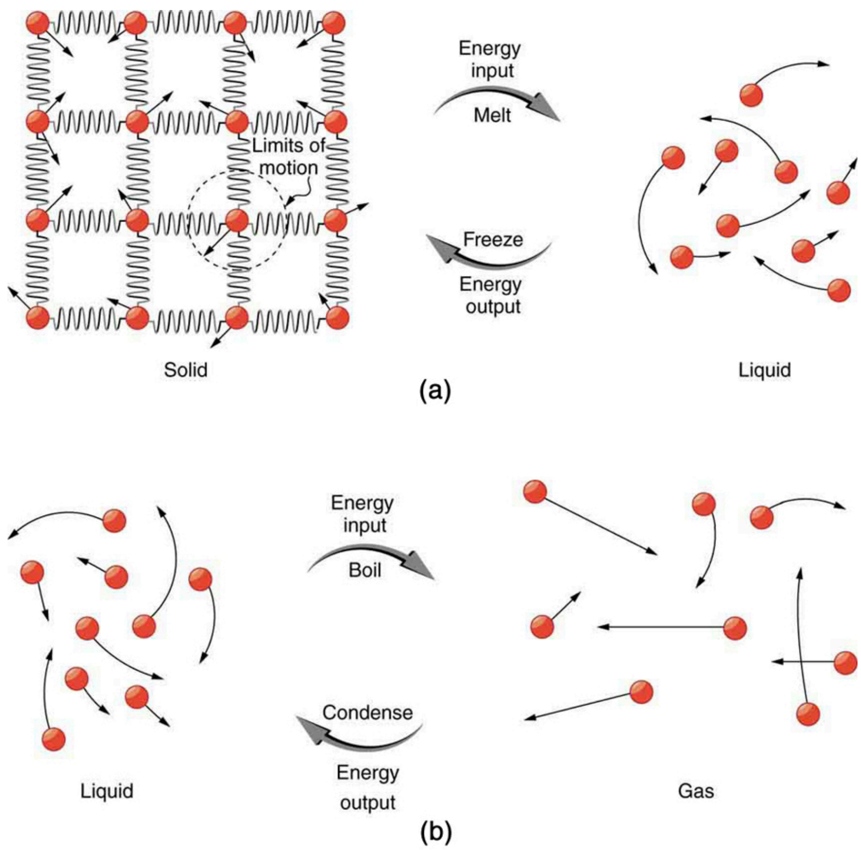


Figure 9.19 (a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_f and L_v depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_f and L_v are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. **Table 9.2** lists representative values of L_f and L_v, together with melting and boiling points.

The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at 0ºC to produce a kilogram of water at $0°\text{C}$. Using the equation for a change in temperature and the value for water from **Table 9.2**, we find that $Q = mL_\text{f} = (1.0\text{ kg})(334\text{ kJ/kg}) = 334\text{ kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from 0ºC to 79.8ºC. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point (100ºC at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

Table 9.2 Heats of Fusion and Vaporization [4]

		L_f			L_v	
Substance	**Melting point (ºC)**	**kJ/kg**	**kcal/kg**	**Boiling point (°C)**	**kJ/kg**	**kcal/kg**
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[5]	539[6]
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above 35.0°C , which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at -20°C (**Figure 9.20**). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \text{cal/g}\cdot^\circ\text{C}$ until it reaches 0°C . Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at 0°C during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \text{cal/g}\cdot^\circ\text{C}$. At 100°C , the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \text{cal/g}\cdot^\circ\text{C}$.

4. Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).
5. At 37.0°C (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg
6. At 37.0°C (body temperature), the heat of vaporization L_v for water is 2430 kJ/kg or 580 kcal/kg

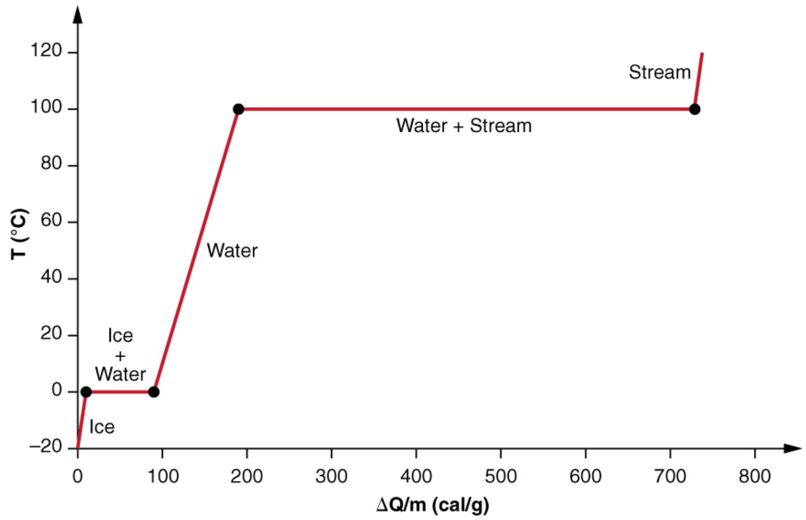


Figure 9.20 A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at $0ºC$ and $100ºC$ reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below $100ºC$ is less than that at $100ºC$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100ºC$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

Example 9.6 Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at $20ºC$ with mass $m_{\text{soda}} = 0.25\text{ kg}$. The ice is at $0ºC$ and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of $0ºC$. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at $0ºC$, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}}. \tag{9.29}$$

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0ºC)$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20ºC)$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0ºC) = -m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20ºC). \tag{9.30}$$

Bring all terms involving T_{f} on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_{f}:

$$T_{\text{f}} = \frac{m_{\text{soda}}c_{\text{W}}(20ºC) - m_{\text{ice}}L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}}}. \tag{9.31}$$

Solution

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3\times6.0\text{ g} = 0.018\text{ kg}$ and the mass of soda is $m_{\text{soda}} = 0.25\text{ kg}$.
2. Calculate the terms in the numerator:

$$m_{\text{soda}}c_{\text{W}}(20ºC) = (0.25\text{ kg})(4186\text{ J/kg}\cdot ºC)(20ºC) = 20{,}930\text{ J} \tag{9.32}$$

and

$$m_{\text{ice}}L_{\text{f}} = (0.018\ \text{kg})(334{,}000\ \text{J/kg}) = 6012\ \text{J}. \tag{9.33}$$

3. Calculate the denominator:

$$(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}} = (0.25\ \text{kg} + 0.018\ \text{kg})(4186\ \text{K/(kg·°C)} = 1122\ \text{J/°C}. \tag{9.34}$$

4. Calculate the final temperature:

$$T_{\text{f}} = \frac{20{,}930\ \text{J} - 6012\ \text{J}}{1122\ \text{J/°C}} = 13\text{°C}. \tag{9.35}$$

Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is -6°C . However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_{\text{v}}$.

Figure 9.21 Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The air cannot hold as much water as it did at room temperature, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

Real-World Application

Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point (0°C) . Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.

Figure 9.22 The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below 0°C . Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of "smoke" from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

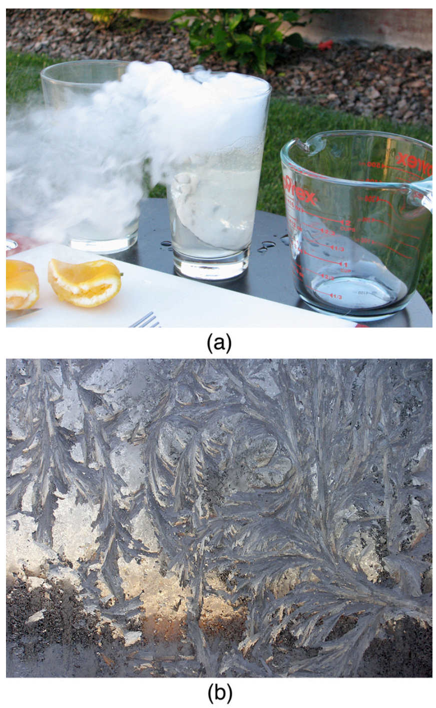


Figure 9.23 Direct transitions between solid and vapor are common, sometimes useful, and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = mL_{\text{s}}$, where L_{s} is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_{s} is analogous to L_{f} and L_{v} , and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Check Your Understanding

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Solution

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0°C. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

9.7 The First Law of Thermodynamics

Figure 9.24 This boiling tea kettle represents energy in motion. The water in the kettle is turning to water vapor because heat is being transferred from the stove to the kettle. As the entire system gets hotter, work is done—from the evaporation of the water to the whistling of the kettle. (credit: Gina Hamilton)

If we are interested in how heat transfer is converted into doing work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. The **first law of thermodynamics** states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

$$\Delta U = Q - W. \tag{9.36}$$

Here ΔU is the *change in internal energy* U of the system. Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfer into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work. (See Figure 9.25.) We will now examine Q, W, and ΔU further.

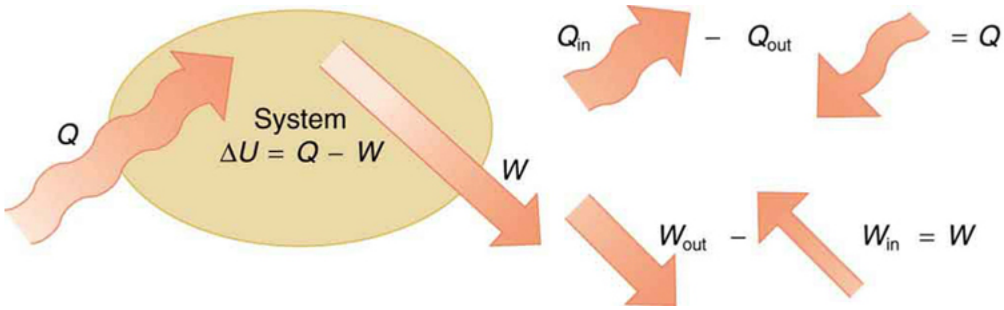


Figure 9.25 The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the system. Q is positive for net heat transfer *into* the system. W is the total work done on and by the system. W is positive when more work is done *by* the system than on it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

Making Connections: Law of Thermodynamics and Law of Conservation of Energy

The first law of thermodynamics is actually the law of conservation of energy stated in a form most useful in thermodynamics. The first law gives the relationship between heat transfer, work done, and the change in internal energy of a system.

Heat *Q* and Work *W*

Heat transfer (Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system.

The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process, involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results.For example, both can cause a temperature increase. Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work. This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy U of a system. Internal energy is a form of energy completely different from either heat or work.

Internal Energy *U*

We can think about the internal energy of a system in two different but consistent ways. The first is the atomic and molecular view, which examines the system on the atomic and molecular scale. The **internal energy** U of a system is the sum of the kinetic and potential energies of its atoms and molecules. Recall that kinetic plus potential energy is called mechanical energy. Thus internal energy is the sum of atomic and molecular mechanical energy. Because it is impossible to keep track of all individual atoms and molecules, we must deal with averages and distributions. A second way to view the internal energy of a system is in terms of its macroscopic characteristics, which are very similar to atomic and molecular average values.

Macroscopically, we define the change in internal energy ΔU to be that given by the first law of thermodynamics:

$$\Delta U = Q - W. \tag{9.37}$$

Many detailed experiments have verified that $\Delta U = Q - W$, where ΔU is the change in total kinetic and potential energy of all atoms and molecules in a system. It has also been determined experimentally that the internal energy U of a system depends only on the state of the system and *not how it reached that state.* More specifically, U is found to be a function of a few macroscopic quantities (pressure, volume, and temperature, for example), independent of past history such as whether there has been heat transfer or work done. This independence means that if we know the state of a system, we can calculate changes in its internal energy U from a few macroscopic variables.

Making Connections: Macroscopic and Microscopic

In thermodynamics, we often use the macroscopic picture when making calculations of how a system behaves, while the atomic and molecular picture gives underlying explanations in terms of averages and distributions. We shall see this again in later sections of this chapter. For example, in the topic of entropy, calculations will be made using the atomic and molecular view.

9.8 The First Law of Thermodynamics and Heat Engine Processes

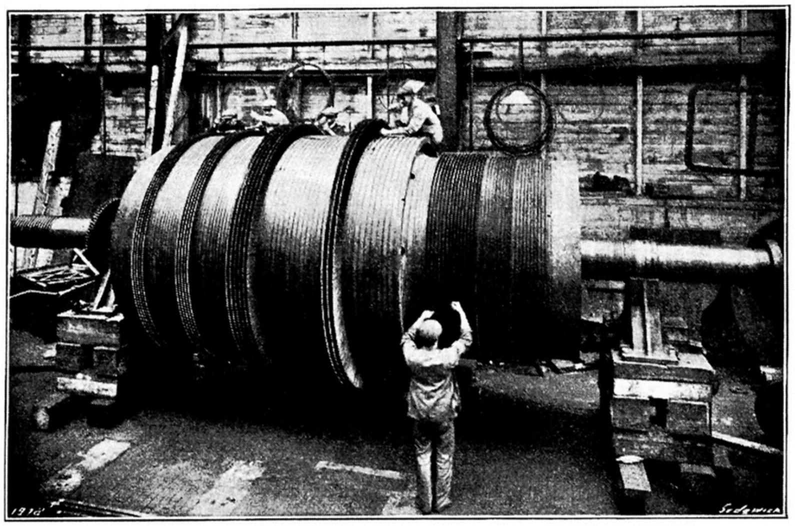


Figure 9.26 Beginning with the Industrial Revolution, humans have harnessed power through the use of the first law of thermodynamics, before we even understood it completely. This photo, of a steam engine at the Turbinia Works, dates from 1911, a mere 61 years after the first explicit statement of the first law of thermodynamics by Rudolph Clausius. (credit: public domain; author unknown)

One of the most important things we can do with heat transfer is to use it to do work for us. Such a device is called a **heat engine**. Car engines and steam turbines that generate electricity are examples of heat engines. Figure 9.27 shows schematically how the first law of thermodynamics applies to the typical heat engine.

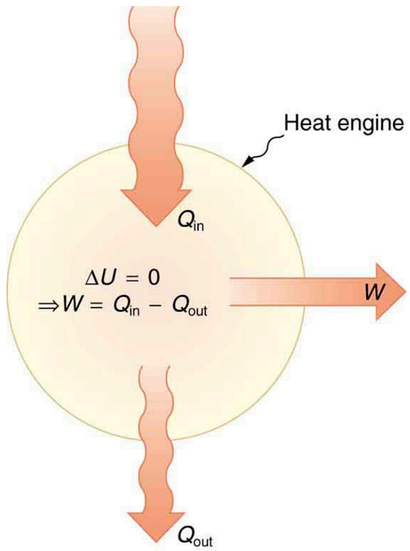


Figure 9.27 Schematic representation of a heat engine, governed, of course, by the first law of thermodynamics (and other laws of thermodynamics we will discuss later).

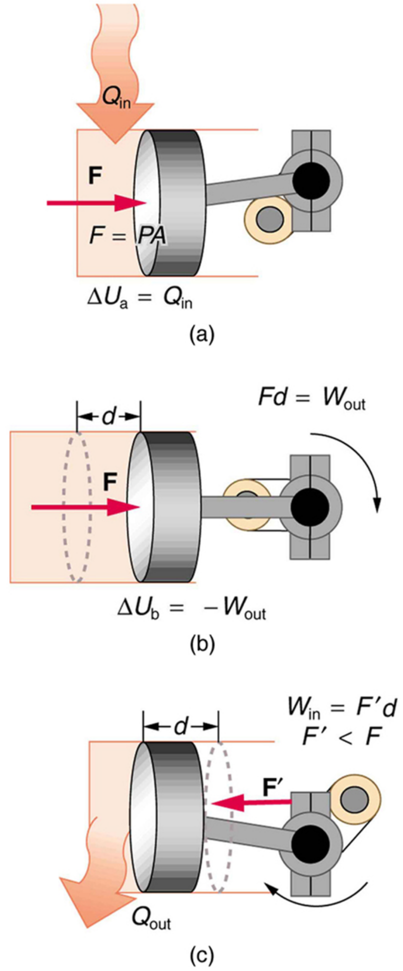


Figure 9.28 (a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease when it expands, indicating that the gas's internal energy has been decreased by doing work. (c) Heat transfer to the environment further reduces pressure in the gas so that the piston can be more easily returned to its starting position.

The illustrations above show one of the ways in which heat transfer does work. Fuel combustion produces heat transfer to a gas in a cylinder, increasing the pressure of the gas and thereby the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Heat transfer to the gas cylinder results in work being done. To repeat this process, the piston needs to be returned to its starting point. Heat transfer now occurs from the gas to the surroundings so that its pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance. Variations of this process are employed daily in hundreds of millions of heat engines. Here, we consider some of the thermodynamic processes on which heat engines are based.

Work Done by a Gas

A process by which a gas does work on a piston at constant pressure is called an **isobaric process**. Since the pressure is constant, the force exerted is constant and the work done is given as

$$P\Delta V. \tag{9.38}$$

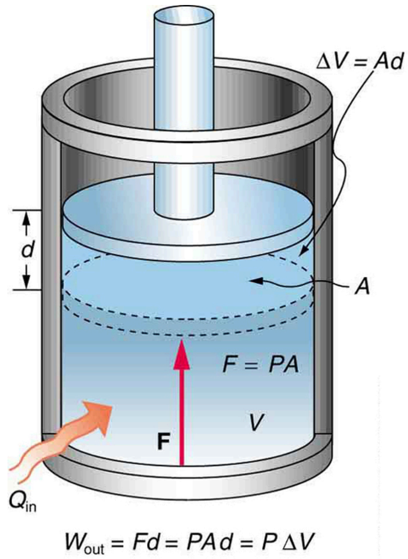


Figure 9.29 An isobaric expansion of a gas requires heat transfer to keep the pressure constant. Since pressure is constant, the work done is $P\Delta V$.

Recall from mechanics that work done by a force F on an object undergoing displacement d is

$$W = Fd. \tag{9.39}$$

See the symbols as shown in **Figure 9.29**. Now force is pressure times area ($F = PA$), and so

$$W = PAd. \tag{9.40}$$

Because the volume of a cylinder is its cross-sectional area A times its length d , we see that $Ad = \Delta V$, the change in volume; thus,

$$W = P\Delta V \text{ (isobaric process).} \tag{9.41}$$

Note that if ΔV is positive, then W is positive, meaning that positive work is done by the gas on the outside world.

(Note that the pressure involved in this work that we've called P is the pressure of the gas *inside* the tank. If we call the pressure outside the tank P_{ext} , an expanding gas would be working *against* the external pressure; the work done would therefore be $W = -P_{\text{ext}}\Delta V$ (isobaric process). There are some—especially chemists—who use this definition of work, and not the definition based on internal pressure, as the basis of the First Law of Thermodynamics. This definition reverses the sign conventions for work, and results in a statement of the first law that becomes $\Delta U = Q + W$. In this textbook, we will use the *physics* convention of using work done by the system on the surrounding, not the other way around.)

This is the key lesson from the above derivation: *a gas expanding under pressure does work on its surrounding, and unless additional energy is added through heat transfer, the internal energy of the gas decreases*. We will examine the experimental results that come about as a consequence of this fact later.

Thermodynamic Processes

We introduced the **isobaric process** above in discussing work done by a gas. Isobaric process is an example of a **thermodynamic process**. A thermodynamic process describes a change that happens to a gas, which results in change in its pressure (P), volume (V), and/or temperature (T). An isobaric process is a thermodynamic process that takes place under constant pressure (so the volume and temperature of the gas may change in an isobaric process).

There are three more named thermodynamic processes. These processes are given special names because, like the isobaric process, they occur under some restrictions, which gives them their special properties, as described briefly below. These three additional named thermodynamic processes are: isochoric, isothermal, and adiabatic processes.

An **isochoric process** is a thermodynamic process in which no change in volume takes place. Because the work done by a gas is proportional to the change in volume, in an isochoric process, no work is done by (or on) the gas. Instead, in an isochoric process, a heat transfer takes place, and the energy from the heat transfer goes into increasing (or decreasing) the internal energy of the gas, increasing (or decreasing) its temperature.

An **isothermal process** is a thermodynamic process in which no change in temperature takes place. A gas expanding isothermally, for example, does work on the surrounding, but its internal energy (as represented by the temperature) does not change, because enough heat flows in to balance out the energy expended in doing work. This is consistent with the first law of thermodynamics ($0 = \Delta U = Q - W$, because $Q = W$). An isothermal process occurs if a thermodynamic process in a gas occurs slowly enough so that the gas remains in thermal equilibrium with the surrounding at all times.

The adiabatic process is, in some sense, the opposite of an isothermal process. In an **adiabatic process**, no heat transfer takes place (that is, $Q = 0$). This may happen because the gas is well-insulated from the surrounding. It may also happen because the process occurs so quickly that no significant heat transfer can take place. In an adiabatic expansion, for example, the internal energy of the gas decreases, because of the work done by the gas in expansion. This is perhaps the clearest experimental evidence one can observe that it takes work for a gas to expand under pressure.

Figure 9.30 illustrates these three processes on a plot of pressure and volume (a PV diagram). In an isothermal process, as the gas expands, the pressure decreases. This can be predicted from the ideal gas law ($PV = NkT$). Since the temperature is constant, if volume increases, the pressure must decrease, to keep PV constant. You can also see that in an adiabatic process, the pressure decreases with expanding volume more steeply than an isothermal process, because in an adiabatic process, the temperature is not constant, but it decreases. So with increasing V , the pressure decrease even more rapidly, so that PV actually decreases (for decreasing temperature).

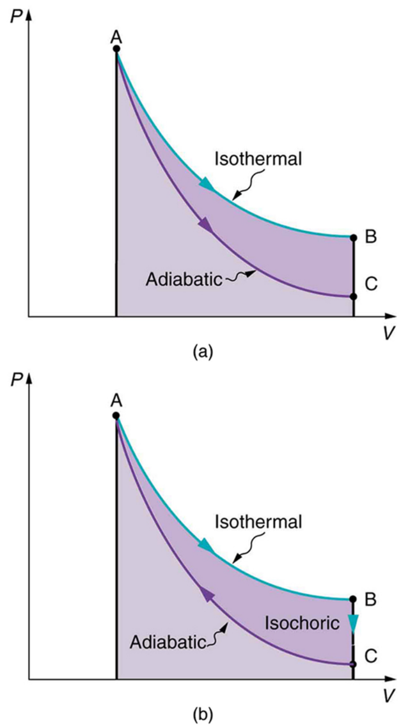


Figure 9.30 (a) The upper curve is an isothermal process ($\Delta T = 0$), whereas the lower curve is an adiabatic process ($Q = 0$). Both start from the same point A, but the isothermal process does more work than the adiabatic because heat transfer into the gas takes place to keep its temperature constant. This keeps the pressure higher all along the isothermal path than along the adiabatic path, producing more work. The adiabatic path thus ends up with a lower pressure and temperature at point C, even though the final volume is the same as for the isothermal process. (b) The cycle ABCA produces a net work output.

Reversible Processes

Both isothermal and adiabatic processes such as shown in **Figure 9.30** are reversible in principle. A **reversible process** is one in which both the system and its environment can return to exactly the states they were in by following the reverse path. The reverse isothermal and adiabatic paths are BA and CA, respectively. Real macroscopic processes are never exactly reversible. In the previous examples, our system is a gas (like that in **Figure 9.29**), and its environment is the piston, cylinder, and the rest of the universe. If there are any energy-dissipating mechanisms, such as friction or turbulence, then heat transfer to the environment occurs for either direction of the piston. So, for example, if the path BA is followed and there is friction, then the gas will be returned to its original state but the environment will not—it will have been heated in both directions. Reversibility requires the direction of heat transfer to reverse for the reverse path. Since dissipative mechanisms cannot be completely eliminated, real processes cannot be reversible.

There must be reasons that real macroscopic processes cannot be reversible. We can imagine them going in reverse. For example, heat transfer occurs spontaneously from hot to cold and never spontaneously the reverse. Yet it would not violate the first law of thermodynamics for this to happen. In fact, all spontaneous processes, such as bubbles bursting, never go in reverse. There is a second thermodynamic law that forbids them from going in reverse. When we study this law, we will learn something about nature and also find that such a law limits the efficiency of heat engines. We will find that heat engines with the greatest possible theoretical efficiency would have to use reversible processes, and even they cannot convert all heat transfer into doing work. **Table 9.3** summarizes the simpler thermodynamic processes and their definitions.

Table 9.3 Summary of Simple Thermodynamic Processes

Isobaric	Constant pressure $W = P\Delta V$
Isochoric	Constant volume $W = 0$
Isothermal	Constant temperature $Q = W$
Adiabatic	No heat transfer $Q = 0$

9.9 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

Figure 9.31 These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform the distinctive alligator-like shape they formed when the picture was taken in the summer of 2009. (credit: Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only—that is, they are irreversible, under a given set of conditions. Here are some examples of irreversible processes seen in day-to-day life: a glass breaking, heat transferring from a hot object to a cold object, conversation of kinetic energy into thermal energy, and a puff of gas expanding from corner of a vacuum chamber into the entire chamber. If you saw a video of glass un-breaking, or two objects in thermal contact where one gets hotter and the other gets colder, or a hot stationary object spontaneously cooling off and moving in some direction, or a gas in a chamber "regrouping" into a corner, you would say this is a video run backwards. This is because you intuitively recognize these are examples of **irreversible process**, and these processes in nature only take place in the forward direction, not reverse (See **Figure 9.32**).

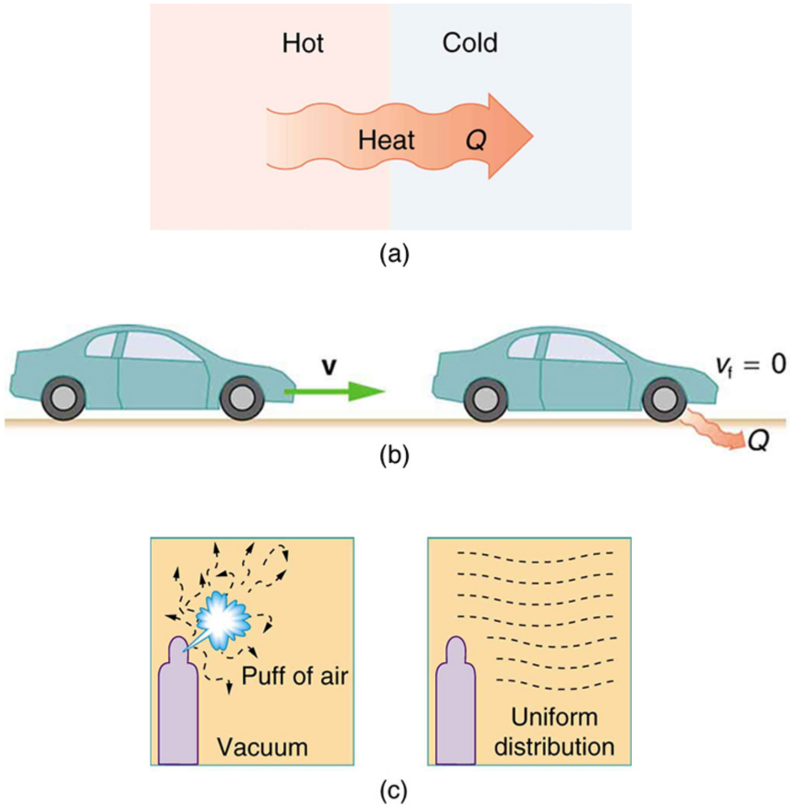


Figure 9.32 Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold and not from cold to hot. (b) The brakes of this car convert its kinetic energy to heat transfer to the environment. The reverse process is impossible. (c) The burst of gas let into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will never return them to the corner.

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur—none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but which in fact are equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes.

The already familiar direction of heat transfer from hot to cold is the basis of our first version of the **second law of thermodynamics**.

The Second Law of Thermodynamics (first expression)

Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction.

Another way of stating this: It is impossible for any process to have as its sole result heat transfer from a cooler to a hotter object.

Heat Engines

Now let us consider a device that uses heat transfer to do work. As noted in the previous section, such a device is called a heat engine, and one is shown schematically in **Figure 9.33**(b). Gasoline and diesel engines, jet engines, and steam turbines are all heat engines that do work by using part of the heat transfer from some source. Heat transfer from the hot object (or hot reservoir) is denoted as Q_h, while heat transfer into the cold object (or cold reservoir) is Q_c, and the work done by the engine is W. The temperatures of the hot and cold reservoirs are T_h and T_c, respectively.

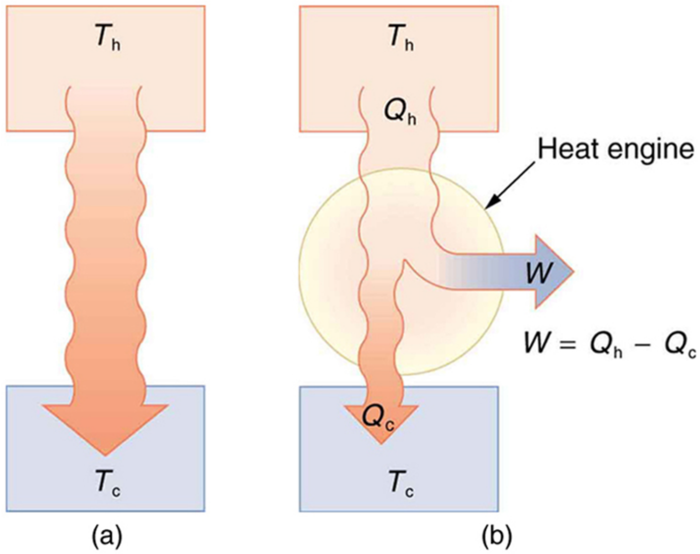


Figure 9.33 (a) Heat transfer occurs spontaneously from a hot object to a cold one, consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the heat transfer to do work. The hot and cold objects are called the hot and cold reservoirs. Q_{h} is the heat transfer out of the hot reservoir, W is the work output, and Q_{c} is the heat transfer into the cold reservoir.

Because the hot reservoir is heated externally, which is energy intensive, it is important that the work is done as efficiently as possible. In fact, we would like W to equal Q_{h} , and for there to be no heat transfer to the environment ($Q_{\text{c}} = 0$).

Unfortunately, this is impossible. The **second law of thermodynamics** also states, with regard to using heat transfer to do work (the second expression of the second law):

The Second Law of Thermodynamics (second expression)

It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes. The second law, just stated in its second form, clearly states that such engines cannot have perfect conversion of heat transfer into work done. Before going into the underlying reasons for the limits on converting heat transfer into work, we need to explore the relationships among W , Q_{h} , and Q_{c} , and to define the efficiency of a cyclical heat engine. As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy U is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that

$$\Delta U = Q - W, \tag{9.42}$$

where Q is the *net* heat transfer during the cycle ($Q = Q_{\text{h}} - Q_{\text{c}}$) and W is the net work done by the system. Since $\Delta U = 0$ for a complete cycle, we have

$$0 = Q - W, \tag{9.43}$$

so that

$$W = Q. \tag{9.44}$$

Thus the net work done by the system equals the net heat transfer into the system, or

$$W = Q_{\text{h}} - Q_{\text{c}} \text{ (cyclical process)}, \tag{9.45}$$

just as shown schematically in **Figure 9.33**(b). The problem is that in all processes, there is some heat transfer Q_{c} to the environment—and usually a very significant amount at that.

In the conversion of energy to work, we are always faced with the problem of getting less out than we put in. We define *conversion efficiency* Eff to be the ratio of useful work output to the energy input (or, in other words, the ratio of what we get to what we spend). In that spirit, we define the efficiency of a heat engine to be its net work output W divided by heat transfer to the engine Q_{h} ; that is,

$$Eff = \frac{W}{Q_h}. \tag{9.46}$$

Since $W = Q_h - Q_c$ in a cyclical process, we can also express this as

$$Eff = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \text{ (cyclical process)}, \tag{9.47}$$

making it clear that an efficiency of 1, or 100%, is possible only if there is no heat transfer to the environment ($Q_c = 0$). Note that all Q s are positive. The direction of heat transfer is indicated by a plus or minus sign. For example, Q_c is out of the system and so is preceded by a minus sign.

Example 9.7 Daily Work Done by a Coal-Fired Power Station, Its Efficiency and Carbon Dioxide Emissions

A coal-fired power station is a huge heat engine. It uses heat transfer from burning coal to do work to turn turbines, which are used to generate electricity. In a single day, a large coal power station has $2.50\times10^{14}\text{ J}$ of heat transfer from coal and $1.48\times10^{14}\text{ J}$ of heat transfer into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station? (c) In the combustion process, the following chemical reaction occurs: $C + O_2 \rightarrow CO_2$. This implies that every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of carbon dioxide into the atmosphere. Assuming that 1 kg of coal can provide $2.5\times10^{6}\text{ J}$ of heat transfer upon combustion, how much CO_2 is emitted per day by this power plant?

Strategy for (a)

We can use $W = Q_h - Q_c$ to find the work output W, assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators, and then condensed back to water to start the cycle again.

Solution for (a)

Work output is given by:

$$W = Q_h - Q_c. \tag{9.48}$$

Substituting the given values:

$$\begin{aligned} W &= 2.50\times10^{14}\text{ J} - 1.48\times10^{14}\text{ J} \\ &= 1.02\times10^{14}\text{ J}. \end{aligned} \tag{9.49}$$

Strategy for (b)

The efficiency can be calculated with $Eff = \frac{W}{Q_h}$ since Q_h is given and work W was found in the first part of this example.

Solution for (b)

Efficiency is given by: $Eff = \frac{W}{Q_h}$. The work W was just found to be $1.02 \times 10^{14}\text{ J}$, and Q_h is given, so the efficiency is

$$\begin{aligned} Eff &= \frac{1.02\times10^{14}\text{ J}}{2.50\times10^{14}\text{ J}} \\ &= 0.408\text{, or }40.8\% \end{aligned} \tag{9.50}$$

Strategy for (c)

The daily consumption of coal is calculated using the information that each day there is $2.50\times10^{14}\text{ J}$ of heat transfer from coal. In the combustion process, we have $C + O_2 \rightarrow CO_2$. So every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of CO_2 into the atmosphere.

Solution for (c)

The daily coal consumption is

$$\frac{2.50\times10^{14}\ \text{J}}{2.50\times10^{6}\ \text{J/kg}} = 1.0\times10^{8}\ \text{kg}. \tag{9.51}$$

Assuming that the coal is pure and that all the coal goes toward producing carbon dioxide, the carbon dioxide produced per day is

$$1.0\times10^{8}\ \text{kg coal}\times\frac{44\ \text{kg CO}_2}{12\ \text{kg coal}} = 3.7\times10^{8}\ \text{kg CO}_2. \tag{9.52}$$

This is 370,000 metric tons of CO_2 produced every day.

Discussion

If all the work output is converted to electricity in a period of one day, the average power output is 1180 MW (this is left to you as an end-of-chapter problem). This value is about the size of a large-scale conventional power plant. The efficiency found is acceptably close to the value of 42% given for coal power stations. It means that fully 59.2% of the energy is heat transfer to the environment, which usually results in warming lakes, rivers, or the ocean near the power station, and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the heat transfer to the environment could be, and sometimes is, used for heating homes or for industrial processes. The generally low cost of energy has not made it economical to make better use of the waste heat transfer from most heat engines. Coal-fired power plants produce the greatest amount of CO_2 per unit energy output (compared to natural gas or oil), making coal the least efficient fossil fuel.

With the information given in **Example 9.7**, we can find characteristics such as the efficiency of a heat engine without any knowledge of how the heat engine operates, but looking further into the mechanism of the engine will give us greater insight. **Figure 9.34** illustrates the operation of the common four-stroke gasoline engine. The four steps shown complete this heat engine's cycle, bringing the gasoline-air mixture back to its original condition.

The **Otto cycle** shown in **Figure 9.35**(a) is used in four-stroke internal combustion engines, although in fact the true Otto cycle paths do not correspond exactly to the strokes of the engine.

The adiabatic process AB corresponds to the nearly adiabatic compression stroke of the gasoline engine. In both cases, work is done on the system (the gas mixture in the cylinder), increasing its temperature and pressure. Along path BC of the Otto cycle, heat transfer Q_h into the gas occurs at constant volume, causing a further increase in pressure and temperature. This process corresponds to burning fuel in an internal combustion engine, and takes place so rapidly that the volume is nearly constant. Path CD in the Otto cycle is an adiabatic expansion that does work on the outside world, just as the power stroke of an internal combustion engine does in its nearly adiabatic expansion. The work done by the system along path CD is greater than the work done on the system along path AB, because the pressure is greater, and so there is a net work output. Along path DA in the Otto cycle, heat transfer Q_c from the gas at constant volume reduces its temperature and pressure, returning it to its original state. In an internal combustion engine, this process corresponds to the exhaust of hot gases and the intake of an air-gasoline mixture at a considerably lower temperature. In both cases, heat transfer into the environment occurs along this final path.

The net work done by a cyclical process is the area inside the closed path on a PV diagram, such as that inside path ABCDA in **Figure 9.35**. Note that in every imaginable cyclical process, it is absolutely necessary for heat transfer from the system to occur in order to get a net work output. In the Otto cycle, heat transfer occurs along path DA. If no heat transfer occurs, then the return path is the same, and the net work output is zero. The lower the temperature on the path AB, the less work has to be done to compress the gas. The area inside the closed path is then greater, and so the engine does more work and is thus more efficient. Similarly, the higher the temperature along path CD, the more work output there is. (See **Figure 9.36**.) So efficiency is related to the temperatures of the hot and cold reservoirs. In the next section, we shall see what the absolute limit to the efficiency of a heat engine is, and how it is related to temperature.

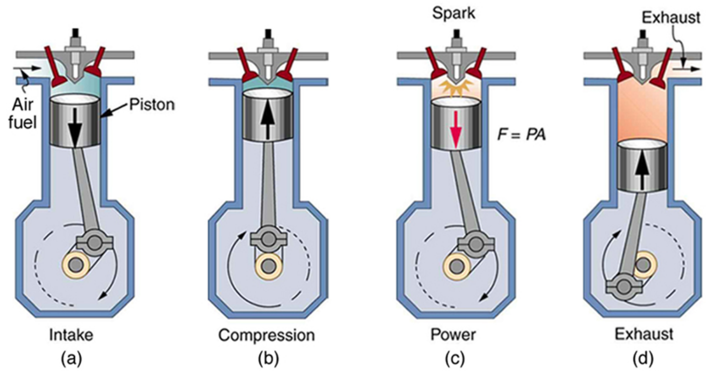


Figure 9.34 In the four-stroke internal combustion gasoline engine, heat transfer into work takes place in the cyclical process shown here. The piston is connected to a rotating crankshaft, which both takes work out of and does work on the gas in the cylinder. (a) Air is mixed with fuel during the intake stroke. (b) During the compression stroke, the air-fuel mixture is rapidly compressed in a nearly adiabatic process, as the piston rises with the valves closed. Work is done on the gas. (c) The power stroke has two distinct parts. First, the air-fuel mixture is ignited, converting chemical potential energy into thermal energy almost instantaneously, which leads to a great increase in pressure. Then the piston descends, and the gas does work by exerting a force through a distance in a nearly adiabatic process. (d) The exhaust stroke expels the hot gas to prepare the engine for another cycle, starting again with the intake stroke.

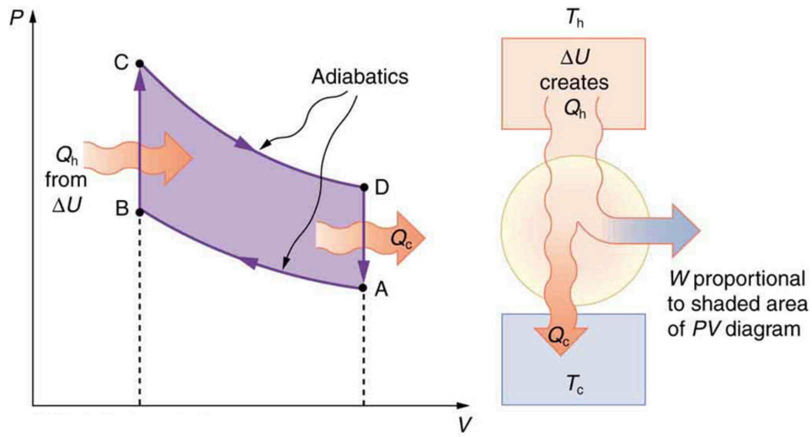


Figure 9.35 PV diagram for a simplified Otto cycle, analogous to that employed in an internal combustion engine. Point A corresponds to the start of the compression stroke of an internal combustion engine. Paths AB and CD are adiabatic and correspond to the compression and power strokes of an internal combustion engine, respectively. Paths BC and DA are isochoric and accomplish similar results to the ignition and exhaust-intake portions, respectively, of the internal combustion engine's cycle. Work is done on the gas along path AB, but more work is done by the gas along path CD, so that there is a net work output.

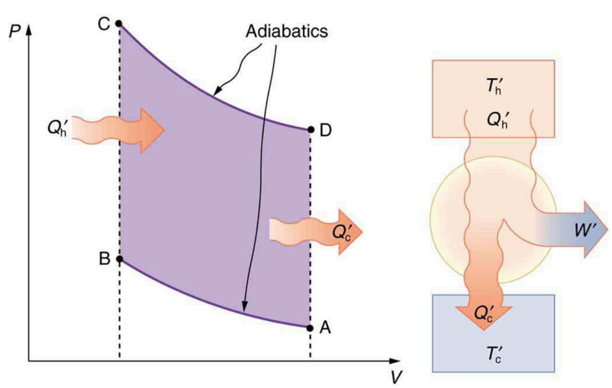


Figure 9.36 This Otto cycle produces a greater work output than the one in **Figure 9.35**, because the starting temperature of path CD is higher and the starting temperature of path AB is lower. The area inside the loop is greater, corresponding to greater net work output.

9.10 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

Figure 9.37 This novelty toy, known as the drinking bird, is an example of Carnot's engine. It contains methylene chloride (mixed with a dye) in the abdomen, which boils at a very low temperature—about 100°F. To operate, one gets the bird's head wet. As the water evaporates, fluid moves up into the head, causing the bird to become top-heavy and dip forward back into the water. This cools down the methylene chloride in the head, and it moves back into the abdomen, causing the bird to become bottom heavy and tip up. Except for a very small input of energy—the original head-wetting—the bird becomes a perpetual motion machine of sorts. (credit: Arabesk.nl, Wikimedia Commons)

We know from the second law of thermodynamics that a heat engine cannot be 100% efficient, since there must always be some heat transfer Q_{c} to the environment, which is often called waste heat. How efficient, then, can a heat engine be? This question was answered at a theoretical level in 1824 by a young French engineer, Sadi Carnot (1796–1832), in his study of the then-emerging heat engine technology crucial to the Industrial Revolution. He devised a theoretical cycle, now called the **Carnot cycle**, which is the most efficient cyclical process possible. The second law of thermodynamics can be restated in terms of the Carnot cycle, and so what Carnot actually discovered was this fundamental law. Any heat engine employing the Carnot cycle is called a **Carnot engine**.

What is crucial to the Carnot cycle—and, in fact, defines it—is that only reversible processes are used. Irreversible processes involve dissipative factors, such as friction and turbulence. This increases heat transfer Q_{c} to the environment and reduces the efficiency of the engine. Obviously, then, reversible processes are superior.

Carnot Engine

Stated in terms of reversible processes, the **second law of thermodynamics** has a third form:

A Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures. Furthermore, all engines employing only reversible processes have this same maximum efficiency when operating between the same given temperatures.

Figure 9.38 shows the PV diagram for a Carnot cycle. The cycle comprises two isothermal and two adiabatic processes. Recall that both isothermal and adiabatic processes are, in principle, reversible.

Carnot also determined the efficiency of a perfect heat engine—that is, a Carnot engine. It is always true that the efficiency of a cyclical heat engine is given by:

$$Eff = \frac{Q_{\text{h}} - Q_{\text{c}}}{Q_{\text{h}}} = 1 - \frac{Q_{\text{c}}}{Q_{\text{h}}}. \tag{9.53}$$

What Carnot found was that for a perfect heat engine, the ratio $Q_{\text{c}} / Q_{\text{h}}$ equals the ratio of the absolute temperatures of the heat reservoirs. That is, $Q_{\text{c}} / Q_{\text{h}} = T_{\text{c}} / T_{\text{h}}$ for a Carnot engine, so that the maximum or **Carnot efficiency** Eff_{C} is given by

$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}, \tag{9.54}$$

where T_{h} and T_{c} are in kelvins (or any other absolute temperature scale). No real heat engine can do as well as the Carnot efficiency—an actual efficiency of about 0.7 of this maximum is usually the best that can be accomplished. But the ideal Carnot engine, like the drinking bird above, while a fascinating novelty, has zero power. This makes it unrealistic for any applications.

Carnot's interesting result implies that 100% efficiency would be possible only if $T_c = 0 \text{ K}$ —that is, only if the cold reservoir were at absolute zero, a practical and theoretical impossibility. But the physical implication is this—the only way to have all heat transfer go into doing work is to remove *all* thermal energy, and this requires a cold reservoir at absolute zero.

It is also apparent that the greatest efficiencies are obtained when the ratio T_c / T_h is as small as possible. Just as discussed for the Otto cycle in the previous section, this means that efficiency is greatest for the highest possible temperature of the hot reservoir and lowest possible temperature of the cold reservoir. (This setup increases the area inside the closed loop on the PV diagram; also, it seems reasonable that the greater the temperature difference, the easier it is to divert the heat transfer to work.) The actual reservoir temperatures of a heat engine are usually related to the type of heat source and the temperature of the environment into which heat transfer occurs. Consider the following example.

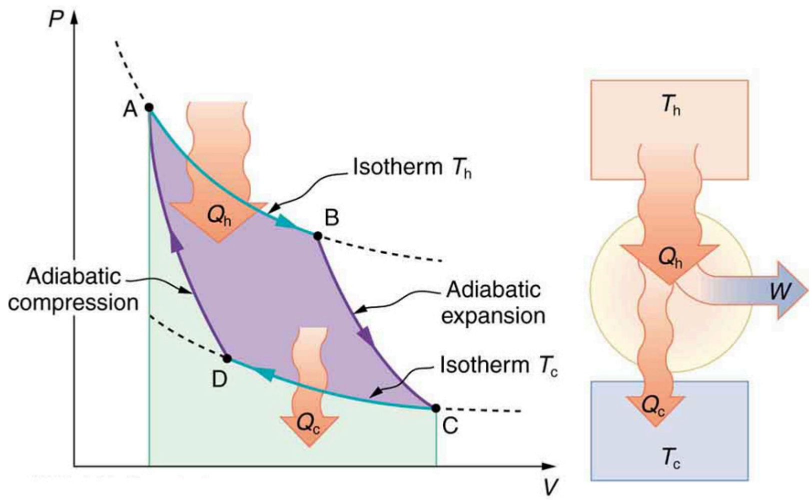


Figure 9.38 PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Q_h occurs into the working substance during the isothermal path AB, which takes place at constant temperature T_h. Heat transfer Q_c occurs out of the working substance during the isothermal path CD, which takes place at constant temperature T_c. The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures T_h and T_c. Any heat engine using reversible processes and operating between these two temperatures will have the same maximum efficiency as the Carnot engine.

Example 9.8 Maximum Theoretical Efficiency for a Nuclear Reactor

A nuclear power reactor has pressurized water at 300°C. (Higher temperatures are theoretically possible but practically not, due to limitations with materials used in the reactor.) Heat transfer from this water is a complex process (see **Figure 9.39**). Steam, produced in the steam generator, is used to drive the turbine-generators. Eventually the steam is condensed to water at 27°C and then heated again to start the cycle over. Calculate the maximum theoretical efficiency for a heat engine operating between these two temperatures.

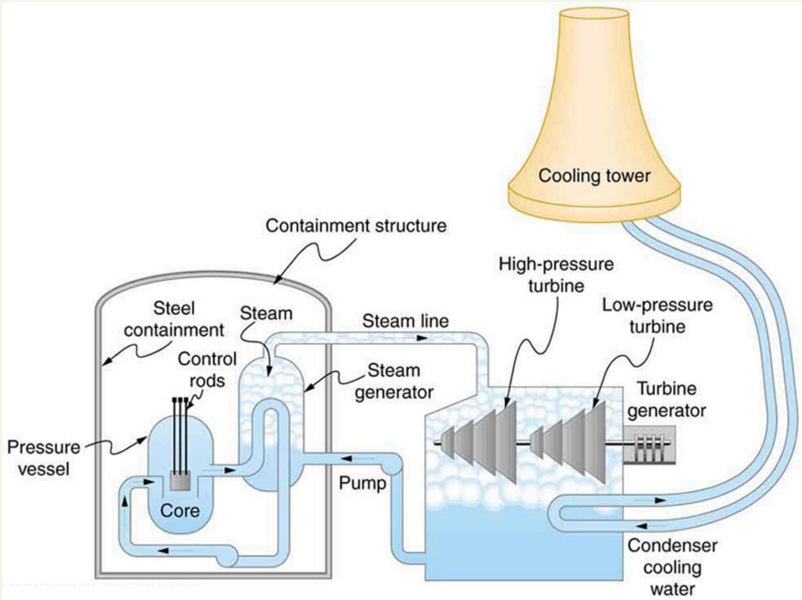


Figure 9.39 Schematic diagram of a pressurized water nuclear reactor and the steam turbines that convert work into electrical energy. Heat exchange is used to generate steam, in part to avoid contamination of the generators with radioactivity. Two turbines are used because this is less expensive than operating a single generator that produces the same amount of electrical energy. The steam is condensed to liquid before being returned to the heat exchanger, to keep exit steam pressure low and aid the flow of steam through the turbines (equivalent to using a lower-temperature cold reservoir). The considerable energy associated with condensation must be dissipated into the local environment; in this example, a cooling tower is used so there is no direct heat transfer to an aquatic environment. (Note that the water going to the cooling tower does not come into contact with the steam flowing over the turbines.)

Strategy

Since temperatures are given for the hot and cold reservoirs of this heat engine, $Eff_C = 1 - \frac{T_c}{T_h}$ can be used to calculate the Carnot (maximum theoretical) efficiency. Those temperatures must first be converted to kelvins.

Solution

The hot and cold reservoir temperatures are given as $300ºC$ and $27.0ºC$, respectively. In kelvins, then, $T_h = 573\text{ K}$ and $T_c = 300\text{ K}$, so that the maximum efficiency is

$$Eff_C = 1 - \frac{T_c}{T_h}. \tag{9.55}$$

Thus,

$$\begin{aligned} Eff_C &= 1 - \frac{300\text{ K}}{573\text{ K}} \\ &= 0.476,\text{ or }47.6\%. \end{aligned} \tag{9.56}$$

Discussion

A typical nuclear power station's actual efficiency is about 35%, a little better than 0.7 times the maximum possible value, a tribute to superior engineering. Electrical power stations fired by coal, oil, and natural gas have greater actual efficiencies (about 42%), because their boilers can reach higher temperatures and pressures. The cold reservoir temperature in any of these power stations is limited by the local environment. **Figure 9.40** shows (a) the exterior of a nuclear power station and (b) the exterior of a coal-fired power station. Both have cooling towers into which water from the condenser enters the tower near the top and is sprayed downward, cooled by evaporation.

(a)

(b)

Figure 9.40 (a) A nuclear power station (credit: BlatantWorld.com) and (b) a coal-fired power station. Both have cooling towers in which water evaporates into the environment, representing Q_{c} . The nuclear reactor, which supplies Q_{h} , is housed inside the dome-shaped containment buildings. (credit: Robert & Mihaela Vicol, publicphoto.org)

Since all real processes are irreversible, the actual efficiency of a heat engine can never be as great as that of a Carnot engine, as illustrated in **Figure 9.41**(a). Even with the best heat engine possible, there are always dissipative processes in peripheral equipment, such as electrical transformers or car transmissions. These further reduce the overall efficiency by converting some of the engine's work output back into heat transfer, as shown in **Figure 9.41**(b).

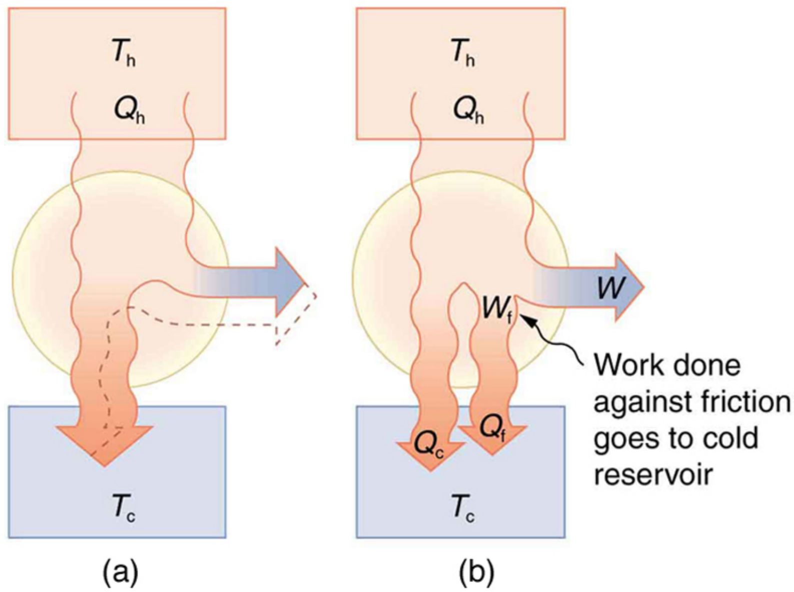


Figure 9.41 Real heat engines are less efficient than Carnot engines. (a) Real engines use irreversible processes, reducing the heat transfer to work. Solid lines represent the actual process; the dashed lines are what a Carnot engine would do between the same two reservoirs. (b) Friction and other dissipative processes in the output mechanisms of a heat engine convert some of its work output into heat transfer to the environment.

9.11 Applications of Thermodynamics: Heat Pumps and Refrigerators

Figure 9.42 Almost every home contains a refrigerator. Most people don't realize they are also sharing their homes with a heat pump. (credit: Id1337x, Wikimedia Commons)

Heat pumps, air conditioners, and refrigerators utilize heat transfer from cold to hot. They are heat engines run backward. We say backward, rather than reverse, because except for Carnot engines, all heat engines, though they can be run backward, cannot truly be reversed. Heat transfer occurs from a cold reservoir Q_c and into a hot one. This requires work input W, which is also converted to heat transfer. Thus the heat transfer to the hot reservoir is $Q_h = Q_c + W$. (Note that Q_h, Q_c, and W are positive, with their directions indicated on schematics rather than by sign.) A heat pump's mission is for heat transfer Q_h to occur into a warm environment, such as a home in the winter. The mission of air conditioners and refrigerators is for heat transfer Q_c to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment. (Actually, a heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. In this section we will concentrate on its heating mode.)

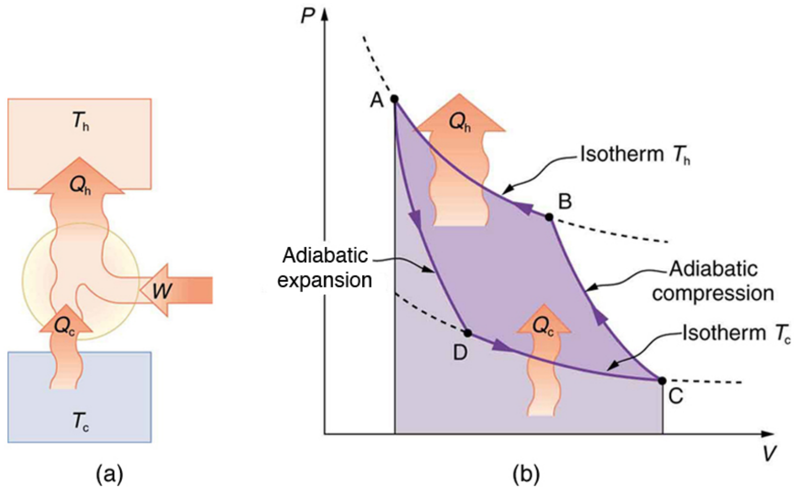


Figure 9.43 Heat pumps, air conditioners, and refrigerators are heat engines operated backward. The one shown here is based on a Carnot (reversible) engine. (a) Schematic diagram showing heat transfer from a cold reservoir to a warm reservoir with a heat pump. The directions of W, Q_h, and Q_c are opposite what they would be in a heat engine. (b) PV diagram for a Carnot cycle similar to that in **Figure 9.44** but reversed, following path ADCBA. The area inside the loop is negative, meaning there is a net work input. There is heat transfer Q_c into the system from a cold reservoir along path DC, and heat transfer Q_h out of the system into a hot reservoir along path BA.

Heat Pumps

The great advantage of using a heat pump to keep your home warm, rather than just burning fuel, is that a heat pump supplies $Q_h = Q_c + W$. Heat transfer is from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_c from the outside at no cost; in many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage is that the work input (required by the second law of thermodynamics) is sometimes more expensive than simply burning fuel, especially if the work is done by electrical energy.

The basic components of a heat pump in its heating mode are shown in **Figure 9.44**. A working fluid such as a non-CFC refrigerant is used. In the outdoor coils (the evaporator), heat transfer Q_c occurs to the working fluid from the cold outdoor air, turning it into a gas.

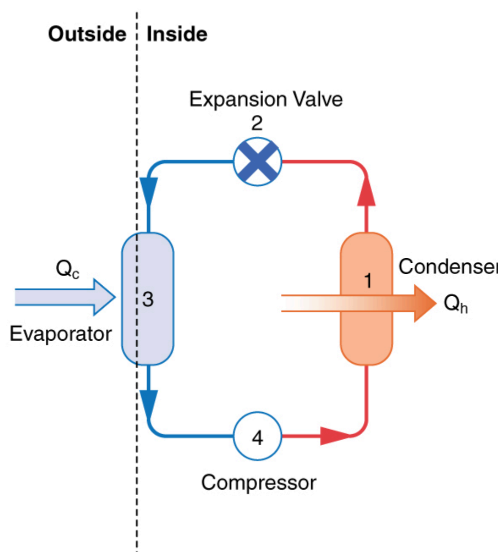


Figure 9.44 A simple heat pump has four basic components: (1) condenser, (2) expansion valve, (3) evaporator, and (4) compressor. In the heating mode, heat transfer Q_c occurs to the working fluid in the evaporator (3) from the colder outdoor air, turning it into a gas. The electrically driven compressor (4) increases the temperature and pressure of the gas and forces it into the condenser coils (1) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfer from the gas to the room occurs as the gas condenses to a liquid. The working fluid is then cooled as it flows back through an expansion valve (2) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfer to the room occurs and the gas condenses to a liquid. The liquid then flows back through a pressure-reducing valve to the outdoor evaporator coils, being cooled through expansion. (In a cooling cycle, the evaporator and condenser coils exchange roles and the flow direction of the fluid is reversed.)

The quality of a heat pump is judged by how much heat transfer Q_{h} occurs into the warm space compared with how much work input W is required. In the spirit of taking the ratio of what you get to what you spend, we define a **heat pump's coefficient of performance** (COP_{hp}) to be

$$COP_{\text{hp}} = \frac{Q_{\text{h}}}{W}. \tag{9.57}$$

Since the efficiency of a heat engine is $Eff = W / Q_{\text{h}}$, we see that $COP_{\text{hp}} = 1 / Eff$, an important and interesting fact. First, since the efficiency of any heat engine is less than 1, it means that COP_{hp} is always greater than 1—that is, a heat pump always has more heat transfer Q_{h} than work put into it. Second, it means that heat pumps work best when temperature differences are small. The efficiency of a perfect, or Carnot, engine is $Eff_{\text{C}} = 1 - (T_{\text{c}} / T_{\text{h}})$; thus, the smaller the temperature difference, the smaller the efficiency and the greater the COP_{hp} (because $COP_{\text{hp}} = 1 / Eff$). In other words, heat pumps do not work as well in very cold climates as they do in more moderate climates.

Friction and other irreversible processes reduce heat engine efficiency, but they do *not* benefit the operation of a heat pump—instead, they reduce the work input by converting part of it to heat transfer back into the cold reservoir before it gets into the heat pump.

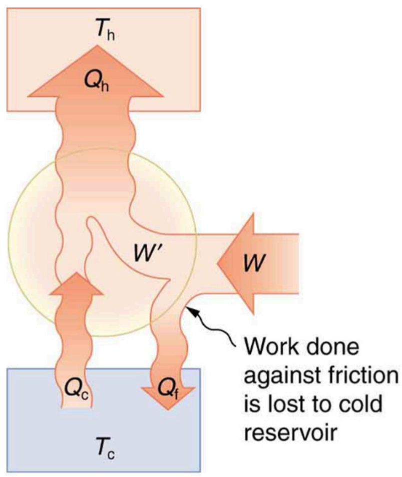


Figure 9.45 When a real heat engine is run backward, some of the intended work input (W) goes into heat transfer before it gets into the heat engine, thereby reducing its coefficient of performance COP_{hp} . In this figure, W' represents the portion of W that goes into the heat pump, while the remainder of W is lost in the form of frictional heat (Q_f) to the cold reservoir. If all of W had gone into the heat pump, then Q_{h} would have been greater. The best heat pump uses adiabatic and isothermal processes, since, in theory, there would be no dissipative processes to reduce the heat transfer to the hot reservoir.

Example 9.9 The Best *COP* hp of a Heat Pump for Home Use

A heat pump used to warm a home must employ a cycle that produces a working fluid at temperatures greater than typical indoor temperature so that heat transfer to the inside can take place. Similarly, it must produce a working fluid at temperatures that are colder than the outdoor temperature so that heat transfer occurs from outside. Its hot and cold reservoir temperatures therefore cannot be too close, placing a limit on its COP_{hp} . (See **Figure 9.46**.) What is the best coefficient of performance possible for such a heat pump, if it has a hot reservoir temperature of 45.0ºC and a cold reservoir temperature of -15.0ºC ?

Strategy

A Carnot engine reversed will give the best possible performance as a heat pump. As noted above, $COP_{\text{hp}} = 1 / Eff$, so that we need to first calculate the Carnot efficiency to solve this problem.

Solution

Carnot efficiency in terms of absolute temperature is given by**:**

$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}. \tag{9.58}$$

The temperatures in kelvins are $T_{\text{h}} = 318\text{ K}$ and $T_{\text{c}} = 258\text{ K}$, so that

$$Eff_{\text{C}} = 1 - \frac{258\text{ K}}{318\text{ K}} = 0.1887. \tag{9.59}$$

Thus, from the discussion above,

$$COP_{\text{hp}} = \frac{1}{Eff} = \frac{1}{0.1887} = 5.30, \tag{9.60}$$

or

$$COP_{\text{hp}} = \frac{Q_{\text{h}}}{W} = 5.30, \tag{9.61}$$

so that

$$Q_{\text{h}} = 5.30\text{ W}. \tag{9.62}$$

Discussion

This result means that the heat transfer by the heat pump is 5.30 times as much as the work put into it. It would cost 5.30 times as much for the same heat transfer by an electric room heater as it does for that produced by this heat pump. This is not a violation of conservation of energy. Cold ambient air provides 4.3 J per 1 J of work from the electrical outlet.

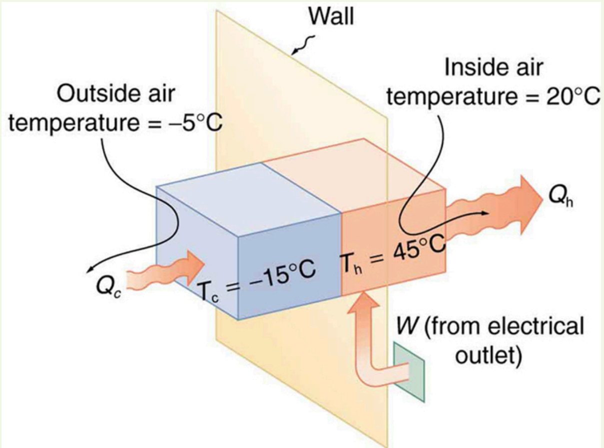


Figure 9.46 Heat transfer from the outside to the inside, along with work done to run the pump, takes place in the heat pump of the example above**.** Note that the cold temperature produced by the heat pump is lower than the outside temperature, so that heat transfer into the working fluid occurs. The pump's compressor produces a temperature greater than the indoor temperature in order for heat transfer into the house to occur.

Real heat pumps do not perform quite as well as the ideal one in the previous example; their values of COP_{hp} range from about 2 to 4. This range means that the heat transfer Q_{h} from the heat pumps is 2 to 4 times as great as the work W put into them. Their economical feasibility is still limited, however, since W is usually supplied by electrical energy that costs more per joule than heat transfer by burning fuels like natural gas. Furthermore, the initial cost of a heat pump is greater than that of many furnaces, so that a heat pump must last longer for its cost to be recovered. Heat pumps are most likely to be economically superior where winter temperatures are mild, electricity is relatively cheap, and other fuels are relatively expensive. Also, since they can cool as well as heat a space, they have advantages where cooling in summer months is also desired. Thus some of the best locations for heat pumps are in warm summer climates with cool winters. **Figure 9.47** shows a heat pump, called a "*reverse cycle"* or "*split-system cooler"* in some countries.

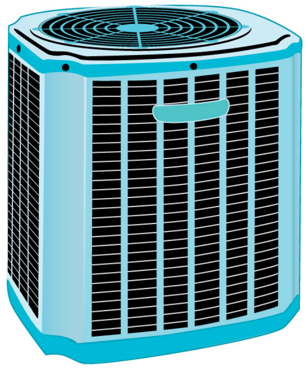


Figure 9.47 In hot weather, heat transfer occurs from air inside the room to air outside, cooling the room. In cool weather, heat transfer occurs from air outside to air inside, warming the room. This switching is achieved by reversing the direction of flow of the working fluid.

Air Conditioners and Refrigerators

Air conditioners and refrigerators are designed to cool something down in a warm environment. As with heat pumps, work input is required for heat transfer from cold to hot, and this is expensive. The quality of air conditioners and refrigerators is judged by how much heat transfer Q_c occurs from a cold environment compared with how much work input W is required. What is considered the benefit in a heat pump is considered waste heat in a refrigerator. We thus define the **coefficient of performance** (COP_{ref}) of an air conditioner or refrigerator to be

$$COP_{ref} = \frac{Q_c}{W}. \tag{9.63}$$

Noting again that $Q_h = Q_c + W$, we can see that an air conditioner will have a lower coefficient of performance than a heat pump, because $COP_{hp} = Q_h / W$ and Q_h is greater than Q_c. In this module's Problems and Exercises, you will show that

$$COP_{ref} = COP_{hp} - 1 \tag{9.64}$$

for a heat engine used as either an air conditioner or a heat pump operating between the same two temperatures. Real air conditioners and refrigerators typically do remarkably well, having values of COP_{ref} ranging from 2 to 6. These numbers are better than the COP_{hp} values for the heat pumps mentioned above, because the temperature differences are smaller, but they are less than those for Carnot engines operating between the same two temperatures.

A type of COP rating system called the "energy efficiency rating" (EER) has been developed. This rating is an example where non-SI units are still used and relevant to consumers. To make it easier for the consumer, Australia, Canada, New Zealand, and the U.S. use an Energy Star Rating out of 5 stars—the more stars, the more energy efficient the appliance. EERs are expressed in mixed units of British thermal units (Btu) per hour of heating or cooling divided by the power input in watts. Room air conditioners are readily available with EERs ranging from 6 to 12. Although not the same as the COPs just described, these EERs are good for comparison purposes—the greater the EER, the cheaper an air conditioner is to operate (but the higher its purchase price is likely to be).

The EER of an air conditioner or refrigerator can be expressed as

$$EER = \frac{Q_c / t_1}{W / t_2}, \tag{9.65}$$

where Q_c is the amount of heat transfer from a cold environment in British thermal units, t_1 is time in hours, W is the work input in joules, and t_2 is time in seconds.

Problem-Solving Strategies for Thermodynamics

1. *Examine the situation to determine whether heat, work, or internal energy are involved.* Look for any system where the primary methods of transferring energy are heat and work. Heat engines, heat pumps, refrigerators, and air conditioners are examples of such systems.
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful. Maximum efficiency means a Carnot engine is involved. Efficiency is not the same as the coefficient of performance.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Be sure to distinguish heat transfer into a system from heat transfer out of the system, as well as work input from work output. In many situations, it is useful to determine the type of process, such as isothermal or adiabatic.

5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. *Substitute the known quantities along with their units into the appropriate equation and obtain numerical solutions complete with units.*
7. *Check the answer to see if it is reasonable: Does it make sense?* For example, efficiency is always less than 1, whereas coefficients of performance are greater than 1.

9.12 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

Figure 9.48 The ice in this drink is slowly melting. Eventually the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics. (credit: Jon Sullivan, PDPhoto.org)

There is yet another way of expressing the second law of thermodynamics. This version relates to a concept called **entropy**. By examining it, we shall see that the directions associated with the second law—heat transfer from hot to cold, for example—are related to the tendency in nature for systems to become disordered and for less energy to be available for use as work. The entropy of a system can in fact be shown to be a measure of its disorder and of the unavailability of energy to do work.

Making Connections: Entropy, Energy, and Work

Recall that the simple definition of energy is the ability to do work. Entropy is a measure of how much energy is not available to do work. Although all forms of energy are interconvertible, and all can be used to do work, it is not always possible, even in principle, to convert the entire available energy into work. That unavailable energy is of interest in thermodynamics, because the field of thermodynamics arose from efforts to convert heat to work.

We can see how entropy is defined by recalling our discussion of the Carnot engine. We noted that for a Carnot cycle, and hence for any reversible processes, $Q_c / Q_h = T_c / T_h$. Rearranging terms yields

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h} \tag{9.66}$$

for any reversible process. Q_c and Q_h are absolute values of the heat transfer at temperatures T_c and T_h , respectively. This ratio of Q / T is defined to be the **change in entropy** ΔS for a reversible process,

$$\Delta S = \left(\frac{Q}{T}\right)_{\text{rev}}, \tag{9.67}$$

where Q is the heat transfer, which is positive for heat transfer into and negative for heat transfer out of, and T is the absolute temperature at which the reversible process takes place. The SI unit for entropy is joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature, avoiding the need to use integral calculus to find ΔS .

The definition of ΔS is strictly valid only for reversible processes, such as used in a Carnot engine. However, we can find ΔS precisely even for real, irreversible processes. The reason is that the entropy S of a system, like internal energy U , depends only on the state of the system and not how it reached that condition. Entropy is a property of state. Thus the change in entropy ΔS of a system between state 1 and state 2 is the same no matter how the change occurs. We just need to find or imagine a reversible process that takes us from state 1 to state 2 and calculate ΔS for that process. That will be the change in entropy for any process going from state 1 to state 2. (See **Figure 9.49**.)

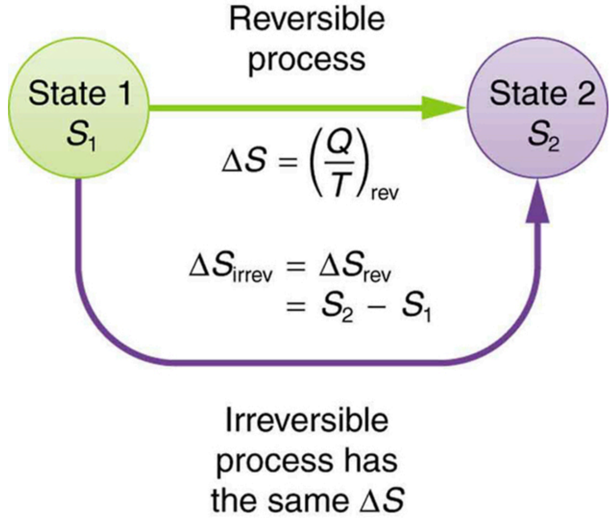


Figure 9.49 When a system goes from state 1 to state 2, its entropy changes by the same amount ΔS, whether a hypothetical reversible path is followed or a real irreversible path is taken.

Now let us take a look at the change in entropy of a Carnot engine and its heat reservoirs for one full cycle. The hot reservoir has a loss of entropy $\Delta S_{\text{h}} = -Q_{\text{h}} / T_{\text{h}}$, because heat transfer occurs out of it (remember that when heat transfers out, then Q has a negative sign). The cold reservoir has a gain of entropy $\Delta S_{\text{c}} = Q_{\text{c}} / T_{\text{c}}$, because heat transfer occurs into it. (We assume the reservoirs are sufficiently large that their temperatures are constant.) So the total change in entropy is

$$\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}. \tag{9.68}$$

Thus, since we know that $Q_{\text{h}} / T_{\text{h}} = Q_{\text{c}} / T_{\text{c}}$ for a Carnot engine,

$$\Delta S_{\text{tot}} = -\frac{Q_{\text{h}}}{T_{\text{h}}} + \frac{Q_{\text{c}}}{T_{\text{c}}} = 0. \tag{9.69}$$

This result, which has general validity, means that *the total change in entropy for a system in any reversible process is zero.*

The entropy of various parts of the system may change, but the total change is zero. Furthermore, the system does not affect the entropy of its surroundings, since heat transfer between them does not occur. Thus the reversible process changes neither the total entropy of the system nor the entropy of its surroundings. Sometimes this is stated as follows: *Reversible processes do not affect the total entropy of the universe.* Real processes are not reversible, though, and they do change total entropy. We can, however, use hypothetical reversible processes to determine the value of entropy in real, irreversible processes. The following example illustrates this point.

Example 9.10 Entropy Increases in an Irreversible (Real) Process

Spontaneous heat transfer from hot to cold is an irreversible process. Calculate the total change in entropy if 4000 J of heat transfer occurs from a hot reservoir at $T_{\text{h}} = 600\text{ K}(327^{\text{o}}\text{ C})$ to a cold reservoir at $T_{\text{c}} = 250\text{ K}(-23^{\text{o}}\text{ C})$, assuming there is no temperature change in either reservoir. (See **Figure 9.50**.)

Strategy

How can we calculate the change in entropy for an irreversible process when $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$ is valid only for reversible processes? Remember that the total change in entropy of the hot and cold reservoirs will be the same whether a reversible or irreversible process is involved in heat transfer from hot to cold. So we can calculate the change in entropy of the hot reservoir for a hypothetical reversible process in which 4000 J of heat transfer occurs from it; then we do the same for a hypothetical reversible process in which 4000 J of heat transfer occurs to the cold reservoir. This produces the same changes in the hot and cold reservoirs that would occur if the heat transfer were allowed to occur irreversibly between them, and so it also produces the same changes in entropy.

Solution

We now calculate the two changes in entropy using $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$. First, for the heat transfer from the hot reservoir,

$$\Delta S_{\text{h}} = \frac{-Q_{\text{h}}}{T_{\text{h}}} = \frac{-4000\text{ J}}{600\text{ K}} = -6.67\text{ J/K}. \tag{9.70}$$

And for the cold reservoir,

$$\Delta S_{\text{c}} = \frac{Q_{\text{c}}}{T_{\text{c}}} = \frac{4000\text{ J}}{250\text{ K}} = 16.0\text{ J/K}. \tag{9.71}$$

Thus the total is

$$\begin{aligned} \Delta S_{\text{tot}} &= \Delta S_{\text{h}} + \Delta S_{\text{c}} \\ &= (-6.67 + 16.0)\text{ J/K} \\ &= 9.33\text{ J/K}. \end{aligned} \tag{9.72}$$

Discussion

There is an *increase* in entropy for the system of two heat reservoirs undergoing this irreversible heat transfer. We will see that this means there is a loss of ability to do work with this transferred energy. Entropy has increased, and energy has become unavailable to do work.

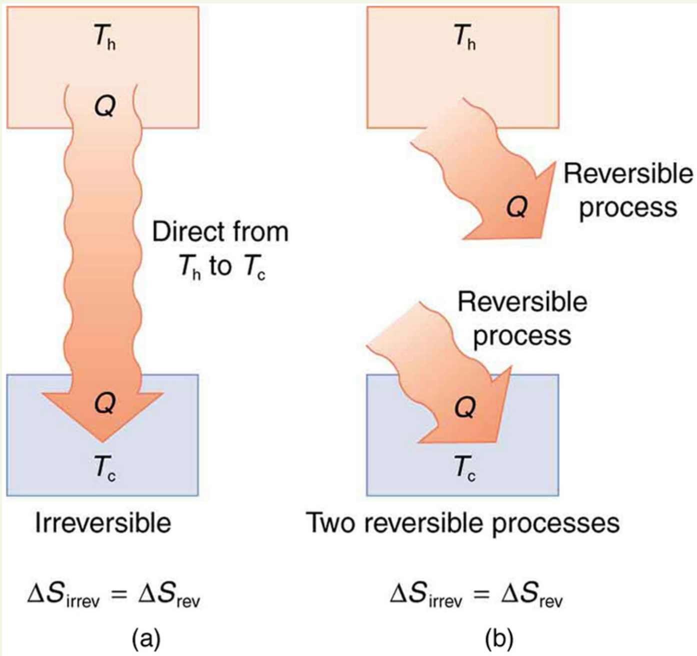


Figure 9.50 (a) Heat transfer from a hot object to a cold one is an irreversible process that produces an overall increase in entropy. (b) The same final state and, thus, the same change in entropy is achieved for the objects if reversible heat transfer processes occur between the two objects whose temperatures are the same as the temperatures of the corresponding objects in the irreversible process.

It is reasonable that entropy increases for heat transfer from hot to cold. Since the change in entropy is Q/T , there is a larger change at lower temperatures. The decrease in entropy of the hot object is therefore less than the increase in entropy of the cold object, producing an overall increase, just as in the previous example. This result is very general:

There is an increase in entropy for any system undergoing an irreversible process.

With respect to entropy, there are only two possibilities: entropy is constant for a reversible process, and it increases for an irreversible process. There is a fourth version of **the second law of thermodynamics stated in terms of entropy**:

The total entropy of a system either increases or remains constant in any process; it never decreases.

For example, heat transfer cannot occur spontaneously from cold to hot, because entropy would decrease.

Entropy is very different from energy. Entropy is *not* conserved but increases in all real processes. Reversible processes (such as in Carnot engines) are the processes in which the most heat transfer to work takes place and are also the ones that keep entropy constant. Thus we are led to make a connection between entropy and the availability of energy to do work.

Order to Disorder

Entropy is related not only to the unavailability of energy to do work—it is also a measure of disorder. This notion was initially postulated by Ludwig Boltzmann in the 1800s. For example, melting a block of ice means taking a highly structured and orderly system of water molecules and converting it into a disorderly liquid in which molecules have no fixed positions. (See **Figure 9.51**.) There is a large increase in entropy in the process, as seen in the following example.

Example 9.11 Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice originally at $0^\circ\ \mathrm{C}$ that is melted to form water at $0^\circ\ \mathrm{C}$.

Strategy

As before, the change in entropy can be calculated from the definition of ΔS once we find the energy Q needed to melt the ice.

Solution

The change in entropy is defined as:

$$\Delta S = \frac{Q}{T}. \tag{9.73}$$

Here Q is the heat transfer necessary to melt 1.00 kg of ice and is given by

$$Q = mL_{\mathrm{f}}, \tag{9.74}$$

where m is the mass and L_{f} is the latent heat of fusion. $L_{\mathrm{f}} = 334\ \mathrm{kJ/kg}$ for water, so that

$$Q = (1.00\ \mathrm{kg})(334\ \mathrm{kJ/kg}) = 3.34\times10^5\ \mathrm{J}. \tag{9.75}$$

Now the change in entropy is positive, since heat transfer occurs into the ice to cause the phase change; thus,

$$\Delta S = \frac{Q}{T} = \frac{3.34\times10^5\ \mathrm{J}}{T}. \tag{9.76}$$

T is the melting temperature of ice. That is, $T = 0^\circ\mathrm{C} = 273\ \mathrm{K}$. So the change in entropy is

$$\begin{aligned}\Delta S &= \frac{3.34\times10^5\ \mathrm{J}}{273\ \mathrm{K}} \\ &= 1.22\times10^3\ \mathrm{J/K}.\end{aligned} \tag{9.77}$$

Discussion

This is a significant increase in entropy accompanying an increase in disorder.

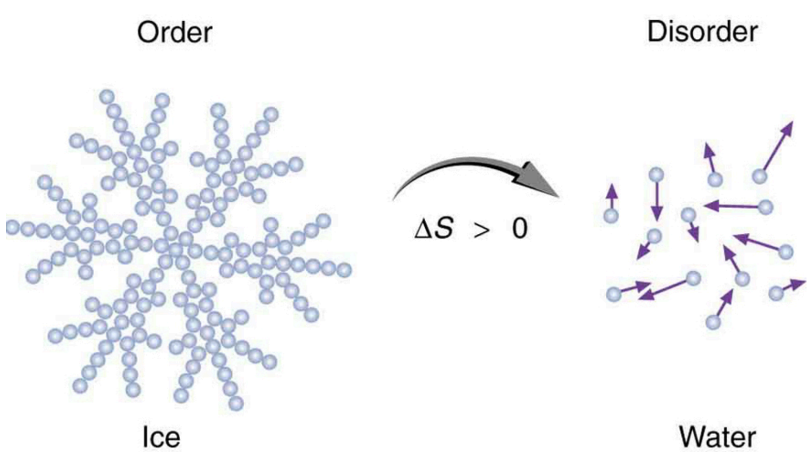


Figure 9.51 When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its entropy increases because heat transfer occurs into it. Entropy is a measure of disorder.

In another easily imagined example, suppose we mix equal masses of water originally at two different temperatures, say $20.0^\circ\ \mathrm{C}$ and $40.0^\circ\ \mathrm{C}$. The result is water at an intermediate temperature of $30.0^\circ\ \mathrm{C}$. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, entropy has increased for the same reason that it did in the example above. Mixing the two bodies of water has the same effect as heat transfer from the hot one and the same heat transfer into the cold one. The mixing decreases the entropy of the hot water but increases the entropy of the cold water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is only one temperature—you cannot run a heat engine with them. The energy that could have been used to run a heat engine is now unavailable to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a uniform temperature.

These three results—entropy, unavailability of energy, and disorder—are not only related but are in fact essentially equivalent.

9.13 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

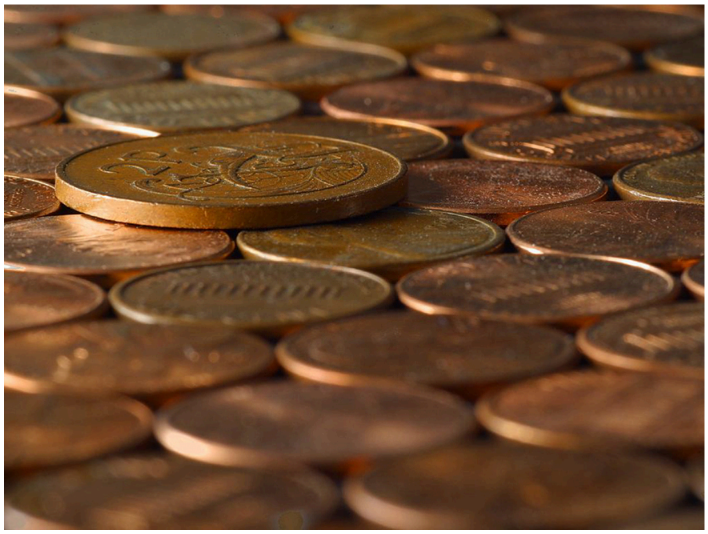


Figure 9.52 When you toss a coin a large number of times, heads and tails tend to come up in roughly equal numbers. Why doesn't heads come up 100, 90, or even 80% of the time? (credit: Jon Sullivan, PDPhoto.org)

The various ways of formulating the second law of thermodynamics tell what happens rather than why it happens. Why should heat transfer occur only from hot to cold? Why should energy become ever less available to do work? Why should the universe become increasingly disorderly? The answer is that it is a matter of overwhelming probability. Disorder is simply vastly more likely than order.

When you watch an emerging rain storm begin to wet the ground, you will notice that the drops fall in a disorganized manner both in time and in space. Some fall close together, some far apart, but they never fall in straight, orderly rows. It is not impossible for rain to fall in an orderly pattern, just highly unlikely, because there are many more disorderly ways than orderly ones. To illustrate this fact, we will examine some random processes, starting with coin tosses.

Coin Tosses

What are the possible outcomes of tossing 5 coins? Each coin can land either heads or tails. On the large scale, we are concerned only with the total heads and tails and not with the order in which heads and tails appear. The following possibilities exist:

$$\begin{gathered}\text{5 heads, 0 tails}\\ \text{4 heads, 1 tail}\\ \text{3 heads, 2 tails}\\ \text{2 heads, 3 tails}\\ \text{1 head, 4 tails}\\ \text{0 head, 5 tails}\end{gathered} \tag{9.78}$$

These are what we call macrostates. A **macrostate** is an overall property of a system. It does not specify the details of the system, such as the order in which heads and tails occur or which coins are heads or tails.

Using this nomenclature, a system of 5 coins has the 6 possible macrostates just listed. Some macrostates are more likely to occur than others. For instance, there is only one way to get 5 heads, but there are several ways to get 3 heads and 2 tails, making the latter macrostate more probable. Table 9.4 lists of all the ways in which 5 coins can be tossed, taking into account the order in which heads and tails occur. Each sequence is called a **microstate**—a detailed description of every element of a system.

Table 9.4 5-Coin Toss

	Individual microstates	Number of microstates
5 heads, 0 tails	HHHHH	1
4 heads, 1 tail	HHHHT, HHHTH, HHTHH, HTHHH, THHHH	5
3 heads, 2 tails	HTHTH, THTHH, HTHHT, THHTH, THHHT HTHTH, THTHH, HTHHT, THHTH, THHHT	10
2 heads, 3 tails	TTTHH, TTHHT, THHTT, HHTTT, TTHTH, THTHT, HTHTT, THTTH, HTTHT, HTTTH	10
1 head, 4 tails	TTTTH, TTTHT, TTHTT, THTTT, HTTTT	5
0 heads, 5 tails	TTTTT	1
		Total: 32

The macrostate of 3 heads and 2 tails can be achieved in 10 ways and is thus 10 times more probable than the one having 5 heads. Not surprisingly, it is equally probable to have the reverse, 2 heads and 3 tails. Similarly, it is equally probable to get 5 tails as it is to get 5 heads. Note that all of these conclusions are based on the crucial assumption that each microstate is equally probable. With coin tosses, this requires that the coins not be asymmetric in a way that favors one side over the other, as with loaded dice. With any system, the assumption that all microstates are equally probable must be valid, or the analysis will be erroneous.

The two most orderly possibilities are 5 heads or 5 tails. (They are more structured than the others.) They are also the least likely, only 2 out of 32 possibilities. The most disorderly possibilities are 3 heads and 2 tails and its reverse. (They are the least structured.) The most disorderly possibilities are also the most likely, with 20 out of 32 possibilities for the 3 heads and 2 tails and its reverse. If we start with an orderly array like 5 heads and toss the coins, it is very likely that we will get a less orderly array as a result, since 30 out of the 32 possibilities are less orderly. So even if you start with an orderly state, there is a strong tendency to go from order to disorder, from low entropy to high entropy. The reverse can happen, but it is unlikely.

Table 9.5 100-Coin Toss

Macrostate		Number of microstates
Heads	Tails	(*W*)
100	0	1
99	1	$1.0{\times}10^2$
95	5	$7.5{\times}10^7$
90	10	$1.7{\times}10^{13}$
75	25	$2.4{\times}10^{23}$
60	40	$1.4{\times}10^{28}$
55	45	$6.1{\times}10^{28}$
51	49	$9.9{\times}10^{28}$
50	50	$1.0{\times}10^{29}$
49	51	$9.9{\times}10^{28}$
45	55	$6.1{\times}10^{28}$
40	60	$1.4{\times}10^{28}$
25	75	$2.4{\times}10^{23}$
10	90	$1.7{\times}10^{13}$
5	95	$7.5{\times}10^7$
1	99	$1.0{\times}10^2$
0	100	1
		Total: $1.27{\times}10^{30}$

This result becomes dramatic for larger systems. Consider what happens if you have 100 coins instead of just 5. The most orderly arrangements (most structured) are 100 heads or 100 tails. The least orderly (least structured) is that of 50 heads and 50 tails. There is only 1 way (1 microstate) to get the most orderly arrangement of 100 heads. There are 100 ways (100 microstates) to get the next most orderly arrangement of 99 heads and 1 tail (also 100 to get its reverse). And there are $1.0{\times}10^{29}$ ways to get 50 heads and 50 tails, the least orderly arrangement. **Table 9.5** is an abbreviated list of the various macrostates and the number of microstates for each macrostate. The total number of microstates—the total number of different ways 100 coins can be tossed—is an impressively large $1.27{\times}10^{30}$. Now, if we start with an orderly macrostate like 100 heads and toss the coins, there is a virtual certainty that we will get a less orderly macrostate. If we keep tossing the coins, it is possible, but exceedingly unlikely, that we will ever get back to the most orderly macrostate. If you tossed the coins once each second, you could expect to get either 100 heads or 100 tails once in $2{\times}10^{22}$ years! This period is 1 trillion (10^{12}) times longer than the age of the universe, and so the chances are essentially zero. In contrast, there is an 8% chance of getting 50 heads, a 73% chance of getting from 45 to 55 heads, and a 96% chance of getting from 40 to 60 heads. Disorder is highly likely.

Disorder in a Gas

The fantastic growth in the odds favoring disorder that we see in going from 5 to 100 coins continues as the number of entities in the system increases. Let us now imagine applying this approach to perhaps a small sample of gas. Because counting microstates and macrostates involves statistics, this is called **statistical analysis**. The macrostates of a gas correspond to its macroscopic properties, such as volume, temperature, and pressure; and its microstates correspond to the detailed description of the positions and velocities of its atoms. Even a small amount of gas has a huge number of atoms: 1.0 cm^3 of an ideal gas at 1.0 atm and 0^o C has $2.7{\times}10^{19}$ atoms. So each macrostate has an immense number of microstates. In plain language, this means that there are an immense number of ways in which the atoms in a gas can be arranged, while still having the same pressure, temperature, and so on.

The most likely conditions (or macrostates) for a gas are those we see all the time—a random distribution of atoms in space with a Maxwell-Boltzmann distribution of speeds in random directions, as predicted by kinetic theory. This is the most disorderly and least structured condition we can imagine. In contrast, one type of very orderly and structured macrostate has all of the atoms in one corner of a container with identical velocities. There are very few ways to accomplish this (very few microstates corresponding to it), and so it is exceedingly unlikely ever to occur. (See **Figure 9.53**(b).) Indeed, it is so unlikely that we have a law saying that it is impossible, which has never been observed to be violated—the second law of thermodynamics.

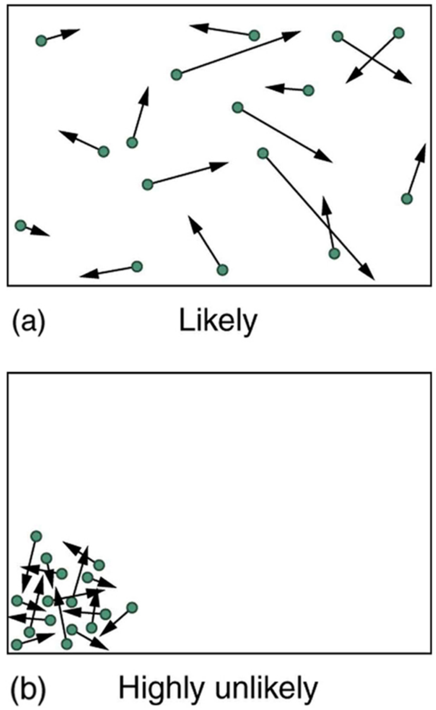


Figure 9.53 (a) The ordinary state of gas in a container is a disorderly, random distribution of atoms or molecules with a Maxwell-Boltzmann distribution of speeds. It is so unlikely that these atoms or molecules would ever end up in one corner of the container that it might as well be impossible. (b) With energy transfer, the gas can be forced into one corner and its entropy greatly reduced. But left alone, it will spontaneously increase its entropy and return to the normal conditions, because they are immensely more likely.

The disordered condition is one of high entropy, and the ordered one has low entropy. With a transfer of energy from another system, we could force all of the atoms into one corner and have a local decrease in entropy, but at the cost of an overall increase in entropy of the universe. If the atoms start out in one corner, they will quickly disperse and become uniformly distributed and will never return to the orderly original state (**Figure 9.53**(b)). Entropy will increase. With such a large sample of atoms, it is possible—but unimaginably unlikely—for entropy to decrease. Disorder is vastly more likely than order.

The arguments that disorder and high entropy are the most probable states are quite convincing. The great Austrian physicist Ludwig Boltzmann (1844–1906)—who, along with Maxwell, made so many contributions to kinetic theory—proved that the entropy of a system in a given state (a macrostate) can be written as

$$S = k\ln W, \tag{9.79}$$

where $k = 1.38\times10^{-23}\ \text{J/K}$ is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate. W is proportional to the probability that the macrostate will occur. Thus entropy is directly related to the probability of a state—the more likely the state, the greater its entropy. Boltzmann proved that this expression for S is equivalent to the definition $\Delta S = Q/T$, which we have used extensively.

Thus the second law of thermodynamics is explained on a very basic level: entropy either remains the same or increases in every process. This phenomenon is due to the extraordinarily small probability of a decrease, based on the extraordinarily larger number of microstates in systems with greater entropy. Entropy *can* decrease, but for any macroscopic system, this outcome is so unlikely that it will never be observed.

Example 9.12 Entropy Increases in a Coin Toss

Suppose you toss 100 coins starting with 60 heads and 40 tails, and you get the most likely result, 50 heads and 50 tails. What is the change in entropy?

Strategy

Noting that the number of microstates is labeled W in **Table 9.5** for the 100-coin toss, we can use $\Delta S = S_\text{f} - S_\text{i} = k\ln W_\text{f}$ - $k\ln W_\text{i}$ to calculate the change in entropy.

Solution

The change in entropy is

$$\Delta S = S_{\text{f}} - S_{\text{i}} = k\ln W_{\text{f}} - k\ln W_{\text{i}}, \tag{9.80}$$

where the subscript i stands for the initial 60 heads and 40 tails state, and the subscript f for the final 50 heads and 50 tails state. Substituting the values for W from **Table 9.5** gives

$$\begin{aligned}\Delta S &= (1.38\times10^{-23}\text{ J/K})[\ln(1.0\times10^{29}) - \ln(1.4\times10^{28})] \\ &= 2.7\times10^{-23}\text{ J/K}\end{aligned} \tag{9.81}$$

Discussion

This increase in entropy means we have moved to a less orderly situation. It is not impossible for further tosses to produce the initial state of 60 heads and 40 tails, but it is less likely. There is about a 1 in 90 chance for that decrease in entropy ($-2.7\times10^{-23}\text{ J/K}$) to occur. If we calculate the decrease in entropy to move to the most orderly state, we get $\Delta S = -92\times10^{-23}\text{ J/K}$. There is about a $1\text{ in }10^{30}$ chance of this change occurring. So while very small decreases in entropy are unlikely, slightly greater decreases are impossibly unlikely. These probabilities imply, again, that for a macroscopic system, a decrease in entropy is impossible. For example, for heat transfer to occur spontaneously from 1.00 kg of 0ºC ice to its 0ºC environment, there would be a decrease in entropy of $1.22\times10^{3}\text{ J/K}$. Given that a $\Delta S\text{ of }10^{-21}\text{ J/K}$ corresponds to about a $1\text{ in }10^{30}$ chance, a decrease of this size (10^{3} J/K) is an *utter* impossibility. Even for a milligram of melted ice to spontaneously refreeze is impossible.

Problem-Solving Strategies for Entropy

1. *Examine the situation to determine if entropy is involved.*
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* You must carefully identify the heat transfer, if any, and the temperature at which the process takes place. It is also important to identify the initial and final states.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* Note that the change in entropy can be determined between any states by calculating it for a reversible process.
6. *Substitute the known value along with their units into the appropriate equation, and obtain numerical solutions complete with units.*
7. *To see if it is reasonable: Does it make sense?* For example, total entropy should increase for any real process or be constant for a reversible process. Disordered states should be more probable and have greater entropy than ordered states.

Glossary

absolute zero: the lowest possible temperature; the temperature at which all molecular motion ceases

adiabatic process: a process in which no heat transfer takes place

Boltzmann constant: k , a physical constant that relates energy to temperature; $k = 1.38\times10^{-23}\text{ J/K}$

Carnot cycle: a cyclical process that uses only reversible processes, the adiabatic and isothermal processes

Carnot efficiency: the maximum theoretical efficiency for a heat engine

Carnot engine: a heat engine that uses a Carnot cycle

Celsius scale: temperature scale in which the freezing point of water is 0ºC and the boiling point of water is 100ºC

change in entropy: the ratio of heat transfer to temperature Q/T

coefficient of performance: for a heat pump, it is the ratio of heat transfer at the output (the hot reservoir) to the work supplied; for a refrigerator or air conditioner, it is the ratio of heat transfer from the cold reservoir to the work supplied

conduction: heat transfer through stationary matter by physical contact

convection: heat transfer by the macroscopic movement of fluid

cyclical process: a process in which the path returns to its original state at the end of every cycle

degree Celsius: unit on the Celsius temperature scale

degree Fahrenheit: unit on the Fahrenheit temperature scale

entropy: a measurement of a system's disorder and its inability to do work in a system

Fahrenheit scale: temperature scale in which the freezing point of water is $32^\circ F$ and the boiling point of water is $212^\circ F$

first law of thermodynamics: states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system

heat: the spontaneous transfer of energy due to a temperature difference

heat engine: a machine that uses heat transfer to do work

heat of sublimation: the energy required to change a substance from the solid phase to the vapor phase

heat pump: a machine that generates heat transfer from cold to hot

human metabolism: conversion of food into heat transfer, work, and stored fat

ideal gas law: the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

internal energy: the sum of the kinetic and potential energies of a system's atoms and molecules

irreversible process: a process which occurs in only one direction in nature; a process that cannot be exactly reversed

isobaric process: constant-pressure process in which a gas does work

isochoric process: a constant-volume process

isothermal process: a constant-temperature process

Kelvin scale: temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

kilocalorie: $1 \text{ kilocalorie} = 1000 \text{ calories}$

latent heat coefficient: a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

macrostate: an overall property of a system

mechanical equivalent of heat: the work needed to produce the same effects as heat transfer

microstate: each sequence within a larger macrostate

Otto cycle: a thermodynamic cycle, consisting of a pair of adiabatic processes and a pair of isochoric processes, that converts heat into work, e.g., the four-stroke engine cycle of intake, compression, ignition, and exhaust

radiation: heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

reversible process: a process in which both the heat engine system and the external environment theoretically can be returned to their original states

second law of thermodynamics: heat transfer flows from a hotter to a cooler object, never the reverse, and some heat energy in any process is lost to available work in a cyclical process

second law of thermodynamics stated in terms of entropy: the total entropy of a system either increases or remains constant; it never decreases

specific heat: the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

statistical analysis: using statistics to examine data, such as counting microstates and macrostates

sublimation: the transition from the solid phase to the vapor phase

temperature: the quantity measured by a thermometer

thermal equilibrium: the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics: law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

Section Summary

9.1 Temperature

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

$$T_{°\text{F}} = \frac{9}{5}T_{°\text{C}} + 32$$

$$T_{°\text{C}} = \frac{5}{9}(T_{°\text{F}} - 32)$$

$$T_{\text{K}} = T_{°\text{C}} + 273.15$$

$$T_{°\text{C}} = T_{\text{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

9.2 The Ideal Gas Law

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:

$$PV = NkT,$$

where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant

$$k = 1.38 \times 10^{-23}\ \text{J/K}.$$

- The ideal gas law is generally valid at temperatures well above the boiling temperature.

9.3 Heat

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5°C and 15.5°C.
- The mechanical equivalent of this heat transfer is $1.00\ \text{kcal} = 4186\ \text{J}$.

9.4 Heat Transfer Methods

- Heat is transferred by three different methods: conduction, convection, and radiation.

9.5 Temperature Change and Heat Capacity

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

9.6 Phase Change and Latent Heat

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:

$$Q = mL,$$

where L is the latent heat coefficient.

9.7 The First Law of Thermodynamics

- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net heat transfer (the sum of all heat transfer into and out of the system), and W is the net work done (the sum of all work done on or by the system).
- Both Q and W are energy in transit; only ΔU represents an independent quantity capable of being stored.
- The internal energy U of a system depends only on the state of the system and not how it reached that state.

9.8 The First Law of Thermodynamics and Heat Engine Processes

- One of the important implications of the first law of thermodynamics is that machines can be harnessed to do work that humans previously did by hand or by external energy supplies such as running water or the heat of the Sun. A machine that uses heat transfer to do work is known as a heat engine.
- There are several simple processes, used by heat engines, that flow from the first law of thermodynamics. Among them are the isobaric, isochoric, isothermal and adiabatic processes.
- These processes differ from one another based on how they affect pressure, volume, temperature, and heat transfer.
- If the work done is performed on the outside environment, work (W) will be a positive value. If the work done is done to the heat engine system, work (W) will be a negative value.
- Some thermodynamic processes, including isothermal and adiabatic processes, are reversible in theory; that is, both the thermodynamic system and the environment can be returned to their initial states. However, because of loss of energy owing to the second law of thermodynamics, complete reversibility does not work in practice.

9.9 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

- The two expressions of the second law of thermodynamics are: (i) Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction; and (ii) It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state. These expressions describe the direction of irreversible processes that occur in nature.
- Cyclical processes are processes that return to their original state at the end of every cycle.
- In a cyclical process, such as a heat engine, the net work done by the system equals the net heat transfer into the system, or $W = Q_\mathrm{h} - Q_\mathrm{c}$, where Q_h is the heat transfer from the hot object (hot reservoir), and Q_c is the heat transfer into the cold object (cold reservoir).
- Efficiency can be expressed as $Eff = \dfrac{W}{Q_\mathrm{h}}$, the ratio of work output divided by the amount of energy input.
- The four-stroke gasoline engine is often explained in terms of the Otto cycle, which is a repeating sequence of processes that convert heat into work.

9.10 Carnot’s Perfect Heat Engine: The Second Law of Thermodynamics Restated

- The Carnot cycle is a theoretical cycle that is the most efficient cyclical process possible. Any engine using the Carnot cycle, which uses only reversible processes (adiabatic and isothermal), is known as a Carnot engine.
- Any engine that uses the Carnot cycle enjoys the maximum theoretical efficiency.
- While Carnot engines are ideal engines, in reality, no engine achieves Carnot’s theoretical maximum efficiency, since dissipative processes, such as friction, play a role. Carnot cycles without heat loss may be possible at absolute zero, but this has never been seen in nature.

9.11 Applications of Thermodynamics: Heat Pumps and Refrigerators

- An artifact of the second law of thermodynamics is the ability to heat an interior space using a heat pump. Heat pumps compress cold ambient air and, in so doing, heat it to room temperature without violation of conservation principles.
- To calculate the heat pump’s coefficient of performance, use the equation $COP_\mathrm{hp} = \dfrac{Q_\mathrm{h}}{W}$.
- A refrigerator is a heat pump; it takes warm ambient air and expands it to chill it.

9.12 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

- Entropy is the loss of energy available to do work.
- Another form of the second law of thermodynamics states that the total entropy of a system either increases or remains constant; it never decreases.
- Change of entropy is zero in a reversible process; it increases in an irreversible process.
- Entropy is also associated with the tendency toward disorder in an *isolated* system.

9.13 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

- Disorder is far more likely than order, which can be seen statistically.

- The entropy of a system in a given state (a macrostate) can be written as

$$S = k \ln W,$$

where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate.

Conceptual Questions

9.1 Temperature

1. What does it mean to say that two systems are in thermal equilibrium?

2. Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

3. When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

4. If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

9.2 The Ideal Gas Law

5. Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

6. A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

9.3 Heat

7. How is heat transfer related to temperature?

8. Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

9. When heat transfers into a system, is the energy stored as heat? Explain briefly.

9.4 Heat Transfer Methods

10. What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0°C hot tub?

Figure 9.54 shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.

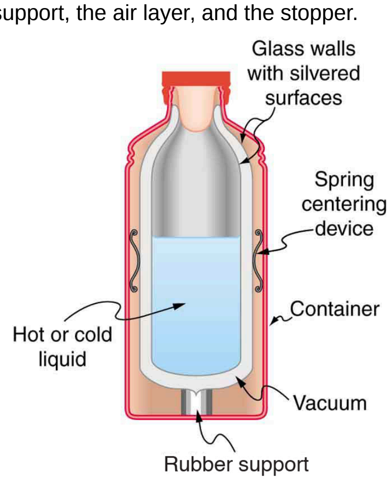


Figure 9.54 The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

9.5 Temperature Change and Heat Capacity

11. What three factors affect the heat transfer that is necessary to change an object's temperature?

12. The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

9.6 Phase Change and Latent Heat

13. Heat transfer can cause temperature and phase changes. What else can cause these changes?

14. How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below 0°C , in the vicinity of large bodies of water?

15. What is the temperature of ice right after it is formed by freezing water?

16. If you place 0°C ice into 0°C water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

17. What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

18. In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about $35^\circ\text{C}(95^\circ\text{F})$. In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

19. In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

20. Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

21. Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

22. When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

23. In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from **Table 9.2**.

9.7 The First Law of Thermodynamics

24. Describe the photo of the tea kettle at the beginning of this section in terms of heat transfer, work done, and internal energy. How is heat being transferred? What is the work done and what is doing it? How does the kettle maintain its internal energy?

25. The first law of thermodynamics and the conservation of energy are clearly related. How do they differ in the types of energy considered?

26. Heat transfer Q and work done W are always energy in transit, whereas internal energy U is energy stored in a system. Give an example of each type of energy, and state specifically how it is either in transit or resides in a system.

27. How do heat transfer and internal energy differ? In particular, which can be stored as such in a system and which cannot?

28. If you run down some stairs and stop, what happens to your kinetic energy and your initial gravitational potential energy?

29. Give an explanation of how food energy (calories) can be viewed as molecular potential energy (consistent with the atomic and molecular definition of internal energy).

30. Identify the type of energy transferred to your body in each of the following as either internal energy, heat transfer, or doing work: (a) basking in sunlight; (b) eating food; (c) riding an elevator to a higher floor.

9.8 The First Law of Thermodynamics and Heat Engine Processes

31. A great deal of effort, time, and money has been spent in the quest for the so-called perpetual-motion machine, which is defined as a hypothetical machine that operates or produces useful work indefinitely and/or a hypothetical machine that produces more work or energy than it consumes. Explain, in terms of heat engines and the first law of thermodynamics, why or why not such a machine is likely to be constructed.

32. One method of converting heat transfer into doing work is for heat transfer into a gas to take place, which expands, doing work on a piston, as shown in the figure below. (a) Is the heat transfer converted directly to work in an isobaric process, or does it go through another form first? Explain your answer. (b) What about in an isothermal process? (c) What about in an adiabatic process (where heat transfer occurred prior to the adiabatic process)?

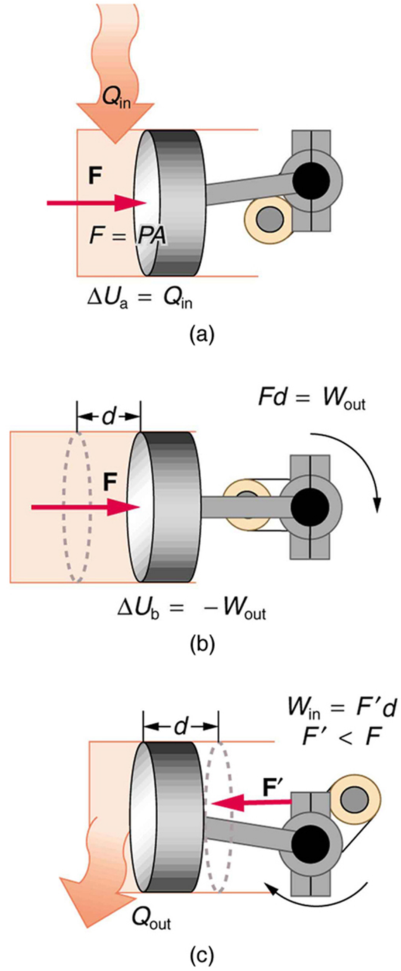


Figure 9.55

33. Would the previous question make any sense for an isochoric process? Explain your answer.

34. We ordinarily say that $\Delta U = 0$ for an isothermal process. Does this assume no phase change takes place? Explain your answer.

35. The temperature of a rapidly expanding gas decreases. Explain why in terms of the first law of thermodynamics. (Hint: Consider whether the gas does work and whether heat transfer occurs rapidly into the gas through conduction.)

36. A real process may be nearly adiabatic if it occurs over a very short time. How does the short time span help the process to be adiabatic?

37. It is unlikely that a process can be isothermal unless it is a very slow process. Explain why. Is the same true for isobaric and isochoric processes? Explain your answer.

9.9 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

38. Imagine you are driving a car up Pike's Peak in Colorado. To raise a car weighing 1000 kilograms a distance of 100 meters would require about a million joules. You could raise a car 12.5 kilometers with the energy in a gallon of gas. Driving up Pike's Peak (a mere 3000-meter climb) should consume a little less than a quart of gas. But other considerations have to be taken into account. Explain, in terms of efficiency, what factors may keep you from realizing your ideal energy use on this trip.

39. Is a temperature difference necessary to operate a heat engine? State why or why not.

40. Definitions of efficiency vary depending on how energy is being converted. Compare the definitions of efficiency for the human body and heat engines. How does the definition of efficiency in each relate to the type of energy being converted into doing work?

41. Why—other than the fact that the second law of thermodynamics says reversible engines are the most efficient—should heat engines employing reversible processes be more efficient than those employing irreversible processes? Consider that dissipative mechanisms are one cause of irreversibility.

9.10 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

42. Think about the drinking bird at the beginning of this section (**Figure 9.37**). Although the bird enjoys the theoretical maximum efficiency possible, if left to its own devices over time, the bird will cease "drinking." What are some of the dissipative processes that might cause the bird's motion to cease?

43. Can improved engineering and materials be employed in heat engines to reduce heat transfer into the environment? Can they eliminate heat transfer into the environment entirely?

44. Does the second law of thermodynamics alter the conservation of energy principle?

9.11 Applications of Thermodynamics: Heat Pumps and Refrigerators

45. Explain why heat pumps do not work as well in very cold climates as they do in milder ones. Is the same true of refrigerators?

46. In some Northern European nations, homes are being built without heating systems of any type. They are very well insulated and are kept warm by the body heat of the residents. However, when the residents are not at home, it is still warm in these houses. What is a possible explanation?

47. Why do refrigerators, air conditioners, and heat pumps operate most cost-effectively for cycles with a small difference between T_h and T_c? (Note that the temperatures of the cycle employed are crucial to its COP.)

48. Grocery store managers contend that there is *less* total energy consumption in the summer if the store is kept at a *low* temperature. Make arguments to support or refute this claim, taking into account that there are numerous refrigerators and freezers in the store.

49. Can you cool a kitchen by leaving the refrigerator door open?

9.12 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

50. Does a gas become more orderly when it liquefies? Does its entropy change? If so, does the entropy increase or decrease? Explain your answer.

51. Explain how water's entropy can decrease when it freezes without violating the second law of thermodynamics. Specifically, explain what happens to the entropy of its surroundings.

52. Is a uniform-temperature gas more or less orderly than one with several different temperatures? Which is more structured? In which can heat transfer result in work done without heat transfer from another system?

53. Give an example of a spontaneous process in which a system becomes less ordered and energy becomes less available to do work. What happens to the system's entropy in this process?

54. What is the change in entropy in an adiabatic process? Does this imply that adiabatic processes are reversible? Can a process be precisely adiabatic for a macroscopic system?

55. Does the entropy of a star increase or decrease as it radiates? Does the entropy of the space into which it radiates (which has a temperature of about 3 K) increase or decrease? What does this do to the entropy of the universe?

56. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

9.13 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

57. Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

Problems & Exercises

9.1 Temperature

1. What is the Fahrenheit temperature of a person with a 39.0°C fever?

2. Frost damage to most plants occurs at temperatures of 28.0°F or lower. What is this temperature on the Kelvin scale?

3. To conserve energy, room temperatures are kept at 68.0°F in the winter and 78.0°F in the summer. What are these temperatures on the Celsius scale?

4. A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

5. The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

6. One of the hottest temperatures ever recorded on the surface of Earth was 134°F in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

7. (a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0°F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

8. (a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

9.2 The Ideal Gas Law

9. The gauge pressure in your car tires is $2.50\times10^5\ \text{N/m}^2$ at a temperature of 35.0°C when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to -40.0°C?

10. Convert an absolute pressure of $7.00\times10^5\ \text{N/m}^2$ to gauge pressure in lb/in^2. (This value was stated to be just less than $90.0\ \text{lb/in}^2$ in **???**. Is it?)

11. Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of 20.0°C. (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is 60.0°C (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

12. Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of 10.0°C and rises to an altitude where its volume is twenty times the original volume and its temperature is -50.0°C? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

13. In the text, it was shown that $N/V = 2.68\times10^{25}\ \text{m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68\times10^{19}\ \text{cm}^{-3}$, as stated. (b) About how many atoms are there in one μm^3 (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

14. An airplane passenger has $100\ \text{cm}^3$ of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50\times10^4\ \text{N/m}^2$?

15. An expensive vacuum system can achieve a pressure as low as $1.00\times10^{-7}\ \text{N/m}^2$ at 20°C. How many atoms are there in a cubic centimeter at this pressure and temperature?

16. The number density of gas atoms at a certain location in the space above our planet is about $1.00\times10^{11}\ \text{m}^{-3}$, and the pressure is $2.75\times10^{-10}\ \text{N/m}^2$ in this space. What is the temperature there?

17. A bicycle tire has a pressure of $7.00\times10^5\ \text{N/m}^2$ at a temperature of 18.0°C and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of $100\ \text{cm}^3$ at atmospheric pressure? Assume tire temperature and volume remain constant.

18. A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40\times10^7\ \text{N/m}^2$ and a temperature of 25.0°C. Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (-78.5°C) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

19. (a) What is the gauge pressure in a 25.0°C car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0°C? Assume the temperature returns to 25.0°C and the volume remains constant.

9.5 Temperature Change and Heat Capacity

20. On a hot day, the temperature of an 80,000-L swimming pool increases by 1.50°C. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.

21. Show that $1 \text{ cal/g} \cdot ^\circ\text{C} = 1 \text{ kcal/kg} \cdot ^\circ\text{C}$.

22. To sterilize a 50.0-g glass baby bottle, we must raise its temperature from 22.0°C to 95.0°C. How much heat transfer is required?

23. The same heat transfer into identical masses of different substances produces different temperature changes. Calculate the final temperature when 1.00 kcal of heat transfers into 1.00 kg of the following, originally at 20.0°C: (a) water; (b) concrete; (c) steel; and (d) mercury.

24. Rubbing your hands together warms them by converting work into thermal energy. If a woman rubs her hands back and forth for a total of 20 rubs, at a distance of 7.50 cm per rub, and with an average frictional force of 40.0 N, what is the temperature increase? The mass of tissues warmed is only 0.100 kg, mostly in the palms and fingers.

25. A 0.250-kg block of a pure material is heated from 20.0°C to 65.0°C by the addition of 4.35 kJ of energy. Calculate its specific heat and identify the substance of which it is most likely composed.

26. Suppose identical amounts of heat transfer into different masses of copper and water, causing identical changes in temperature. What is the ratio of the mass of copper to water?

27. (a) The number of kilocalories in food is determined by calorimetry techniques in which the food is burned and the amount of heat transfer is measured. How many kilocalories per gram are there in a 5.00-g peanut if the energy from burning it is transferred to 0.500 kg of water held in a 0.100-kg aluminum cup, causing a 54.9°C temperature increase? (b) Compare your answer to labeling information found on a package of peanuts and comment on whether the values are consistent.

28. Following vigorous exercise, the body temperature of an 80.0-kg person is 40.0°C. At what rate in watts must the person transfer thermal energy to reduce the the body temperature to 37.0°C in 30.0 min, assuming the body continues to produce energy at the rate of 150 W? $(1 \text{ watt} = 1 \text{ joule/second or } 1 \text{ W} = 1 \text{ J/s})$.

29. Even when shut down after a period of normal use, a large commercial nuclear reactor transfers thermal energy at the rate of 150 MW by the radioactive decay of fission products. This heat transfer causes a rapid increase in temperature if the cooling system fails $(1 \text{ watt} = 1 \text{ joule/second or } 1 \text{ W} = 1 \text{ J/s and } 1 \text{ MW} = 1 \text{ megawatt})$. (a) Calculate the rate of temperature increase in degrees Celsius per second $(^\circ\text{C/s})$ if the mass of the reactor core is $1.60\times10^5 \text{ kg}$ and it has an average specific heat of $0.3349 \text{ kJ/kg}^\circ \cdot \text{C}$. (b) How long would it take to obtain a temperature increase of 2000°C, which could cause some metals holding the radioactive materials to melt? (The initial rate of temperature increase would be greater than that calculated here because the heat transfer is concentrated in a smaller mass. Later, however, the temperature increase would slow down because the $5\times10^5\text{-kg}$ steel containment vessel would also begin to heat up.)

Figure 9.56 Radioactive spent-fuel pool at a nuclear power plant. Spent fuel stays hot for a long time. (credit: U.S. Department of Energy)

9.6 Phase Change and Latent Heat

30. How much heat transfer (in kilocalories) is required to thaw a 0.450-kg package of frozen vegetables originally at 0°C if their heat of fusion is the same as that of water?

31. A bag containing 0°C ice is much more effective in absorbing energy than one containing the same amount of 0°C water.

a. How much heat transfer is necessary to raise the temperature of 0.800 kg of water from 0°C to 30.0°C?
b. How much heat transfer is required to first melt 0.800 kg of 0°C ice and then raise its temperature?
c. Explain how your answer supports the contention that the ice is more effective.

32. (a) How much heat transfer is required to raise the temperature of a 0.750-kg aluminum pot containing 2.50 kg of water from 30.0°C to the boiling point and then boil away 0.750 kg of water? (b) How long does this take if the rate of heat transfer is 500 W $1 \text{ watt} = 1 \text{ joule/second } (1 \text{ W} = 1 \text{ J/s})$?

33. The formation of condensation on a glass of ice water causes the ice to melt faster than it would otherwise. If 8.00 g of condensation forms on a glass containing both water and 200 g of ice, how many grams of the ice will melt as a result? Assume no other heat transfer occurs.

34. On a trip, you notice that a 3.50-kg bag of ice lasts an average of one day in your cooler. What is the average power in watts entering the ice if it starts at 0°C and completely melts to 0°C water in exactly one day $1 \text{ watt} = 1 \text{ joule/second } (1 \text{ W} = 1 \text{ J/s})$?

35. On a certain dry sunny day, a swimming pool's temperature would rise by 1.50°C if not for evaporation. What fraction of the water must evaporate to carry away precisely enough energy to keep the temperature constant?

36. (a) How much heat transfer is necessary to raise the temperature of a 0.200-kg piece of ice from -20.0°C to 130°C, including the energy needed for phase changes?
(b) How much time is required for each stage, assuming a constant 20.0 kJ/s rate of heat transfer?
(c) Make a graph of temperature versus time for this process.

37. In 1986, a gargantuan iceberg broke away from the Ross Ice Shelf in Antarctica. It was approximately a rectangle 160 km long, 40.0 km wide, and 250 m thick.

(a) What is the mass of this iceberg, given that the density of ice is $917\ \text{kg/m}^3$?

(b) How much heat transfer (in joules) is needed to melt it?

(c) How many years would it take sunlight alone to melt ice this thick, if the ice absorbs an average of $100\ \text{W/m}^2$, 12.00 h per day?

38. How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from 95.0°C to 45.0°C? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)

39. (a) It is difficult to extinguish a fire on a crude oil tanker, because each liter of crude oil releases $2.80\times10^7\ \text{J}$ of energy when burned. To illustrate this difficulty, calculate the number of liters of water that must be expended to absorb the energy released by burning 1.00 L of crude oil, if the water has its temperature raised from 20.0°C to 100°C, it boils, and the resulting steam is raised to 300°C. (b) Discuss additional complications caused by the fact that crude oil has a smaller density than water.

40. The energy released from condensation in thunderstorms can be very large. Calculate the energy released into the atmosphere for a small storm of radius 1 km, assuming that 1.0 cm of rain is precipitated uniformly over this area.

41. To help prevent frost damage, 4.00 kg of 0°C water is sprayed onto a fruit tree.

(a) How much heat transfer occurs as the water freezes?

(b) How much would the temperature of the 200-kg tree decrease if this amount of heat transferred from the tree? Take the specific heat to be $3.35\ \text{kJ/kg}\cdot^\circ\text{C}$, and assume that no phase change occurs.

42. A 0.250-kg aluminum bowl holding 0.800 kg of soup at 25.0°C is placed in a freezer. What is the final temperature if 377 kJ of energy is transferred from the bowl and soup, assuming the soup's thermal properties are the same as that of water? Explicitly show how you follow the steps in Problem-Solving Strategies for the Effects of Heat Transfer.

43. A 0.0500-kg ice cube at -30.0°C is placed in 0.400 kg of 35.0°C water in a very well-insulated container. What is the final temperature?

44. If you pour 0.0100 kg of 20.0°C water onto a 1.20-kg block of ice (which is initially at -15.0°C), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.

45. Indigenous people sometimes cook in watertight baskets by placing hot rocks into water to bring it to a boil. What mass of 500°C rock must be placed in 4.00 kg of 15.0°C water to bring its temperature to 100°C, if 0.0250 kg of water escapes as vapor from the initial sizzle? You may neglect the effects of the surroundings and take the average specific heat of the rocks to be that of granite.

46. In some countries, liquid nitrogen is used on dairy trucks instead of mechanical refrigerators. A 3.00-hour delivery trip requires 200 L of liquid nitrogen, which has a density of $808\ \text{kg/m}^3$.

(a) Calculate the heat transfer necessary to evaporate this amount of liquid nitrogen and raise its temperature to 3.00°C. (Use c_p and assume it is constant over the temperature range.) This value is the amount of cooling the liquid nitrogen supplies.

(b) What is this heat transfer rate in kilowatt-hours?

(c) Compare the amount of cooling obtained from melting an identical mass of 0°C ice with that from evaporating the liquid nitrogen.

47. Some gun fanciers make their own bullets, which involves melting and casting the lead slugs. How much heat transfer is needed to raise the temperature and melt 0.500 kg of lead, starting from 25.0°C?

9.8 The First Law of Thermodynamics and Heat Engine Processes

48. Steam to drive an old-fashioned steam locomotive is supplied at a constant gauge pressure of $1.75\times10^6\ \text{N/m}^2$ (about 250 psi) to a piston with a 0.200-m radius. (a) By calculating $P\Delta V$, find the work done by the steam when the piston moves 0.800 m. Note that this is the net work output, since gauge pressure is used. (b) Now find the amount of work by calculating the force exerted times the distance traveled. Is the answer the same as in part (a)?

49. A hand-driven tire pump has a piston with a 2.50-cm diameter and a maximum stroke of 30.0 cm. (a) How much work do you do in one stroke if the average gauge pressure is $2.40\times10^5\ \text{N/m}^2$ (about 35 psi)? (b) What average force do you exert on the piston, neglecting friction and gravitational force?

50. Unreasonable Results

What is wrong with the claim that a cyclical heat engine does 4.00 kJ of work on an input of 24.0 kJ of heat transfer while 16.0 kJ of heat transfers to the environment?

51. (a) A cyclical heat engine, operating between temperatures of $450^\circ\ \text{C}$ and $150^\circ\ \text{C}$ produces 4.00 MJ of work on a heat transfer of 5.00 MJ into the engine. How much heat transfer occurs to the environment? (b) What is unreasonable about the engine? (c) Which premise is unreasonable?

52. Construct Your Own Problem

Consider a car's gasoline engine. Construct a problem in which you calculate the maximum efficiency this engine can have. Among the things to consider are the effective hot and cold reservoir temperatures. Compare your calculated efficiency with the actual efficiency of car engines.

53. Construct Your Own Problem

Consider a car trip into the mountains. Construct a problem in which you calculate the overall efficiency of the car for the trip as a ratio of kinetic and potential energy gained to fuel consumed. Compare this efficiency to the thermodynamic efficiency quoted for gasoline engines and discuss why the thermodynamic efficiency is so much greater. Among the factors to be considered are the gain in altitude and speed, the mass of the car, the distance traveled, and typical fuel economy.

9.9 Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

54. A certain heat engine does 10.0 kJ of work and 8.50 kJ of heat transfer occurs to the environment in a cyclical process. (a) What was the heat transfer into this engine? (b) What was the engine's efficiency?

55. With $2.56\times10^{6}\text{ J}$ of heat transfer into this engine, a given cyclical heat engine can do only $1.50\times10^{5}\text{ J}$ of work. (a) What is the engine's efficiency? (b) How much heat transfer to the environment takes place?

56. (a) What is the work output of a cyclical heat engine having a 22.0% efficiency and $6.00\times10^{9}\text{ J}$ of heat transfer into the engine? (b) How much heat transfer occurs to the environment?

57. (a) What is the efficiency of a cyclical heat engine in which 75.0 kJ of heat transfer occurs to the environment for every 95.0 kJ of heat transfer into the engine? (b) How much work does it produce for 100 kJ of heat transfer into the engine?

58. The engine of a large ship does $2.00\times10^{8}\text{ J}$ of work with an efficiency of 5.00%. (a) How much heat transfer occurs to the environment? (b) How many barrels of fuel are consumed, if each barrel produces $6.00\times10^{9}\text{ J}$ of heat transfer when burned?

59. (a) How much heat transfer occurs to the environment by an electrical power station that uses $1.25\times10^{14}\text{ J}$ of heat transfer into the engine with an efficiency of 42.0%? (b) What is the ratio of heat transfer to the environment to work output? (c) How much work is done?

60. Assume that the turbines at a coal-powered power plant were upgraded, resulting in an improvement in efficiency of 3.32%. Assume that prior to the upgrade the power station had an efficiency of 36% and that the heat transfer into the engine in one day is still the same at $2.50\times10^{14}\text{ J}$. (a) How much more electrical energy is produced due to the upgrade? (b) How much less heat transfer occurs to the environment due to the upgrade?

61. This problem compares the energy output and heat transfer to the environment by two different types of nuclear power stations—one with the normal efficiency of 34.0%, and another with an improved efficiency of 40.0%. Suppose both have the same heat transfer into the engine in one day, $2.50\times10^{14}\text{ J}$. (a) How much more electrical energy is produced by the more efficient power station? (b) How much less heat transfer occurs to the environment by the more efficient power station? (One type of more efficient nuclear power station, the gas-cooled reactor, has not been reliable enough to be economically feasible in spite of its greater efficiency.)

9.10 Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated

62. A certain gasoline engine has an efficiency of 30.0%. What would the hot reservoir temperature be for a Carnot engine having that efficiency, if it operates with a cold reservoir temperature of 200°C?

63. A gas-cooled nuclear reactor operates between hot and cold reservoir temperatures of 700°C and 27.0°C. (a) What is the maximum efficiency of a heat engine operating between these temperatures? (b) Find the ratio of this efficiency to the Carnot efficiency of a standard nuclear reactor (found in **Example 9.8**).

64. (a) What is the hot reservoir temperature of a Carnot engine that has an efficiency of 42.0% and a cold reservoir temperature of 27.0°C? (b) What must the hot reservoir temperature be for a real heat engine that achieves 0.700 of the maximum efficiency, but still has an efficiency of 42.0% (and a cold reservoir at 27.0°C)? (c) Does your answer imply practical limits to the efficiency of car gasoline engines?

65. Steam locomotives have an efficiency of 17.0% and operate with a hot steam temperature of 425°C. (a) What would the cold reservoir temperature be if this were a Carnot engine? (b) What would the maximum efficiency of this steam engine be if its cold reservoir temperature were 150°C?

66. Practical steam engines utilize 450°C steam, which is later exhausted at 270°C. (a) What is the maximum efficiency that such a heat engine can have? (b) Since 270°C steam is still quite hot, a second steam engine is sometimes operated using the exhaust of the first. What is the maximum efficiency of the second engine if its exhaust has a temperature of 150°C? (c) What is the overall efficiency of the two engines? (d) Show that this is the same efficiency as a single Carnot engine operating between 450°C and 150°C. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Thermodynamics**.

67. A coal-fired electrical power station has an efficiency of 38%. The temperature of the steam leaving the boiler is 550°C. What percentage of the maximum efficiency does this station obtain? (Assume the temperature of the environment is 20°C.)

68. Would you be willing to financially back an inventor who is marketing a device that she claims has 25 kJ of heat transfer at 600 K, has heat transfer to the environment at 300 K, and does 12 kJ of work? Explain your answer.

69. Unreasonable Results

(a) Suppose you want to design a steam engine that has heat transfer to the environment at 270°C and has a Carnot efficiency of 0.800. What temperature of hot steam must you use? (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

70. Unreasonable Results

Calculate the cold reservoir temperature of a steam engine that uses hot steam at 450°C and has a Carnot efficiency of 0.700. (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

9.11 Applications of Thermodynamics: Heat Pumps and Refrigerators

71. What is the coefficient of performance of an ideal heat pump that has heat transfer from a cold temperature of -25.0°C to a hot temperature of 40.0°C ?

72. Suppose you have an ideal refrigerator that cools an environment at -20.0°C and has heat transfer to another environment at 50.0°C . What is its coefficient of performance?

73. What is the best coefficient of performance possible for a hypothetical refrigerator that could make liquid nitrogen at -200°C and has heat transfer to the environment at 35.0°C ?

74. In a very mild winter climate, a heat pump has heat transfer from an environment at 5.00°C to one at 35.0°C . What is the best possible coefficient of performance for these temperatures? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Thermodynamics**.

75. (a) What is the best coefficient of performance for a heat pump that has a hot reservoir temperature of 50.0°C and a cold reservoir temperature of -20.0°C ? (b) How much heat transfer occurs into the warm environment if $3.60\times10^{7}\text{ J}$ of work ($10.0\text{kW}\cdot\text{h}$) is put into it? (c) If the cost of this work input is $10.0\text{ cents/kW}\cdot\text{h}$, how does its cost compare with the direct heat transfer achieved by burning natural gas at a cost of 85.0 cents per therm. (A therm is a common unit of energy for natural gas and equals $1.055\times10^{8}\text{ J}$.)

76. (a) What is the best coefficient of performance for a refrigerator that cools an environment at -30.0°C and has heat transfer to another environment at 45.0°C ? (b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment? (c) What is the cost of doing this if the work costs 10.0 cents per $3.60\times10^{6}\text{ J}$ (a kilowatt-hour)? (d) How many kJ of heat transfer occurs into the warm environment? (e) Discuss what type of refrigerator might operate between these temperatures.

77. Suppose you want to operate an ideal refrigerator with a cold temperature of -10.0°C , and you would like it to have a coefficient of performance of 7.00. What is the hot reservoir temperature for such a refrigerator?

78. An ideal heat pump is being considered for use in heating an environment with a temperature of 22.0°C . What is the cold reservoir temperature if the pump is to have a coefficient of performance of 12.0?

79. A 4-ton air conditioner removes $5.06\times10^{7}\text{ J}$ (48,000 British thermal units) from a cold environment in 1.00 h. (a) What energy input in joules is necessary to do this if the air conditioner has an energy efficiency rating (EER) of 12.0? (b) What is the cost of doing this if the work costs 10.0 cents per $3.60\times10^{6}\text{ J}$ (one kilowatt-hour)? (c) Discuss whether this cost seems realistic. Note that the energy efficiency rating (EER) of an air conditioner or refrigerator is defined to be the number of British thermal units of heat transfer from a cold environment per hour divided by the watts of power input.

80. Show that the coefficients of performance of refrigerators and heat pumps are related by $COP_{\text{ref}} = COP_{\text{hp}} - 1$.

Start with the definitions of the COP s and the conservation of energy relationship between Q_h , Q_c , and W .

9.12 Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

81. (a) On a winter day, a certain house loses $5.00\times10^{8}\text{ J}$ of heat to the outside (about 500,000 Btu). What is the total change in entropy due to this heat transfer alone, assuming an average indoor temperature of 21.0° C and an average outdoor temperature of 5.00° C ? (b) This large change in entropy implies a large amount of energy has become unavailable to do work. Where do we find more energy when such energy is lost to us?

82. On a hot summer day, $4.00\times10^{6}\text{ J}$ of heat transfer into a parked car takes place, increasing its temperature from 35.0° C to 45.0° C . What is the increase in entropy of the car due to this heat transfer alone?

83. A hot rock ejected from a volcano's lava fountain cools from 1100° C to 40.0° C , and its entropy decreases by 950 J/K. How much heat transfer occurs from the rock?

84. When $1.60\times10^{5}\text{ J}$ of heat transfer occurs into a meat pie initially at 20.0° C , its entropy increases by 480 J/K. What is its final temperature?

85. The Sun radiates energy at the rate of $3.80\times10^{26}\text{ W}$ from its 5500° C surface into dark empty space (a negligible fraction radiates onto Earth and the other planets). The effective temperature of deep space is -270° C . (a) What is the increase in entropy in one day due to this heat transfer? (b) How much work is made unavailable?

86. (a) In reaching equilibrium, how much heat transfer occurs from 1.00 kg of water at 40.0^{o} C when it is placed in contact with 1.00 kg of 20.0^{o} C water in reaching equilibrium? (b) What is the change in entropy due to this heat transfer? (c) How much work is made unavailable, taking the lowest temperature to be 20.0^{o} C? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Entropy**.

87. What is the decrease in entropy of 25.0 g of water that condenses on a bathroom mirror at a temperature of 35.0^{o} C, assuming no change in temperature and given the latent heat of vaporization to be 2450 kJ/kg?

88. Find the increase in entropy of 1.00 kg of liquid nitrogen that starts at its boiling temperature, boils, and warms to 20.0^{o} C at constant pressure.

89. A large electrical power station generates 1000 MW of electricity with an efficiency of 35.0%. (a) Calculate the heat transfer to the power station, Q_{h}, in one day. (b) How much heat transfer Q_{c} occurs to the environment in one day? (c) If the heat transfer in the cooling towers is from 35.0^{o} C water into the local air mass, which increases in temperature from 18.0^{o} C to 20.0^{o} C, what is the total increase in entropy due to this heat transfer? (d) How much energy becomes unavailable to do work because of this increase in entropy, assuming an 18.0^{o} C lowest temperature? (Part of Q_{c} could be utilized to operate heat engines or for simply heating the surroundings, but it rarely is.)

90. (a) How much heat transfer occurs from 20.0 kg of 90.0^{o} C water placed in contact with 20.0 kg of 10.0^{o} C water, producing a final temperature of 50.0^{o} C? (b) How much work could a Carnot engine do with this heat transfer, assuming it operates between two reservoirs at constant temperatures of 90.0^{o} C and 10.0^{o} C? (c) What increase in entropy is produced by mixing 20.0 kg of 90.0^{o} C water with 20.0 kg of 10.0^{o} C water? (d) Calculate the amount of work made unavailable by this mixing using a low temperature of 10.0^{o} C, and compare it with the work done by the Carnot engine. Explicitly show how you follow the steps in the **Problem-Solving Strategies for Entropy**. (e) Discuss how everyday processes make increasingly more energy unavailable to do work, as implied by this problem.

9.13 Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

91. Using **Table 9.5**, verify the contention that if you toss 100 coins each second, you can expect to get 100 heads or 100 tails once in 2×10^{22} years; calculate the time to two-digit accuracy.

92. What percent of the time will you get something in the range from 60 heads and 40 tails through 40 heads and 60 tails when tossing 100 coins? The total number of microstates in that range is 1.22×10^{30}. (Consult **Table 9.5**.)

93. (a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and 49 tails? (b) What percent of the total possibilities is this? (Consult **Table 9.5**.)

94. (a) What is the change in entropy if you start with 100 coins in the 45 heads and 55 tails macrostate, toss them, and get 51 heads and 49 tails? (b) What if you get 75 heads and 25 tails? (c) How much more likely is 51 heads and 49 tails than 75 heads and 25 tails? (d) Does either outcome violate the second law of thermodynamics?

95. (a) What is the change in entropy if you start with 10 coins in the 5 heads and 5 tails macrostate, toss them, and get 2 heads and 8 tails? (b) How much more likely is 5 heads and 5 tails than 2 heads and 8 tails? (Take the ratio of the number of microstates to find out.) (c) If you were betting on 2 heads and 8 tails would you accept odds of 252 to 45? Explain why or why not.

Table 9.6 10-Coin Toss

Macrostate		Number of Microstates (*W*)
Heads	Tails	
10	0	1
9	1	10
8	2	45
7	3	120
6	4	210
5	5	252
4	6	210
3	7	120
2	8	45
1	9	10
0	10	1
		Total: 1024

96. (a) If you toss 10 coins, what percent of the time will you get the three most likely macrostates (6 heads and 4 tails, 5 heads and 5 tails, 4 heads and 6 tails)? (b) You can realistically toss 10 coins and count the number of heads and tails about twice a minute. At that rate, how long will it take on average to get either 10 heads and 0 tails or 0 heads and 10 tails?

97. (a) Construct a table showing the macrostates and all of the individual microstates for tossing 6 coins. (Use **Table 9.6** as a guide.) (b) How many macrostates are there? (c) What is the total number of microstates? (d) What percent chance is there of tossing 5 heads and 1 tail? (e) How much more likely are you to toss 3 heads and 3 tails than 5 heads and 1 tail? (Take the ratio of the number of microstates to find out.)

98. In an air conditioner, 12.65 MJ of heat transfer occurs from a cold environment in 1.00 h. (a) What mass of ice melting would involve the same heat transfer? (b) How many hours of operation would be equivalent to melting 900 kg of ice? (c) If ice costs 20 cents per kg, do you think the air conditioner could be operated more cheaply than by simply using ice? Describe in detail how you evaluate the relative costs.

10 ELECTRICITY

Figure 10.1 Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

Chapter Outline

10.1. Static Electricity and Charge: Conservation of Charge
- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.

10.2. Coulomb's Law
- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.

10.3. Electric Field: Concept of a Field Revisited
- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

10.4. Electric Field Lines
- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

10.5. Electric Potential Energy: Potential Difference
- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

10.6. Applications of Electrostatics
- Name several real-world applications of the study of electrostatics.

10.7. Current
- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.

10.8. Ohm's Law: Resistance and Simple Circuits
- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

10.9. Electric Power and Energy
- Calculate the power dissipated by a resistor and power supplied by a power supply.

- Calculate the cost of electricity under various circumstances.

10.10. Resistors in Series and Parallel

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

10.11. Electric Hazards and the Human Body

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

Introduction to Electricity

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See **Figure 10.2**.) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.

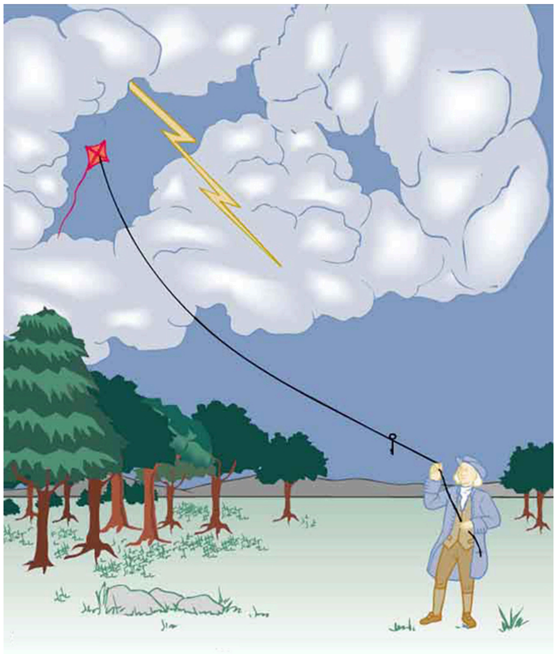


Figure 10.2 When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Our understanding of electricity and the range of phenomena that are electrical in nature has vastly expanded since those early days. For example, atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism. Furthermore, All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

In this chapter, we will develop the concepts of electric fields and electric potential (also known as *voltage*), which are used to describe electric force and electrical energy. Consider, for example, great amounts of electrical energy stored in batteries or transmitted cross-country through power lines, or consider electrical signals sent in our nervous systems at molecular levels, with *ions* crossing cell membranes and transferring information, or the role of electricity in our household appliances and devices, including your laptop and smartphones. We will look at the laws that help us describe electric circuits and begin to explore some of the many applications of electricity.

10.1 Static Electricity and Charge: Conservation of Charge

Figure 10.3 Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see **Figure 10.3**). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. **Figure 10.4** shows how these simple materials can be used to explore the nature of the force between charges.

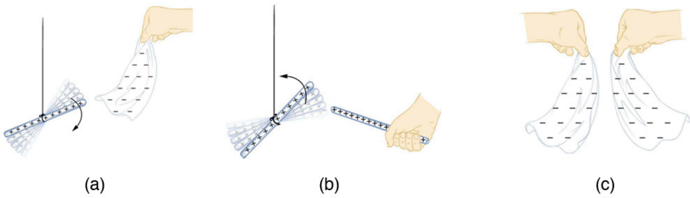


Figure 10.4 A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

Figure 10.5 shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.

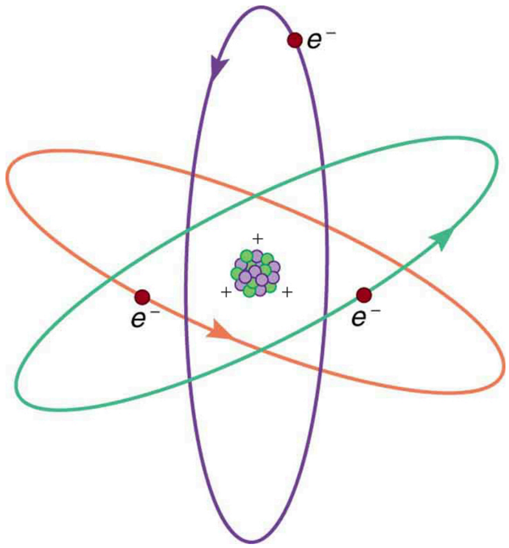


Figure 10.5 This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

$$|\, q_e \,| = 1.60\times 10^{-19}\text{ C.} \tag{10.1}$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

$$1.00\text{ C}\times\frac{1\text{ proton}}{1.60\times 10^{-19}\text{ C}} = 6.25\times 10^{18}\text{ protons.} \tag{10.2}$$

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|\, q_e \,|$, and all observed charges are integral multiples of $|\, q_e \,|$.

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See **Figure 10.6**.) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

Figure 10.6 shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.

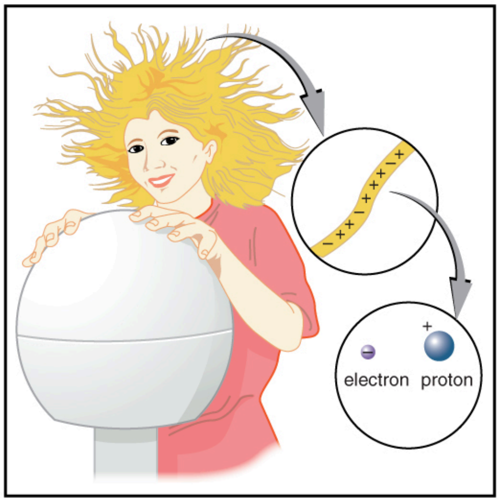


Figure 10.6 When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in **Figure 10.7**. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.

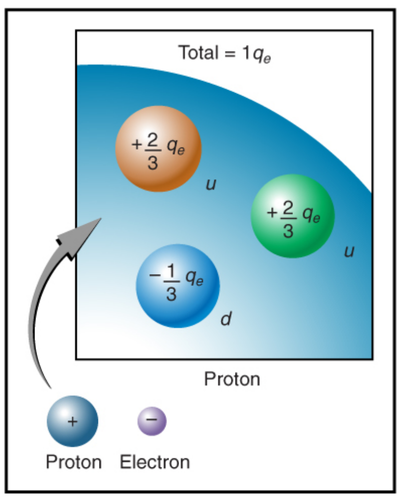


Figure 10.7 Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See **Figure 10.8**.) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.

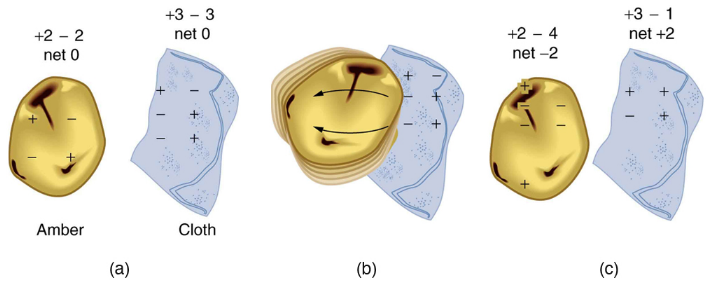


Figure 10.8 When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Law of Conservation of Charge

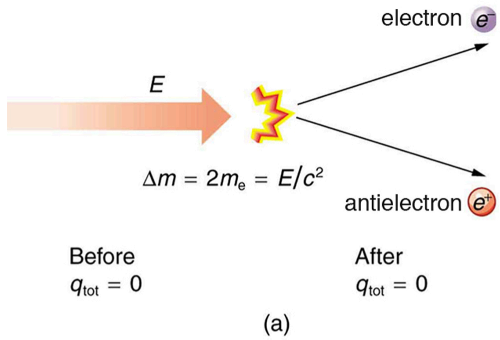
Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm , can be created from energy in the amount $\Delta m = \frac{E}{c^2}$.

Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See **Figure 10.9**.) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy *E*, again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



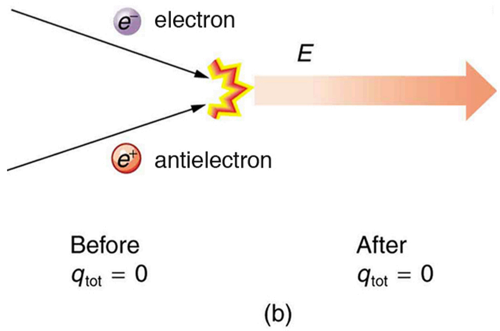


Figure 10.9 (a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

10.2 Coulomb's Law

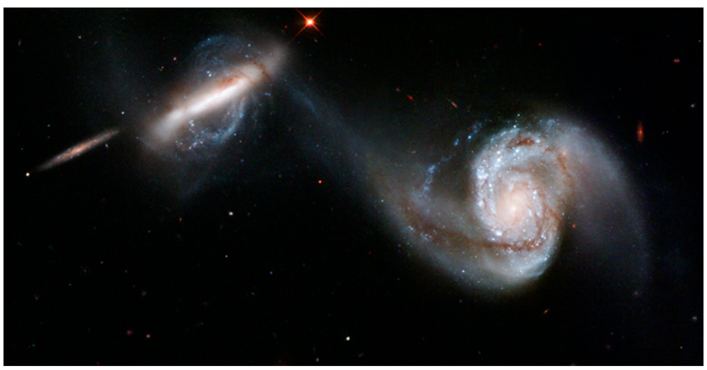


Figure 10.10 This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Coulomb's Law

$$F = k\frac{|q_1 q_2|}{r^2}. \tag{10.3}$$

Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2, separated by a distance

r. In SI units, the constant k is equal to

$$k = 8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \approx 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}. \tag{10.4}$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See **Figure 10.11**.)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1 / r^2\right)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.

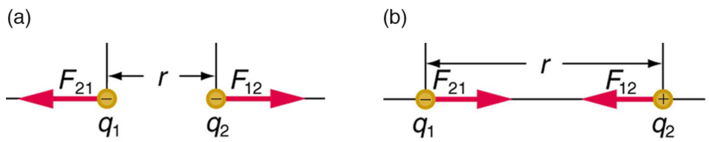


Figure 10.11 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges.

Example 10.1 How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k\frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

$$F = k\frac{|q_1 q_2|}{r^2} \tag{10.5}$$

$$= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2} \tag{10.6}$$

Thus the Coulomb force is

$$F = 8.19 \times 10^{-8} \text{ N}. \tag{10.7}$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of 8.99×10^{22} m / s^2 (verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

$$F_G = G\frac{mM}{r^2}, \tag{10.8}$$

where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

$$F_G = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(0.530 \times 10^{-10} \text{ m})^2} = 3.61 \times 10^{-47} \text{ N} \tag{10.9}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

$$\frac{F}{F_G} = 2.27\times10^{39}. \tag{10.10}$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

10.3 Electric Field: Concept of a Field Revisited

Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to "touch." That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb's law, $F = k|q_1 q_2| / r^2$, its magnitude is given by the equation $F = k|qQ| / r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see **Figure 10.12**). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q .

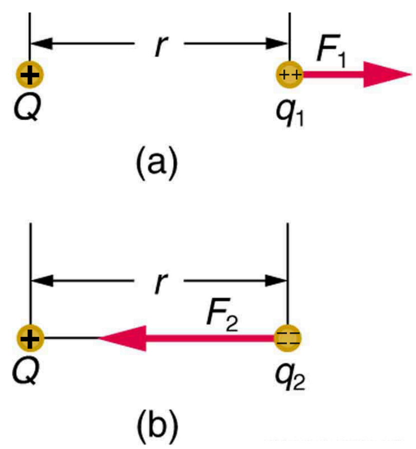


Figure 10.12 The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q . (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1 , and so the force F_2 acting on it is attractive and stronger than F_1 . The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2 as well as the charge Q .

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q . The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

$$\mathbf{E} = \frac{\mathbf{F}}{q}, \tag{10.11}$$

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q . It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is $F = k|qQ| / r^2$. Thus the magnitude of the electric field, E, for a point charge is

$$E = \left|\frac{F}{q}\right| = k\left|\frac{qQ}{qr^2}\right| = k\frac{|Q|}{r^2}. \tag{10.12}$$

Since the test charge cancels, we see that

$$E = k\frac{|Q|}{r^2}. \tag{10.13}$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

Example 10.2 Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ / r^2$.

Solution

Here $Q = 2.00\times10^{-9}$ C and $r = 5.00\times10^{-3}$ m. Entering those values into the above equation gives

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\times\frac{(2.00\times10^{-9}\ \text{C})}{(5.00\times10^{-3}\ \text{m})^2} \\ &= 7.19\times10^5\ \text{N/C}. \end{aligned} \tag{10.14}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

Example 10.3 Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250\ \mu\text{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field $\mathbf{E} = \mathbf{F} / q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250\ \mu\text{C}$ exerted by a field of strength $E = 7.20\times10^5$ N/C is thus,

$$\begin{aligned} F &= -qE \\ &= (0.250\times10^{-6}\ \text{C})(7.20\times10^5\ \text{N/C}) \\ &= 0.180\ \text{N}. \end{aligned} \tag{10.15}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

10.4 Electric Field Lines

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

Figure 10.13 shows two pictorial representations of the same electric field created by a positive point charge Q. Figure 10.13 (b) shows the standard representation using continuous lines. Figure 10.13 (b) shows numerous individual arrows with each arrow representing the force on a test charge q. Field lines are essentially a map of infinitesimal force vectors.

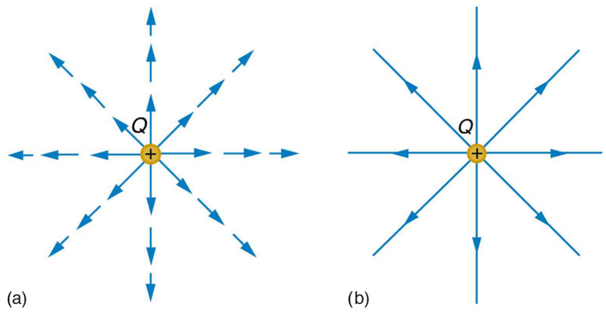


Figure 10.13 Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See Figure 10.14.) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.

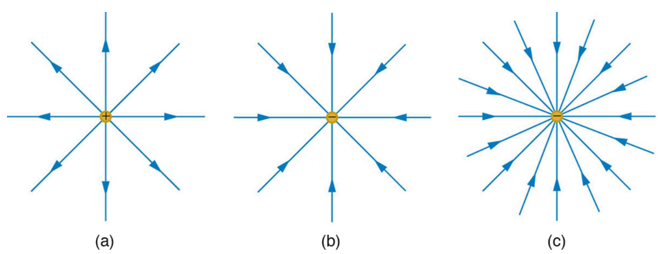


Figure 10.14 The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge.

Figure 10.15 shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See Figure 10.15 and Figure 10.16(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

Figure 10.16(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.

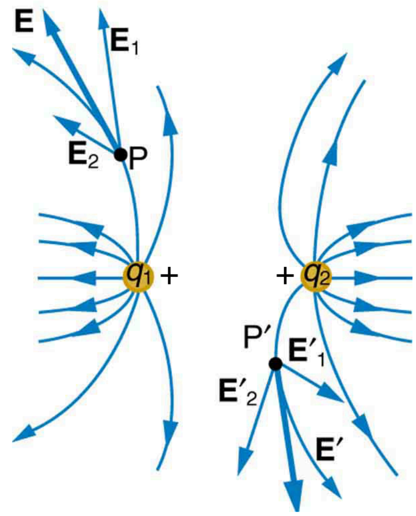


Figure 10.15 Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.

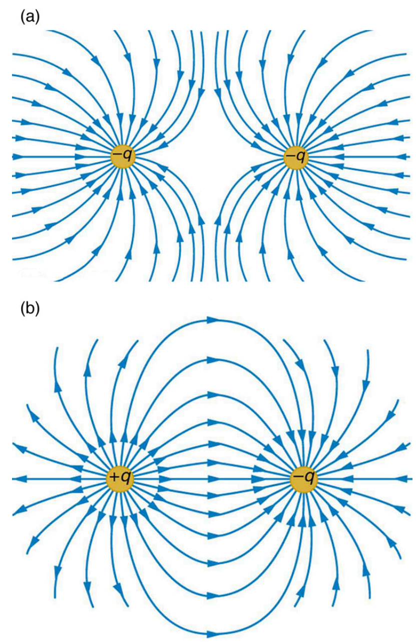


Figure 10.16 (a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

10.5 Electric Potential Energy: Potential Difference

When a free positive charge q is accelerated by an electric field, such as shown in **Figure 10.17**, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

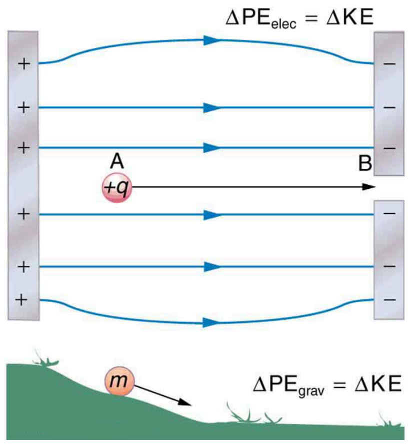


Figure 10.17 A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write $W = -\Delta\text{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Potential Energy

$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta\text{PE}.$ There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE , is proportional to the test charge $q.$ To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

$$V = \frac{\text{PE}}{q}. \tag{10.16}$$

Electric Potential

This is the electric potential energy per unit charge.

$$V = \frac{\text{PE}}{q} \tag{10.17}$$

Since PE is proportional to q , the dependence on q cancels. Thus V does not depend on q . The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}} = \frac{\Delta \mathrm{PE}}{q}. \tag{10.18}$$

The **potential difference** between points A and B, $V_{\mathrm{B}} - V_{\mathrm{A}}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}} \tag{10.19}$$

Potential Difference

The potential difference between points A and B, V_{B} - V_{A}, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\ \mathrm{V} = 1\ \frac{\mathrm{J}}{\mathrm{C}} \tag{10.20}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V. \tag{10.21}$$

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V. \tag{10.22}$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \mathrm{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example 10.4 Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta \mathrm{PE} = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\ \mathrm{C}$ and $\Delta V = 12.0\ \mathrm{V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned}\Delta \mathrm{PE}_{\text{cycle}} &= (5000\ \mathrm{C})(12.0\ \mathrm{V}) \\ &= (5000\ \mathrm{C})(12.0\ \mathrm{J/C}) \\ &= 6.00\times10^{4}\ \mathrm{J}.\end{aligned} \tag{10.23}$$

Similarly, for the car battery, $q = 60{,}000\ \mathrm{C}$ and

$$\begin{aligned}\Delta \mathrm{PE}_{\text{car}} &= (60{,}000\ \mathrm{C})(12.0\ \mathrm{V}) \\ &= 7.20\times10^{5}\ \mathrm{J}.\end{aligned} \tag{10.24}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in **Figure 10.18**. The change in potential is $\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}} = +12\ \mathrm{V}$ and the charge q is negative, so that $\Delta \mathrm{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

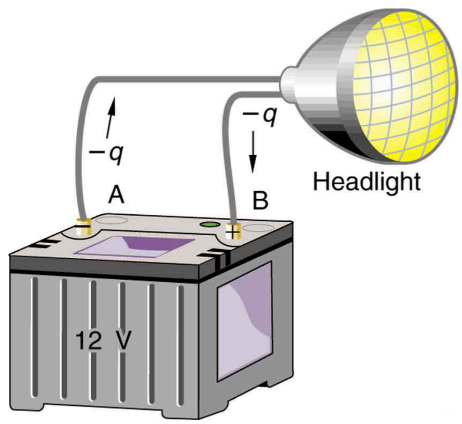


Figure 10.18 A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example 10.5 How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta \mathrm{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta \mathrm{PE} = -30.0\ \mathrm{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \mathrm{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta \mathrm{PE} = q\Delta V$:

$$q = \frac{\Delta \mathrm{PE}}{\Delta V}. \tag{10.25}$$

Entering the values for $\Delta \mathrm{PE}$ and ΔV , we get

$$q = \frac{-30.0\ \mathrm{J}}{+12.0\ \mathrm{V}} = \frac{-30.0\ \mathrm{J}}{+12.0\ \mathrm{J/C}} = -2.50\ \mathrm{C}. \tag{10.26}$$

The number of electrons $\mathrm{n_e}$ is the total charge divided by the charge per electron. That is,

$$\mathrm{n_e} = \frac{-2.50\ \mathrm{C}}{-1.60\times 10^{-19}\ \mathrm{C/e^-}} = 1.56\times 10^{19}\ \mathrm{electrons}. \tag{10.27}$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. **Figure 10.19** shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta\text{PE} = q\Delta V,$ we can think of the joule as a coulomb-volt.

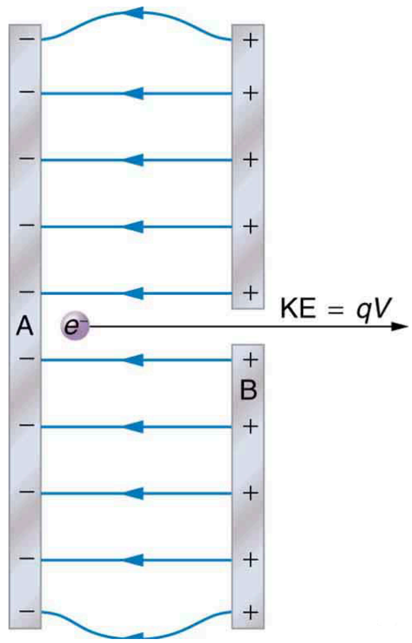


Figure 10.19 A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned} \tag{10.28}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, and kilowatt-hours for electrical energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30{,}000\text{ eV} \div 5\text{ eV per molecule} = 6000\text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

$$\text{KE} + \text{PE} = \text{constant} \tag{10.29}$$

or

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} , \tag{10.30}$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example 10.6 Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\text{KE}_\text{i} = 0, \text{KE}_\text{f} = ½mv^2, \text{PE}_\text{i} = qV, \text{and PE}_\text{f} = 0.$

Solution

Conservation of energy states that

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}. \tag{10.31}$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}. \tag{10.32}$$

We solve this for v:

$$v = \sqrt{\frac{2qV}{m}}. \tag{10.33}$$

Entering values for $q, V, \text{and } m$ gives

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\text{ C}\right)(-100\text{ J/C})}{9.11\times10^{-31}\text{ kg}}} \\ &= 5.93\times10^{6}\text{ m/s.} \end{aligned} \tag{10.34}$$

Discussion

Note that both the charge and the initial voltage are negative, as in **Figure 10.19**. We know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns.

10.6 Applications of Electrostatics

The study of **electrostatics** has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. **Figure 10.20** shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.

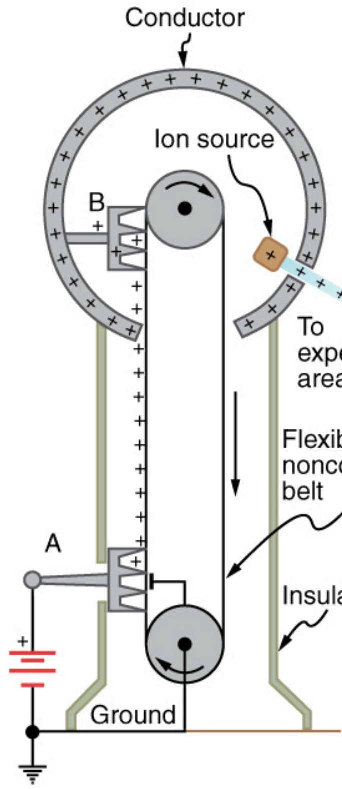


Figure 10.20 Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in **Figure 10.21**.

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.

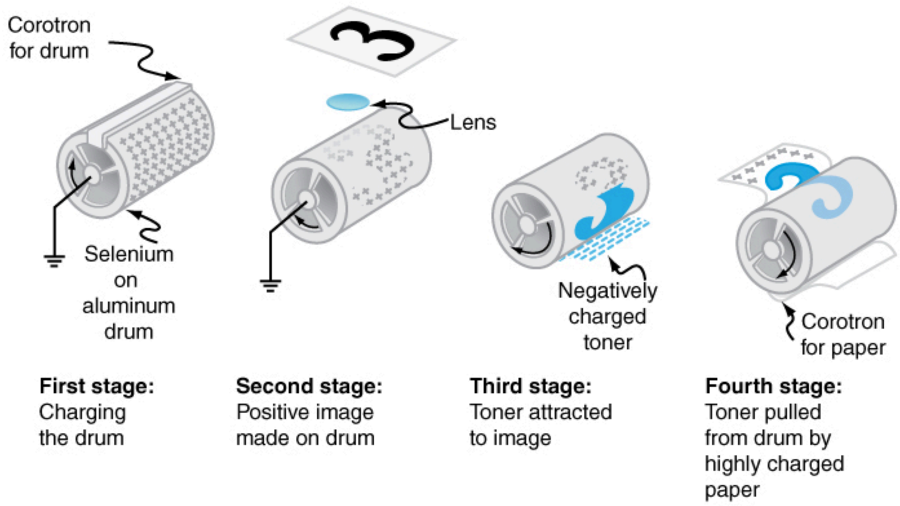


Figure 10.21 Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in **Figure 10.22**. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

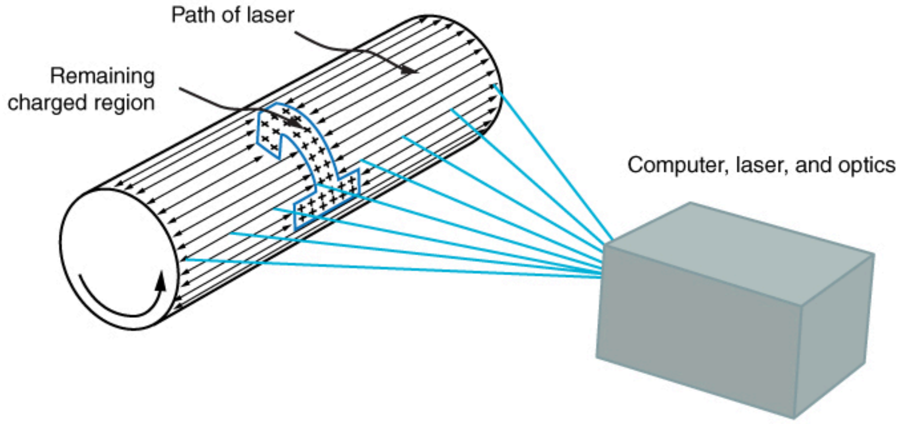


Figure 10.22 In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See **Figure 10.23**.)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)

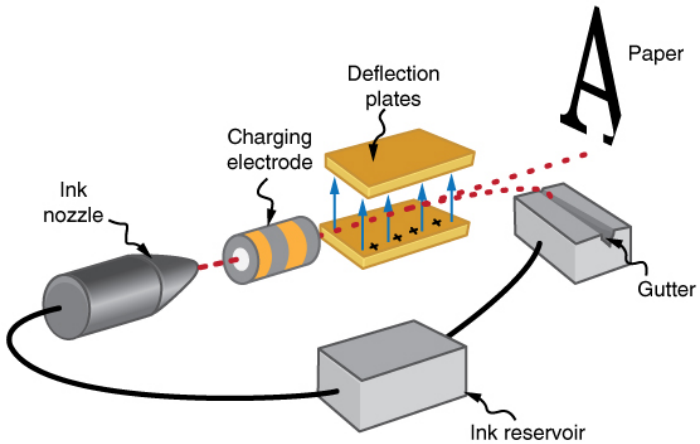


Figure 10.23 The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See **Figure 10.24**.)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.

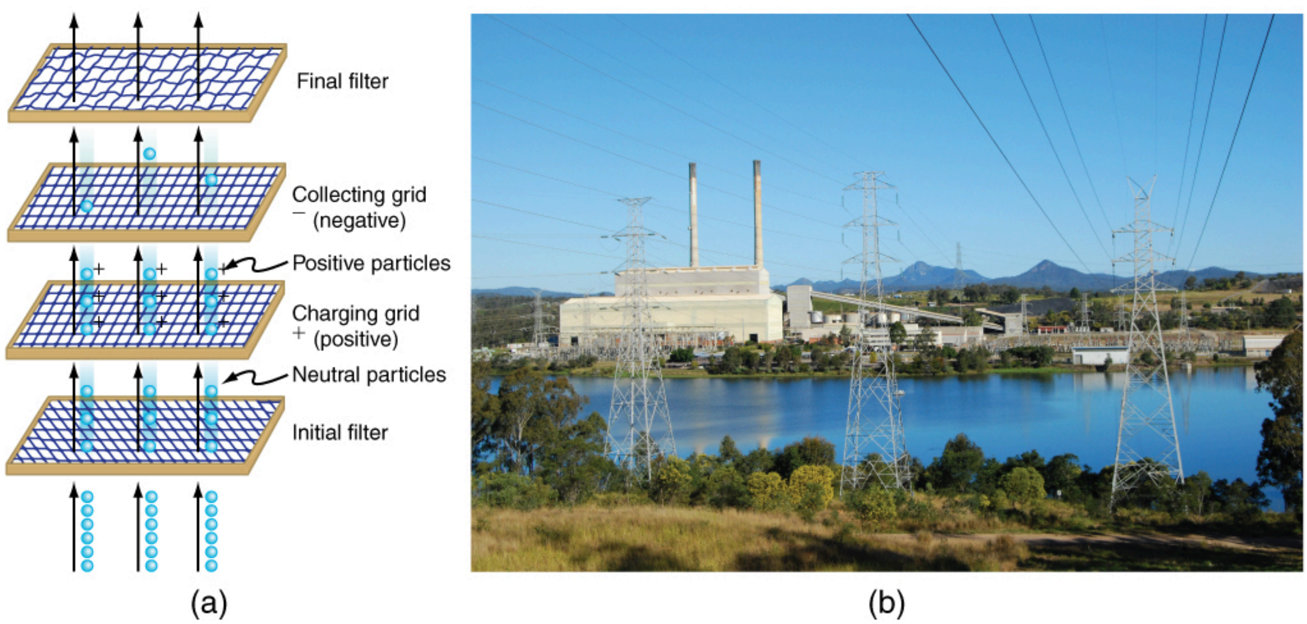


Figure 10.24 (a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

10.7 Current

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

$$I = \frac{\Delta Q}{\Delta t}, \tag{10.35}$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See **Figure 10.25**.) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

$$1 \text{ A} = 1 \text{ C/s} \tag{10.36}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

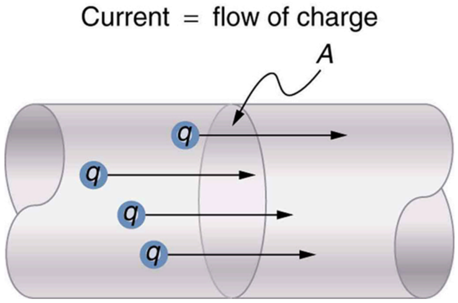


Figure 10.25 The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example 10.7 Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720 \text{ C}}{4.00 \text{ s}} = 180 \text{ C/s} \\ &= 180 \text{ A}. \end{aligned} \tag{10.37}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q / \Delta t$ for time Δt, and entering the known values for charge and current gives

$$\begin{aligned} \Delta t &= \frac{\Delta Q}{I} = \frac{1.00 \text{ C}}{0.300 \times 10^{-3} \text{ C/s}} \\ &= 3.33 \times 10^{3} \text{ s}. \end{aligned} \tag{10.38}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It’s because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

Figure 10.26 shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in **Figure 10.26** (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.

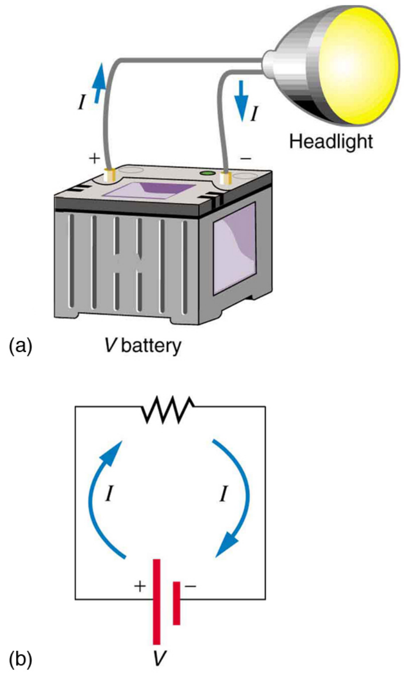


Figure 10.26 (a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide variety of similar circuits.

Note that the direction of current flow in **Figure 10.26** is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. **Figure 10.27** illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in **Figure 10.27**. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.

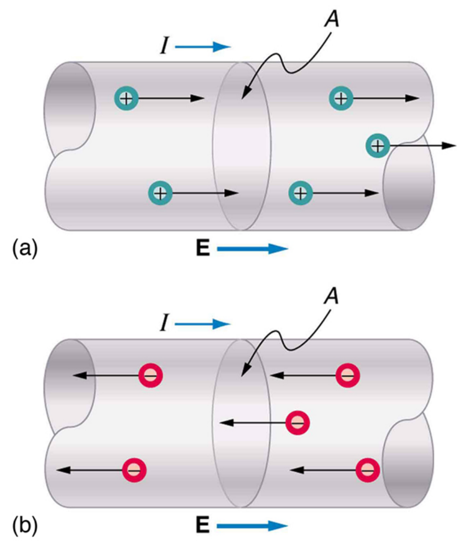


Figure 10.27 Current I is the rate at which charge moves through an area A, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example 10.8 Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the **Example 10.7** example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3}\ \text{C/s}$ .Since each electron (e^-) has a charge of $-1.60 \times 10^{-19}\ \text{C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}}. \tag{10.39}$$

We divide this by the charge per electron, so that

$$\begin{aligned} \frac{e^-}{\text{s}} &= \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}} \times \frac{1\ e^-}{-1.60 \times 10^{-19}\ \text{C}} \\ &= 1.88 \times 10^{15}\ \frac{e^-}{\text{s}}. \end{aligned} \tag{10.40}$$

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

10.8 Ohm's Law: Resistance and Simple Circuits

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

$$I \propto V. \tag{10.41}$$

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

$$I \propto \frac{1}{R}. \tag{10.42}$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

$$I = \frac{V}{R}. \tag{10.43}$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

$$1\ \Omega = 1\frac{V}{A}. \tag{10.44}$$

Figure 10.28 shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.

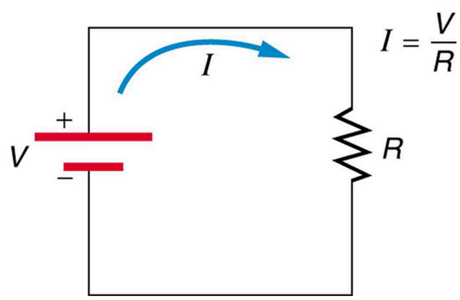


Figure 10.28 A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example 10.9 Calculating Resistance: An Automobile Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

$$R = \frac{V}{I} = \frac{12.0\text{ V}}{2.50\text{ A}} = 4.80\ \Omega. \tag{10.45}$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. Resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\ \Omega$, whereas the resistance of the human heart is about $10^3\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed.

Additional insight is gained by solving $I = V/R$ for $V,$ yielding

$$V = IR. \tag{10.46}$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I . The phrase IR *drop* is often used for this voltage. For instance, the headlight in **Example 10.9** has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\mathrm{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See **Figure 10.29**.)

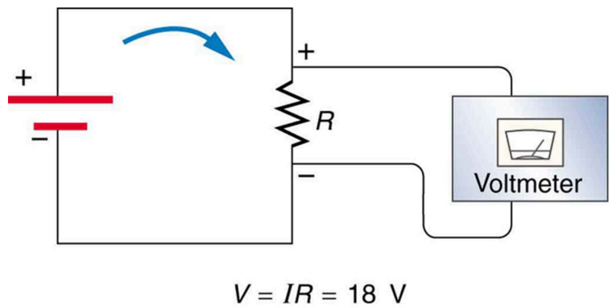


Figure 10.29 The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

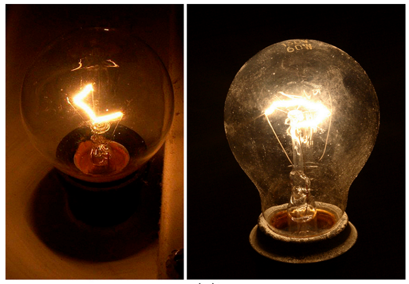
Making Connections: Conservation of Energy

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

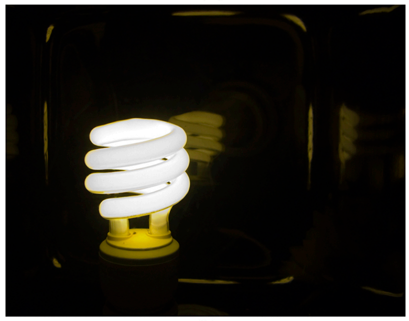
10.9 Electric Power and Energy

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See **Figure 10.30**(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

Figure 10.30 (a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\mathrm{PE} = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the charge moves through). Power is the rate at which energy is moved, and so electric power is

$$P = \frac{PE}{t} = \frac{qV}{t}. \tag{10.47}$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

$$P = IV. \tag{10.48}$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \mathrm{A} \cdot \mathrm{V} = 1\ \mathrm{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20\ \mathrm{A})(12\ \mathrm{V}) = 240\ \mathrm{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \mathrm{kA} \cdot \mathrm{V} = 1\ \mathrm{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

$$P = IV \tag{10.49}$$

$$P = \frac{V^2}{R} \tag{10.50}$$

$$P = I^2R. \tag{10.51}$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example 10.10 Calculating Power Dissipation and Current

Consider the example given in "Ohm's Law: Resistance and Simple Circuits." Then find the power dissipated by the car headlight.

Strategy

For the headlight, we know voltage and current, so we can use $P = IV$ to find the power.

Solution

Entering the known values of current and voltage for the hot headlight, we obtain

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}. \tag{10.52}$$

Discussion

The 30 W dissipated by the hot headlight is typical.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E / t$, we see that

$$E = Pt \tag{10.53}$$

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6{\times}10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See **Figure 10.30**(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example 10.11 Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

$$E = Pt = (60 \text{ W})(1000 \text{ h}) = 60{,}000 \text{ W} \cdot \text{h}. \tag{10.54}$$

In kilowatt-hours, this is

$$E = 60.0 \text{ kW} \cdot \text{h}. \tag{10.55}$$

Now the electricity cost is

$$\text{cost} = (60.0\ \text{kW}\cdot\text{h})(\$0.12/\text{kW}\cdot\text{h}) = \$7.20. \tag{10.56}$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

10.10 Resistors in Series and Parallel

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in **Figure 10.31**. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

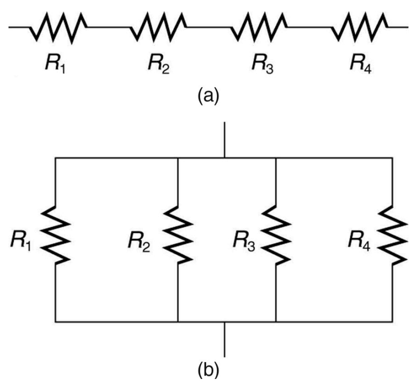


Figure 10.31 (a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in **Figure 10.31**(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

Figure 10.32 shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)

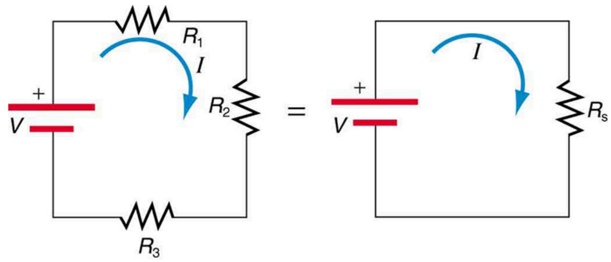


Figure 10.32 Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in **Figure 10.32**.

According to **Ohm's law**, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation $V = IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R.

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

$$V = V_1 + V_2 + V_3. \tag{10.57}$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE = qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is

$$qV_1 + qV_2 + qV_3. \tag{10.58}$$

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3). \tag{10.59}$$

Note that for the equivalent single series resistance R_s, we have

$$V = IR_s. \tag{10.60}$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_s = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

$$R_s = R_1 + R_2 + R_3 + ..., \tag{10.61}$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example 10.12 Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in **Figure 10.32** is 12.0 V, and the resistances are $R_1 = 1.00 \ \Omega$, $R_2 = 6.00 \ \Omega$, and $R_3 = 13.0 \ \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned} \tag{10.62}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

$$I = \frac{V}{R_s} = \frac{12.0\ \text{V}}{20.0\ \ \Omega} = 0.600\ \text{A}. \tag{10.63}$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

$$V_1 = IR_1 = (0.600\ \text{A})(1.0\ \ \Omega\ ) = 0.600\ \text{V}. \tag{10.64}$$

Similarly,

$$V_2 = IR_2 = (0.600\ \text{A})(6.0\ \ \Omega\ ) = 3.60\ \text{V} \tag{10.65}$$

and

$$V_3 = IR_3 = (0.600\ \text{A})(13.0\ \ \Omega\ ) = 7.80\ \text{V}. \tag{10.66}$$

Discussion for (c)

The three IR drops add to $12.0\ \text{V}$, as predicted:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\ \text{V} = 12.0\ \text{V}. \tag{10.67}$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

$$P_1 = I^2 R_1 = (0.600\ \text{A})^2 (1.00\ \ \Omega\ ) = 0.360\ \text{W}. \tag{10.68}$$

Similarly,

$$P_2 = I^2 R_2 = (0.600\ \text{A})^2 (6.00\ \ \Omega\ ) = 2.16\ \text{W} \tag{10.69}$$

and

$$P_3 = I^2 R_3 = (0.600\ \text{A})^2 (13.0\ \ \Omega\ ) = 4.68\ \text{W}. \tag{10.70}$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

$$P = (0.600\ \text{A})(12.0\ \text{V}) = 7.20\ \text{W}. \tag{10.71}$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\ \text{W} = 7.20\ \text{W}. \tag{10.72}$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + ...$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

Figure 10.33 shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See **Figure 10.33**(b).)

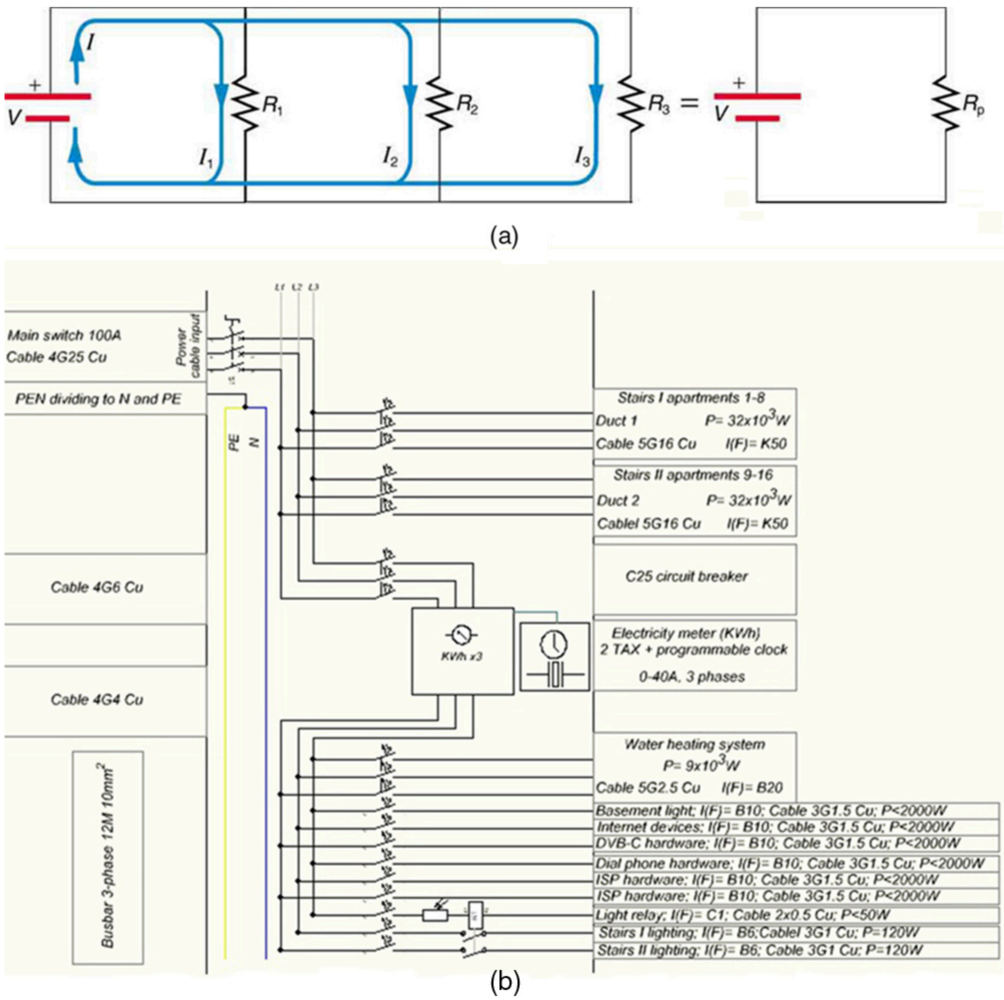


Figure 10.33 (a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

$$I = I_1 + I_2 + I_3. \tag{10.73}$$

Substituting the expressions for the individual currents gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \tag{10.74}$$

Note that Ohm's law for the equivalent single resistance gives

$$I = \frac{V}{R_\text{p}} = V\left(\frac{1}{R_\text{p}}\right). \tag{10.75}$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{.3}} + ... \tag{10.76}$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example 10.13 Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in **Figure 10.33** be the same as the previously considered series connection: $V = 12.0\text{ V}$, $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

$$\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega}. \tag{10.77}$$

Thus,

$$\frac{1}{R_\text{p}} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}. \tag{10.78}$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p. This yields

$$R_\text{p} = \frac{1}{1.2436}\ \Omega = 0.8041\ \Omega. \tag{10.79}$$

The total resistance with the correct number of significant digits is $R_\text{p} = 0.804\ \Omega$.

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

$$I = \frac{V}{R_\text{p}} = \frac{12.0\text{ V}}{0.8041\ \Omega} = 14.92\text{ A}. \tag{10.80}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

$$I_1 = \frac{V}{R_1} = \frac{12.0\text{ V}}{1.00\ \Omega} = 12.0\text{ A}. \tag{10.81}$$

Similarly,

$$I_2 = \frac{V}{R_2} = \frac{12.0\ \text{V}}{6.00\ \ \Omega} = 2.00\ \text{A} \tag{10.82}$$

and

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \text{V}}{13.0\ \ \Omega} = 0.92\ \text{A}. \tag{10.83}$$

Discussion for (c)

The total current is the sum of the individual currents:

$$I_1 + I_2 + I_3 = 14.92\ \text{A}. \tag{10.84}$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0\ \text{V})^2}{1.00\ \ \Omega} = 144\ \text{W}. \tag{10.85}$$

Similarly,

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0\ \text{V})^2}{6.00\ \ \Omega} = 24.0\ \text{W} \tag{10.86}$$

and

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0\ \text{V})^2}{13.0\ \ \Omega} = 11.1\ \text{W}. \tag{10.87}$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

$$P = IV = (14.92\ \text{A})(12.0\ \text{V}) = 179\ \text{W}. \tag{10.88}$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

$$P_1 + P_2 + P_3 = 144\ \text{W} + 24.0\ \text{W} + 11.1\ \text{W} = 179\ \text{W}. \tag{10.89}$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_\text{p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in **Figure 10.34**. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

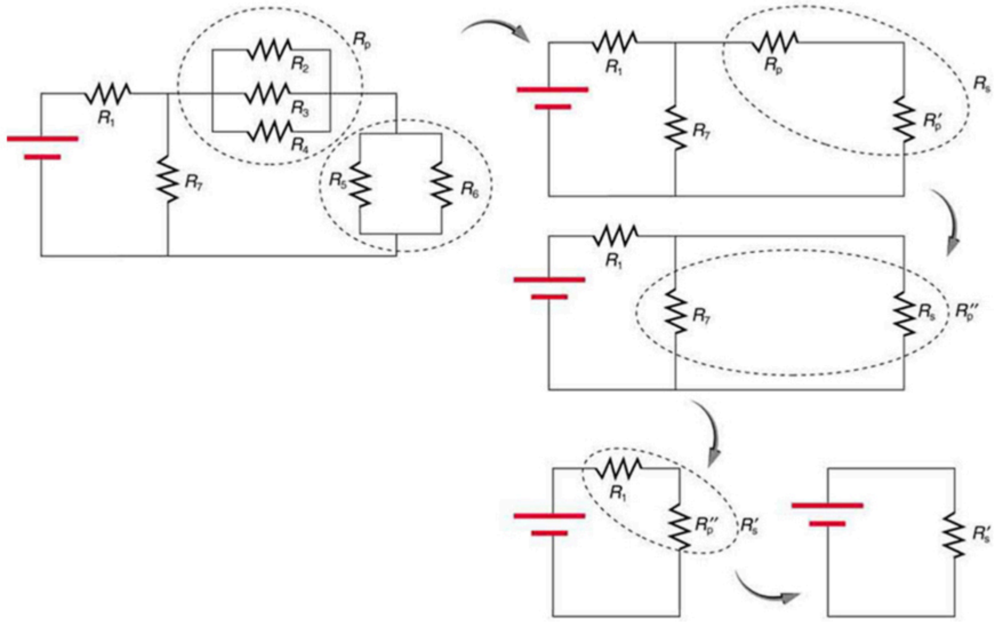


Figure 10.34 This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

The simplest combination of series and parallel resistance, shown in **Figure 10.35**, is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example 10.14 Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

Figure 10.35 shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?

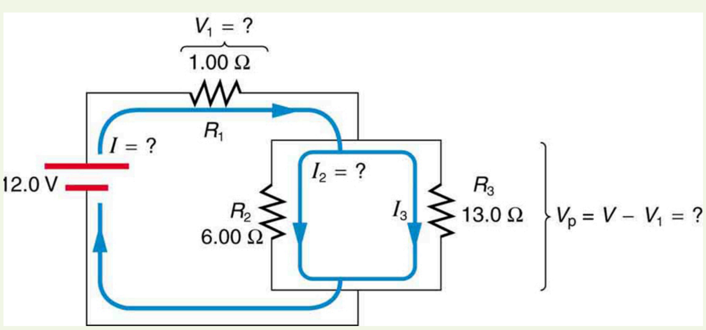


Figure 10.35 These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

$$R_{\text{tot}} = R_1 + R_{\text{p}}. \tag{10.90}$$

First, we find R_p using the equation for resistors in parallel and entering known values:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}. \tag{10.91}$$

Inverting gives

$$R_p = \frac{1}{0.2436}\ \Omega\ = 4.11\ \ \Omega\ . \tag{10.92}$$

So the total resistance is

$$R_{tot} = R_1 + R_p = 1.00\ \ \Omega\ + 4.11\ \ \Omega\ = 5.11\ \ \Omega\ . \tag{10.93}$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1 , we note that the full current I flows through R_1 . Thus its IR drop is

$$V_1 = IR_1. \tag{10.94}$$

We must find I before we can calculate V_1 . The total current I is found using Ohm's law for the circuit. That is,

$$I = \frac{V}{R_{tot}} = \frac{12.0\ \text{V}}{5.11\ \ \Omega} = 2.35\ \text{A}. \tag{10.95}$$

Entering this into the expression above, we get

$$V_1 = IR_1 = (2.35\ \text{A})(1.00\ \ \Omega\ ) = 2.35\ \text{V}. \tag{10.96}$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1 . When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3 .

Strategy and Solution for (c)

To find the current through R_2 , we must first find the voltage applied to it. We call this voltage V_p , because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1 , and so it is

$$V_p = V - V_1 = 12.0\ \text{V} - 2.35\ \text{V} = 9.65\ \text{V}. \tag{10.97}$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65\ \text{V}}{6.00\ \ \Omega} = 1.61\ \text{A}. \tag{10.98}$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

$$P_2 = (I_2)^2 R_2 = (1.61\ \text{A})^2(6.00\ \ \Omega\ ) = 15.5\ \text{W}. \tag{10.99}$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in **Figure 10.36**. The device represented by R_3 has a very low resistance, and so when it is switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.

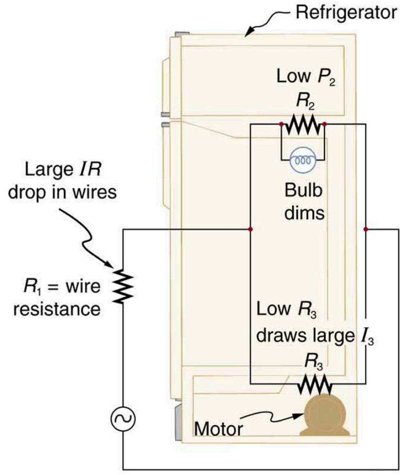


Figure 10.36 Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Check Your Understanding

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in **Kirchhoff's Rules (https://legacy.cnx.org/content/m42359/latest/)** , will allow you to analyze the circuit.

Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

10.11 Electric Hazards and the Human Body

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in **Figure 10.37**. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2 / r$, is very large. For example, if V is 120 V and r is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

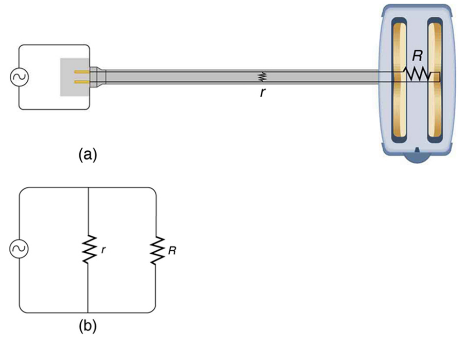


Figure 10.37 A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r. Since $P = V^2 / r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2 / r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_{\text{w}}$, where R_{w} is the resistance of the wires and I the current flowing through them. If either I or R_{w} is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_{\text{w}} = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_{\text{w}} = 200\text{ W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100\text{-}\Omega$ resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\text{ W}$. Fuses and circuit breakers are used to limit excessive currents. (See **Figure 10.38** and **Figure 10.39**.) Each device opens the circuit automatically when a sustained current exceeds safe limits.

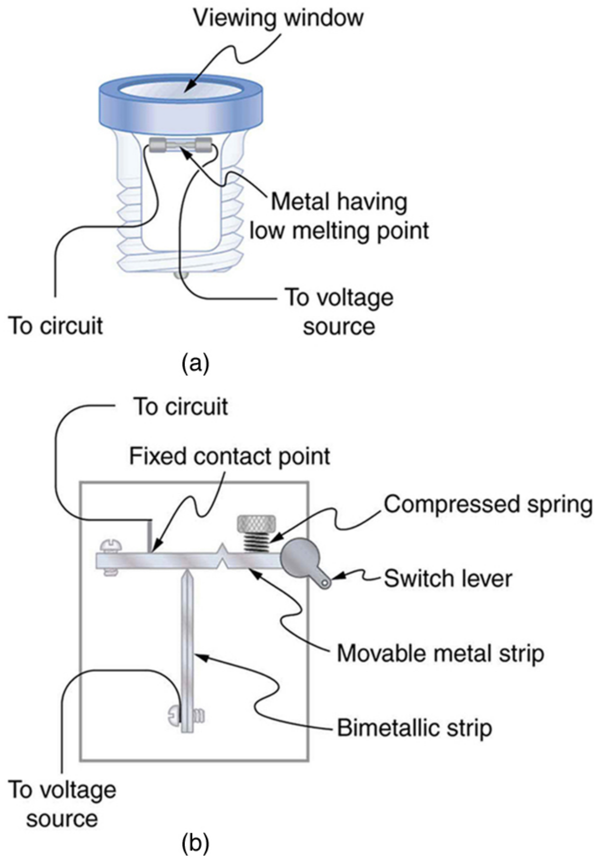


Figure 10.38 (a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.

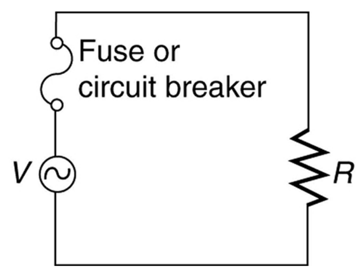


Figure 10.39 Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current

3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

Table 10.1 gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.

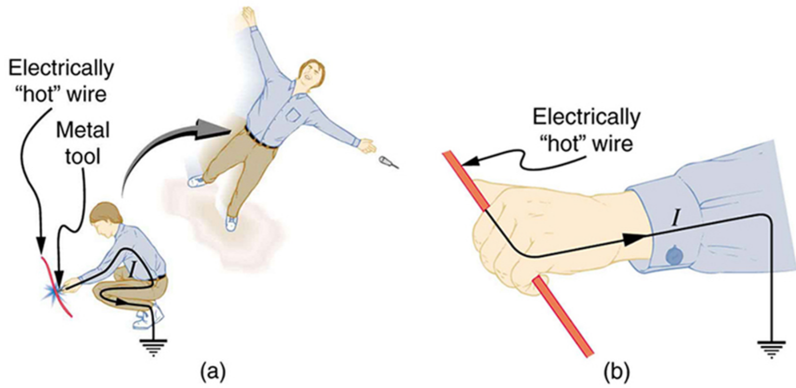


Figure 10.40 An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Table 10.1 Effects of Electrical Shock as a Function of Current[1]

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles contracted, propelling them in a manner not of their own choosing. (See **Figure 10.40**(a).) More frightening, and potentially more dangerous, is the "can't let go" effect illustrated in **Figure 10.40**(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer's hand may close about the victim's wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called "ventricular fibrillation." This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a

1. For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance of about $200\ \text{k}\ \Omega$. If he comes into contact with 120-V AC, a current $I = (120\ \text{V})\,/\,(200\ \text{k}\ \Omega\ ) = 0.6\ \text{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \text{k}\ \Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in **Table 10.1** produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.

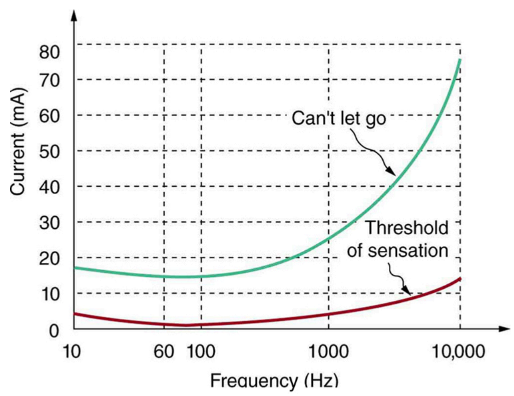


Figure 10.41 Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. **Figure 10.41** presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See **Figure 10.42**.)

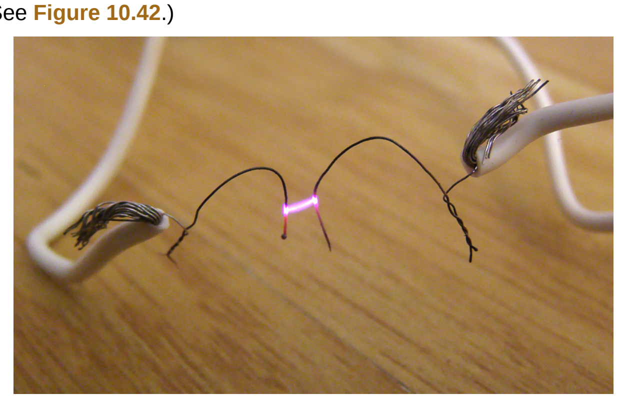


Figure 10.42 Is this electric arc dangerous? The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Glossary

ampere: (amp) the SI unit for current; 1 A = 1 C/s

Coulomb force: another term for the electrostatic force

Coulomb's law: the mathematical equation calculating the electrostatic force vector between two charged particles

current: the flow of charge through an electric circuit past a given point of measurement

electric charge: a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

electric current: the rate at which charge flows, $I = \Delta Q/\Delta t$

electric field: a three-dimensional map of the electric force extended out into space from a point charge

electric field lines: a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

electric potential: potential energy per unit charge

electric power: the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

electromagnetic force: one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

electron: a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

electron volt: the energy given to a fundamental charge accelerated through a potential difference of one volt

electrostatic force: the amount and direction of attraction or repulsion between two charged bodies

electrostatic precipitators: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

electrostatics: the study of electric forces that are static or slow-moving

field: a map of the amount and direction of a force acting on other objects, extending out into space

grounded: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

ink-jet printer: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

Joule's law: the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

laser printer: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

law of conservation of charge: states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

mechanical energy: sum of the kinetic energy and potential energy of a system; this sum is a constant

microshock sensitive: a condition in which a person's skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

ohm: the unit of resistance, given by 1Ω = 1 V/A

ohmic: a type of a material for which Ohm's law is valid

Ohm's law: an empirical relation stating that the current I is proportional to the potential difference V. It is often written as $I = V/R$, where R is the resistance

Ohm's law: the relationship between current, voltage, and resistance within an electrical circuit: $V = IR$

parallel: the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

photoconductor: a substance that is an insulator until it is exposed to light, when it becomes a conductor

point charge: A charged particle, designated Q, generating an electric field

potential difference (or voltage): change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

proton: a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

resistance: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, *R* = *V/I*

resistance: causing a loss of electrical power in a circuit

resistor: a component that provides resistance to the current flowing through an electrical circuit

series: a sequence of resistors or other components wired into a circuit one after the other

shock hazard: when electric current passes through a person

short circuit: also known as a “short,” a low-resistance path between terminals of a voltage source

simple circuit: a circuit with a single voltage source and a single resistor

static electricity: a buildup of electric charge on the surface of an object

test charge: A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

thermal hazard: a hazard in which electric current causes undesired thermal effects

Van de Graaff generator: a machine that produces a large amount of excess charge, used for experiments with high voltage

vector: a quantity with both magnitude and direction

vector addition: mathematical combination of two or more vectors, including their magnitudes, directions, and positions

voltage: the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop: the loss of electrical power as a current travels through a resistor, wire or other component

xerography: a dry copying process based on electrostatics

Section Summary

10.1 Static Electricity and Charge: Conservation of Charge

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is

$$|q_e| = 1.60\times10^{-19}\text{ C.}$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

10.2 Coulomb’s Law

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb’s law gives the magnitude of the force between point charges. It is

$$F = k\frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are two point charges separated by a distance r, and $k \approx 8.99 \times 10^9 \ \mathrm{N \cdot m^2 / C^2}$

- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

10.3 Electric Field: Concept of a Field Revisited

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is

$$\mathbf{E} = k\frac{|Q|}{r^2}.$$

where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

10.4 Electric Field Lines

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

10.5 Electric Potential Energy: Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_\mathrm{B} - V_\mathrm{A}$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1 \ \mathrm{eV} &= \left(1.60\times10^{-19} \ \mathrm{C}\right)(1 \ \mathrm{V}) = \left(1.60\times10^{-19} \ \mathrm{C}\right)(1 \ \mathrm{J/C}) \\ &= 1.60\times10^{-19} \ \mathrm{J}. \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\mathrm{KE} + \mathrm{PE}.$ This sum is a constant.

10.6 Applications of Electrostatics

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

10.7 Current

- Electric current I is the rate at which charge flows, given by

$$I = \frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through an area in time Δt.

- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1 \text{ A} = 1 \text{ C/s}.$
- Current is the flow of free charges, such as electrons and ions.

10.8 Ohm's Law: Resistance and Simple Circuits

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be $I = \frac{V}{R}.$
- Resistance has units of ohms (Ω), related to volts and amperes by $1 \ \Omega = 1 \text{ V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

10.9 Electric Power and Energy

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are

$$P = IV,$$

$$P = \frac{V^2}{R},$$

and

$$P = I^2R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

10.10 Resistors in Series and Parallel

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_s = R_1 + R_2 + R_3 + ...$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

10.11 Electric Hazards and the Human Body

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- **Table 10.1** lists shock hazards as a function of current.
- **Figure 10.41** graphs the threshold current for two hazards as a function of frequency.

Conceptual Questions

10.1 Static Electricity and Charge: Conservation of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

2. Why do most objects tend to contain nearly equal numbers of positive and negative charges?

10.2 Coulomb's Law

3. **Figure 10.43** shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.

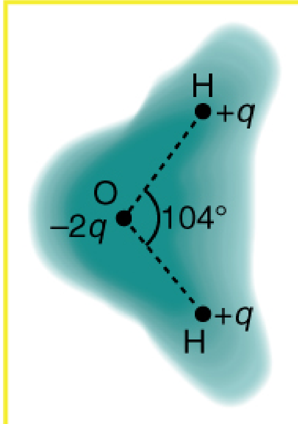


Figure 10.43 Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

4. Using **Figure 10.43**, explain, in terms of Coulomb's law, why a polar molecule (such as in **Figure 10.43**) is attracted by both positive and negative charges.

5. Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

10.3 Electric Field: Concept of a Field Revisited

6. Why must the test charge q in the definition of the electric field be vanishingly small?

7. Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

10.4 Electric Field Lines

8. Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

10.5 Electric Potential Energy: Potential Difference

9. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

10. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

11. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

12. Voltages are always measured between two points. Why?

13. How are units of volts and electron volts related? How do they differ?

10.7 Current

14. Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

15. Car batteries are rated in ampere-hours ($\mathrm{A \cdot h}$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

16. Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

17. In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

18. Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

10.8 Ohm's Law: Resistance and Simple Circuits

19. The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

20. How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

10.9 Electric Power and Energy

21. Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

22. The power dissipated in a resistor is given by $P = V^2 / R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

10.10 Resistors in Series and Parallel

23. A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in **Figure 10.44** has on current when open and when closed.

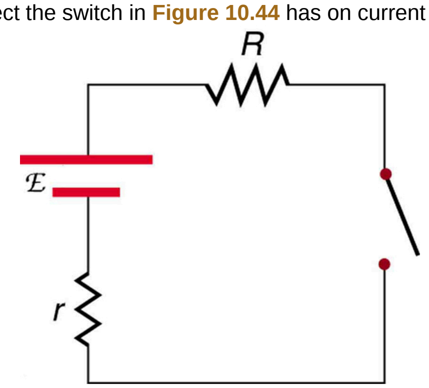


Figure 10.44 A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

24. What is the voltage across the open switch in **Figure 10.44**?

25. There is a voltage across an open switch, such as in **Figure 10.44**. Why, then, is the power dissipated by the open switch small?

26. Why is the power dissipated by a closed switch, such as in **Figure 10.44**, small?

27. A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in **Figure 10.45**. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)

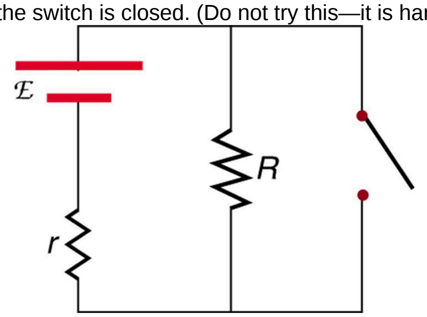


Figure 10.45 A wiring mistake put this switch in parallel with the device represented by R. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

28. Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

29. Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

30. Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

31. If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

32. Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

33. Before World War II, some radios got power through a “resistance cord” that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio’s tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

34. Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

10.11 Electric Hazards and the Human Body

35. Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

36. What are the two major hazards of electricity?

37. Why isn’t a short circuit a shock hazard?

38. What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

39. An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

40. Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

41. Some devices often used in bathrooms, such as hairdryers, often have safety messages saying “Do not use when the bathtub or basin is full of water.” Why is this so?

42. We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

43. Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

44. Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

45. Could a person on intravenous infusion (an IV) be microshock sensitive?

46. In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problems & Exercises

10.1 Static Electricity and Charge: Conservation of Charge

1. Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$?

2. If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

3. To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

4. A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

10.2 Coulomb's Law

5. What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?

6. (a) How strong is the attractive force between a glass rod with a $0.700\ \mu\text{C}$ charge and a silk cloth with a $-0.600\ \mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

7. Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

8. Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

9. How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

10. If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

11. Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms).

12. (a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

13. Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

14. (a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

15. (a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

16. At what distance is the electrostatic force between two protons equal to the weight of one proton?

17. A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

18. (a) Two point charges totaling $8.00\ \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

10.3 Electric Field: Concept of a Field Revisited

19. What is the magnitude and direction of an electric field that exerts a $2.00\times10^{-5}\text{ N}$ upward force on a $-1.75\ \mu\text{C}$ charge?

20. What is the magnitude and direction of the force exerted on a $3.50\ \mu\text{C}$ charge by a 250 N/C electric field that points due east?

21. Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

22. (a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

23. Calculate the initial (from rest) acceleration of a proton in a $5.00\times10^{6}\text{ N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

24. (a) Find the direction and magnitude of an electric field that exerts a $4.80\times10^{-17}\text{ N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

10.4 Electric Field Lines

25. (a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

26. Sketch the electric field lines a long distance from the charge distributions shown in **Figure 10.16** (a) and (b)

27. Figure 10.46 shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes?

(b) Sketch the electric field lines a long distance from the charges shown in the figure.

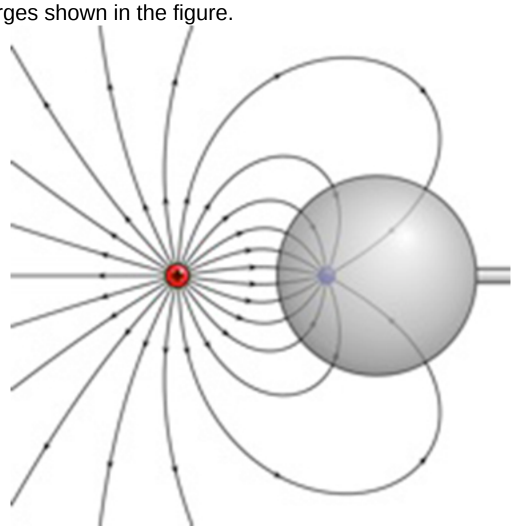


Figure 10.46 The electric field near two charges.

28. Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See Figure 10.46 for a similar situation).

10.5 Electric Potential Energy: Potential Difference

29. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be 1.67×10^{-27} kg.

30. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

31. A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

32. Integrated Concepts

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

33. Integrated Concepts

The temperature near the center of the Sun is thought to be 15 million degrees Celsius $\left(1.5\times10^7\ ^\circ\text{C}\right)$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

34. Integrated Concepts

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

35. Integrated Concepts

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^2 MV. (a) What energy was dissipated? (b) What mass of water could be raised from 15°C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

36. Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from 20.0°C to 90.0°C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

37. Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^2 m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^2 N force for an hour.

38. Integrated Concepts

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

10.7 Current

39. What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

40. A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

41. What is the current when a typical static charge of $0.250\ \mu\text{C}$ moves from your finger to a metal doorknob in $1.00\ \mu\text{s}$?

42. Find the current when 2.00 nC jumps between your comb and hair over a $0.500\text{-}\mu\text{s}$ time interval.

43. A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

44. The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

45. (a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

46. A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

47. The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

48. Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

49. A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

10.8 Ohm's Law: Resistance and Simple Circuits

50. What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

51. Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

52. What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

53. How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

54. (a) Find the voltage drop in an extension cord having a $0.0600\text{-}\ \Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

55. A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00\times10^{9}\ \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

10.9 Electric Power and Energy

56. What is the power of a $1.00\times10^{2}\ \text{MV}$ lightning bolt having a current of $2.00 \times 10^{4}\ \text{A}$?

57. What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

58. A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See **Figure 10.47**.)

Figure 10.47 The strip of solar cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

59. How many watts does a flashlight that has $6.00\times10^{2}\ \text{C}$ pass through it in 0.500 h use if its voltage is 3.00 V?

60. Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600\text{ - }\ \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

61. Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

62. Show that the units $1\ \text{V}^2/\ \Omega\ = 1\text{W}$, as implied by the equation $P = V^2/R$.

63. Show that the units $1\ \text{A}^2 \cdot\ \Omega\ = 1\ \text{W}$, as implied by the equation $P = I^2 R$.

64. Verify the energy unit equivalence that $1\ \text{kW} \cdot \text{h} = 3.60\times10^{6}\ \text{J}$.

65. Electrons in an X-ray tube are accelerated through $1.00\times10^{2}\ \text{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

66. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents/kW} \cdot \text{h}$? See **Figure 10.48**.

Figure 10.48 On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

67. With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\ \text{cents/kW} \cdot \text{h}$, how much does this cost?

68. Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

69. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\text{ A}\cdot\text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

70. A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

71. The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0\text{ cents/kW}\cdot\text{h}$.

10.10 Resistors in Series and Parallel

Note: Data taken from figures can be assumed to be accurate to three significant digits.

72. (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series? (b) In parallel?

73. (a) What is the resistance of a $1.00\times10^{2}\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\ \Omega$ resistor connected in series? (b) In parallel?

74. What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

75. An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

76. Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

77. (a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

78. Referring to the example combining series and parallel circuits and **Figure 10.35**, calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

79. Referring to **Figure 10.35**: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

80. Refer to **Figure 10.36** and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

81. A 240-kV power transmission line carrying $5.00\times10^{2}\text{ A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00\times10^{9}\text{-}\Omega$ resistance. **Figure 10.49**. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the **Problem-Solving Strategies for Series and Parallel Resistors**.

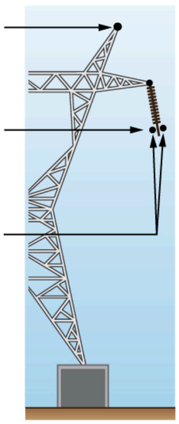


Figure 10.49 High-voltage (240-kV) transmission line carrying $5.00\times10^{2}\text{ A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00\times10^{9}\ \Omega$ of resistance each.

82. Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.

83. Unreasonable Results

Two resistors, one having a resistance of $145\ \Omega$, are connected in parallel to produce a total resistance of $150\ \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

84. Unreasonable Results

Two resistors, one having a resistance of $900\text{ k}\Omega$, are connected in series to produce a total resistance of $0.500\text{ M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

10.11 Electric Hazards and the Human Body

85. (a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

86. What voltage is involved in a 1.44-kW short circuit through a $0.100 - \Omega$ resistance?

87. Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\ \mathrm{k}\ \Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\ \mathrm{k}\ \Omega$.

88. While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

89. Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

90. (a) During surgery, a current as small as $20.0\ \mu\mathrm{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

91. (a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

92. A heart defibrillator passes 10.0 A through a patient’s torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path’s resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

93. Integrated Concepts

A short circuit in a 120-V appliance cord has a $0.500\text{-}\ \Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\ \mathrm{cal/g{\cdot}^{\circ}C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

94. Construct Your Own Problem

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

11 MAGNETISM

Figure 11.1 The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth's magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)

Chapter Outline

11.1. Magnets
- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.

11.2. Ferromagnets and Electromagnets
- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

11.3. Magnetic Fields and Magnetic Field Lines
- Define magnetic field and describe the magnetic field lines of various magnetic fields.

11.4. Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field
- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

11.5. Magnetic Force on a Current-Carrying Conductor
- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

11.6. Motors and Meters
- Describe how motors and meters work in terms of force on a current loop.

11.7. Magnetic Fields Produced by Currents: Ampere's Law
- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

11.8. Induced Voltage and Magnetic Flux
- Describe methods to produce an electromotive force (emf) with a magnetic field or magnet and a loop of wire.

11.9. Faraday's Law of Induction: Lenz's Law
- Calculate voltage, current, and magnetic fields using Faraday's Law.
- Explain the physical results of Lenz's Law

11.10. Transformers
- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

11.11. Alternating Current versus Direct Current
- Explain the differences and similarities between AC and DC current.

- Describe rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Introduction to Magnetism

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists’ understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn’t have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

Figure 11.2 Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)

11.1 Magnets

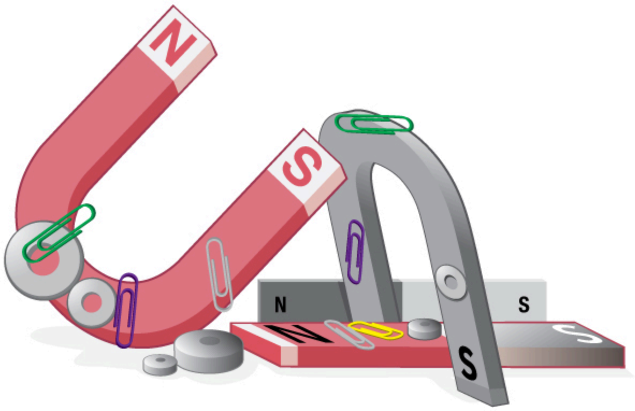


Figure 11.3 Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.

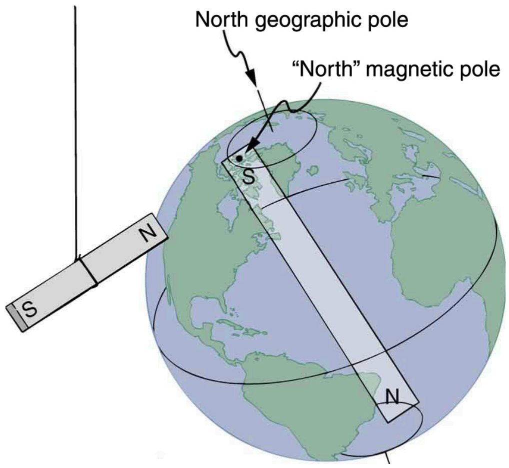


Figure 11.4 One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the North Pole. Thus, "North magnetic pole" is actually a misnomer—it should be called the South magnetic pole.

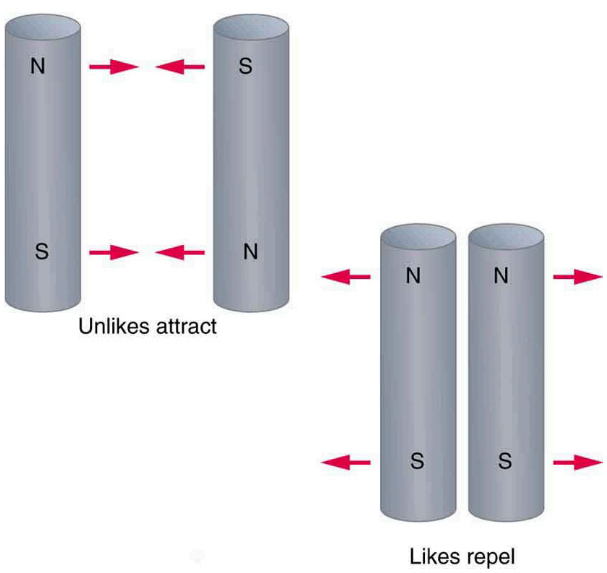


Figure 11.5 Unlike poles attract, whereas like poles repel.

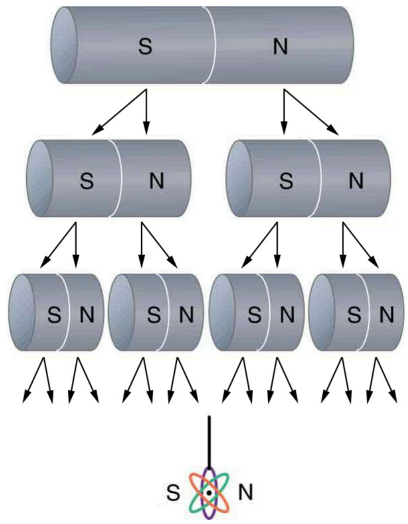


Figure 11.6 North and south poles always occur in pairs. Attempts to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

11.2 Ferromagnets and Electromagnets

Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.

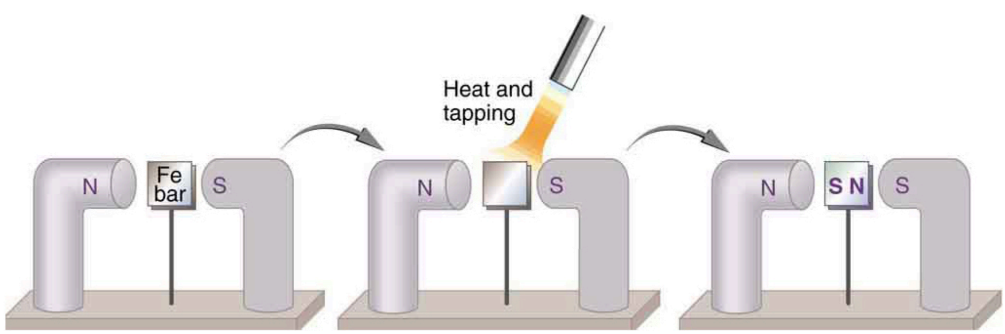


Figure 11.7 An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in **Figure 11.7**. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in **Figure 11.8**. The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in **Figure 11.8**(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.

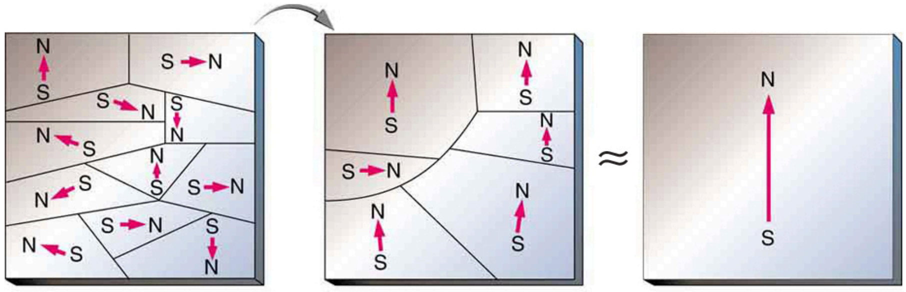


Figure 11.8 (a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K $(770^{\circ}\mathrm{C})$, which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See **Figure 11.9**).

Figure 11.9 Instrument for magnetic resonance imaging (MRI). The device uses a superconducting cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

Figure 11.10 shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.

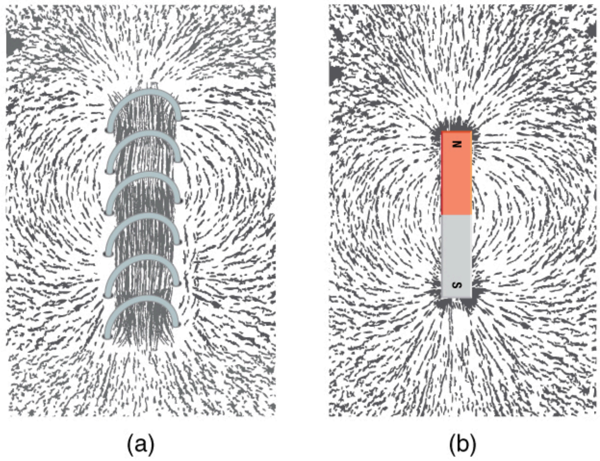


Figure 11.10 Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See **Figure 11.11**.) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

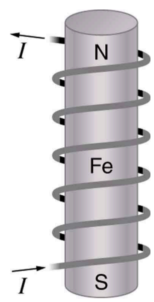


Figure 11.11 An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet.

Figure 11.12 shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.

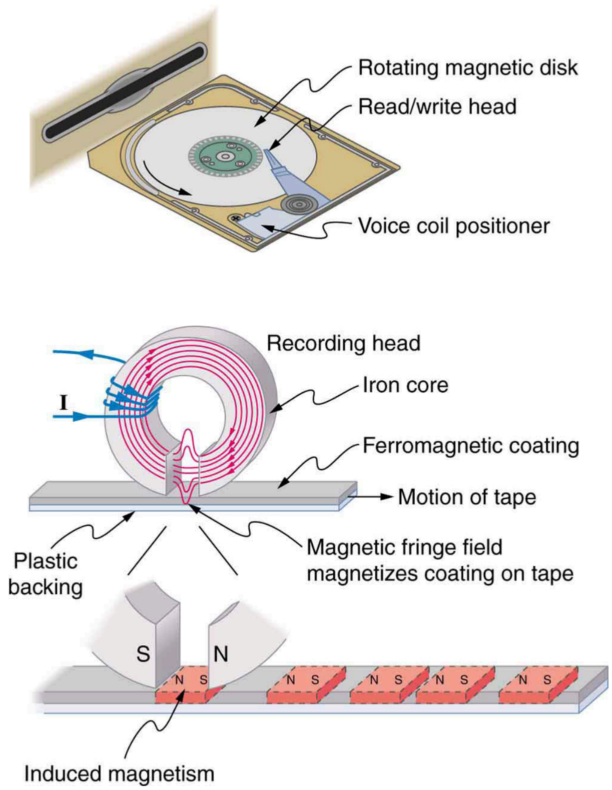


Figure 11.12 An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog (with a varying strength), such as on audiotapes.

Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? **Figure 11.13** shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron’s orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Electric Currents and Magnetism

Electric current is the source of all magnetism.

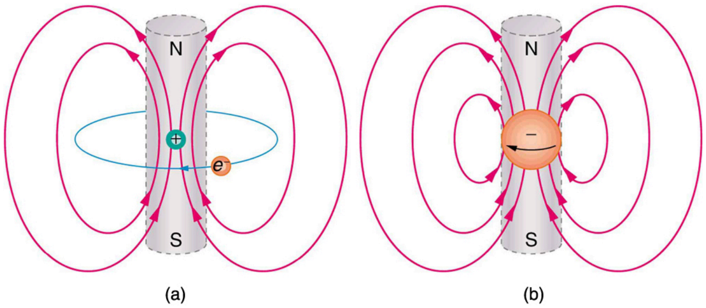


Figure 11.13 (a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

11.3 Magnetic Fields and Magnetic Field Lines

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in **Figure 11.14**, the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B*-field**.

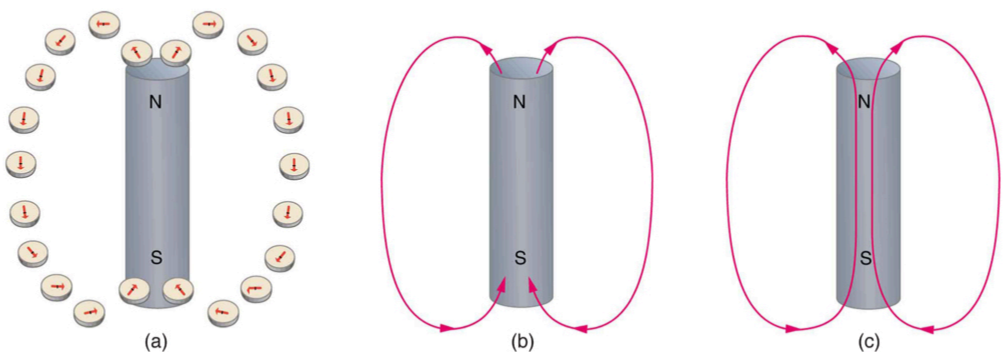


Figure 11.14 Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) **Figure 11.15** shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.

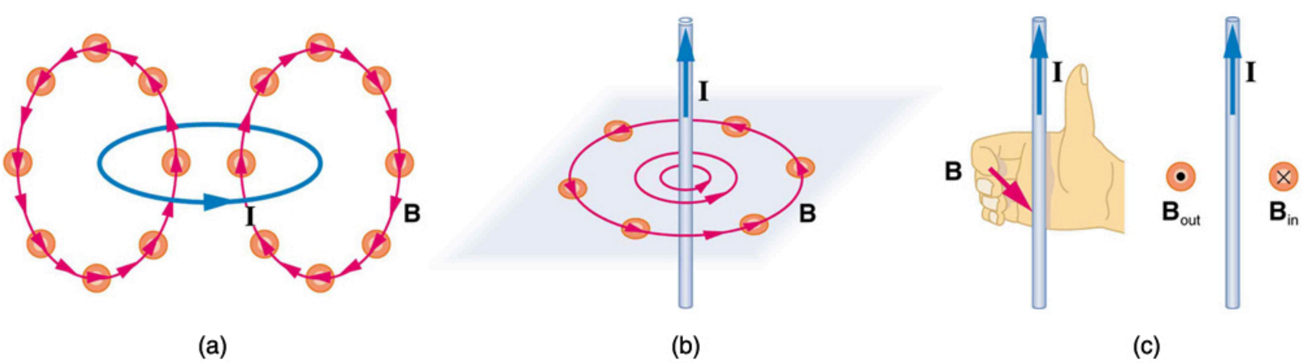


Figure 11.15 Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

11.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Magnetic Force on a Moving Charge

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force.

The magnitude of the magnetic force F on a charge depends on: the quantity of charge q, its speed v, the strength of magnetic field B, and **the direction of motion relative to the magnetic field's direction**. Motion, and its direction, are critical.

The **maximum** force occurs when the direction of motion and the magnetic field's direction are *perpendicular* to one another (i.e. ninety degree angle between directions). $\mathbf{v} \perp \mathbf{B}$ In that situation, the magnitude of the magnetic force is $F = qvB$ The **minimum** force occurs when the direction of motion and the magnetic field's direction are *parallel* to one another (i.e. zero or 180 degree angle between directions). $\mathbf{v} \parallel \mathbf{B}$ In that situation, the magnitude of the magnetic force is $F = 0$

We define the magnetic field strength B in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve for the magnetic field strength.

$$B = \frac{F}{qv} \tag{11.1}$$

So, the tesla is

$$1\text{ T} = \frac{1\text{ N}}{\text{C}\cdot\text{m/s}} = \frac{1\text{ N}}{\text{A}\cdot\text{m}} \tag{11.2}$$

(note that C/s = A).

Another smaller unit, called the **gauss** (G), where $1\text{ G} = 10^{-4}\text{ T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth’s magnetic field on its surface is only about $5\times10^{-5}\text{ T}$, or 0.5 G.

Making Connections: Charges and Magnets

There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Direction of Force: Right Hand Rule 1

The *direction* of the magnetic force $\mathbf{F}$ is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in **Figure 11.16**. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of $\mathbf{v}$, the fingers in the direction of $\mathbf{B}$, and a perpendicular to the palm points in the direction of $\mathbf{F}$. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.

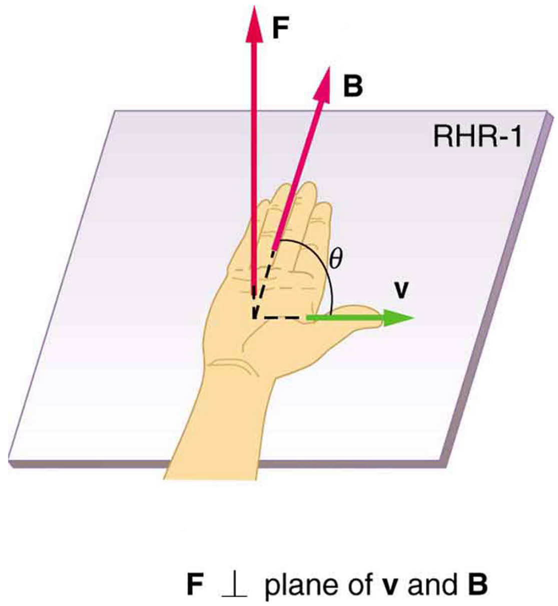


Figure 11.16 Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and depends on the angle between $\mathbf{v}$ and $\mathbf{B}$.

11.5 Magnetic Force on a Current-Carrying Conductor

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.

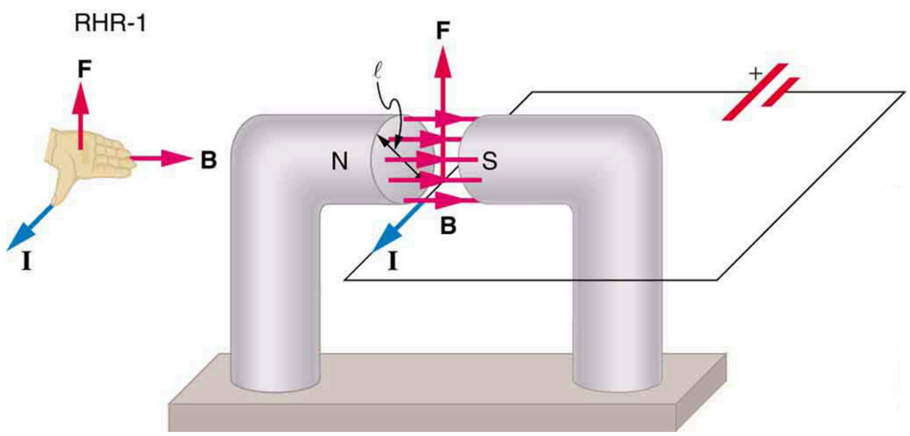


Figure 11.17 The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

The maximum force on a current-carrying conductor occurs when the current direction and the magnetic field's direction are perpendicular to one another (i.e. ninety degree angle between directions). We can derive an expression for the maximum magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity vd is given by $F = qv_d B$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_d B)(N)$, where N is the number of charge carriers in the section of wire of length l. Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (qv_d B)(nAl)$. Gathering terms,

$$F = (nqAv_d)(lB). \tag{11.3}$$

Because $nqAv_\mathrm{d} = I$,

$$F = IlB \tag{11.4}$$

is the equation for *maximum magnetic force on a length* l *of wire carrying a current* I *in a uniform magnetic field* B, as shown in **Figure 11.18**. If we divide both sides of this expression by l, we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB.$ The direction of this force is given by RHR-1, with the thumb in the direction of the current I. Then, with the fingers in the direction of B, a perpendicular to the palm points in the direction of F, as in **Figure 11.18**.

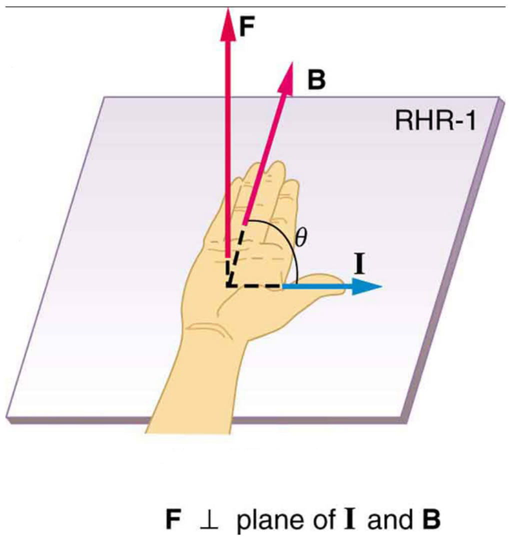


Figure 11.18 The force on a current-carrying wire in a magnetic field is $F = IlB$. Its direction is given by RHR-1.

Example 11.1 Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in **Figure 11.17**, given $B = 1.50\ \mathrm{T}$, $l = 5.00\ \mathrm{cm}$, and $I = 20.0\ \mathrm{A}$.

Strategy

The force can be found with the given information by using $F = IlB$ because the angle between I and B is 90 $^\circ$.

Solution

Entering the given values into $F = IlB$ yields

$$F = IlB = (20.0\ \mathrm{A})(0.0500\ \mathrm{m})(1.50\ \mathrm{T}). \tag{11.5}$$

The units for tesla are $1\ \mathrm{T} = \frac{\mathrm{N}}{\mathrm{A \cdot m}}$; thus,

$$F = 1.50\ \mathrm{N}. \tag{11.6}$$

Discussion

This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See **Figure 11.19**.)

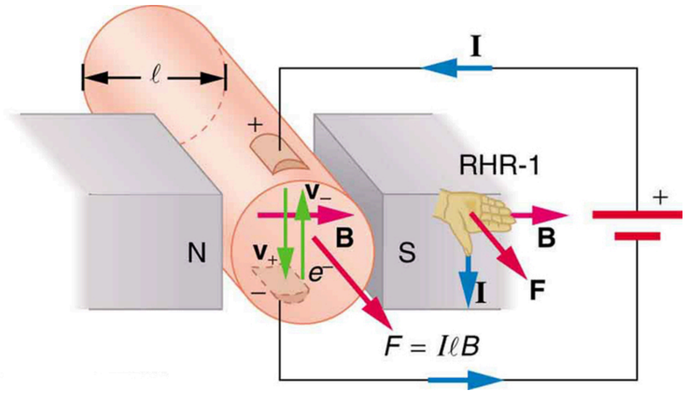


Figure 11.19 Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See **Figure 11.20**.) Existing MHD drives are heavy and inefficient—much development work is needed.

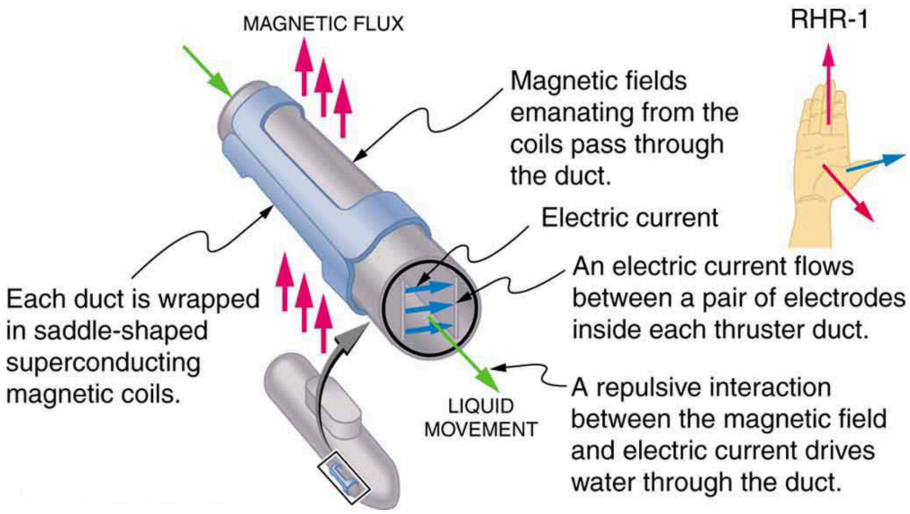


Figure 11.20 An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

11.6 Motors and Meters

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts force on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See **Figure 11.21**.)

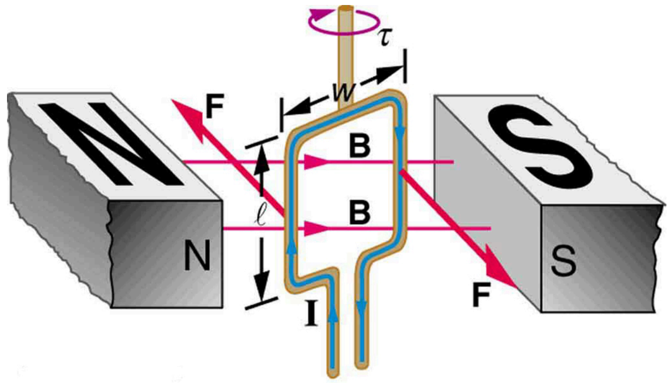


Figure 11.21 Force on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise rotation as viewed from above.

As the coil rotates, the force decreases to zero at $\theta = 0$. The force then *reverses* its direction once the coil rotates past $\theta = 0$. This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See **Figure 11.22**.)

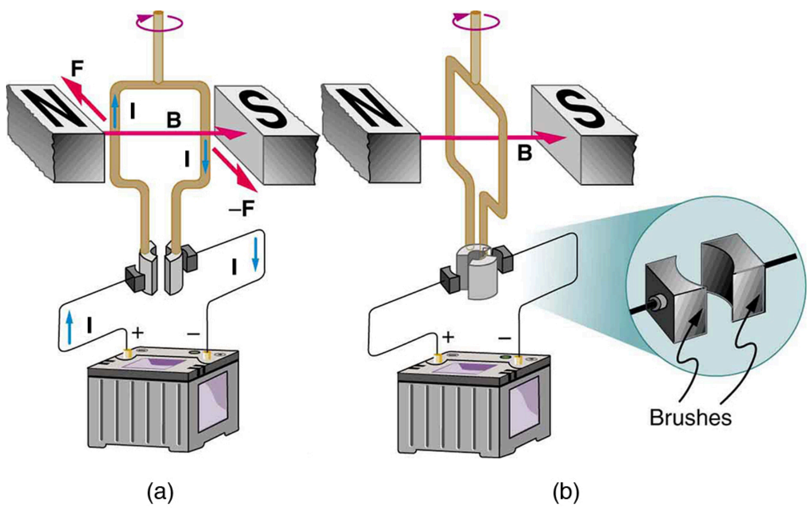


Figure 11.22 (a) As the momentum of the coil carries it through $\theta = 0$, the brushes reverse the current to keep the motion clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the motion.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic force on a current-carrying loop. **Figure 11.23** shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. A linear spring exerts a counter-force that balances the current-produced force. This makes the needle deflection proportional to I. If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A, high magnetic field B, and low-resistance coils.

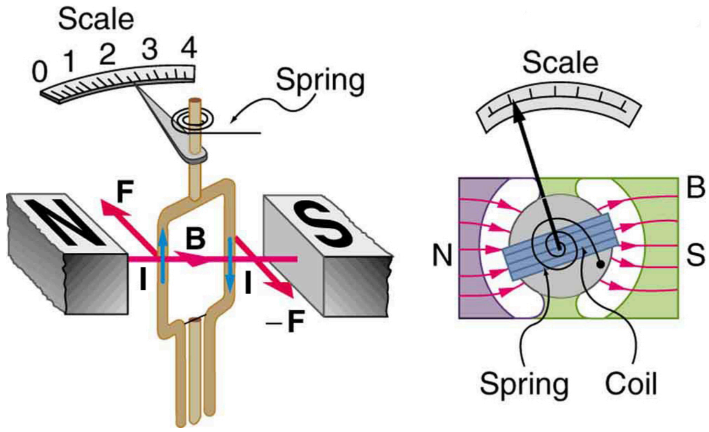


Figure 11.23 Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the force does not depend on θ and the deflection against the return spring is proportional only to the current I.

11.7 Magnetic Fields Produced by Currents: Ampere's Law

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in **Figure 11.24**. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.

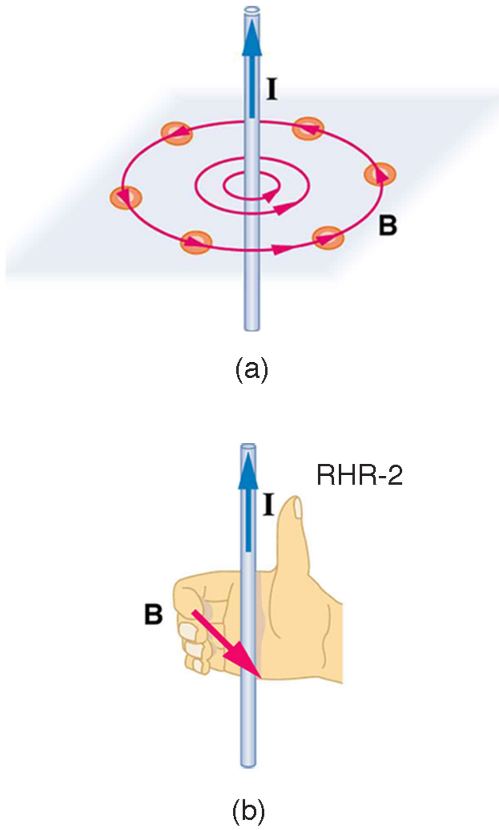


Figure 11.24 (a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude) produced by a long straight current-carrying wire** is found by experiment to be

$$B = \frac{\mu_0 I}{2\pi r} \text{ (long straight wire)}, \tag{11.7}$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

Example 11.2 Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0\times10^{-5}\ \text{T}$, and so here B due to the wire is taken to be $1.0\times10^{-4}\ \text{T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I, since all other quantities are known.

Solution

Solving for I and entering known values gives

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(5.0\times10^{-2}\ \text{m}\right)\left(1.0\times10^{-4}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} \\ &= 25\ \text{A}. \end{aligned} \tag{11.8}$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines, while concentrating on the fields created in certain important situations.

Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in **Figure 11.25**. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in "Magnetic Fields and Magnetic Field Lines" are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

$$B = \frac{\mu_0 I}{2R} \text{ (at center of loop)}, \tag{11.9}$$

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I/(2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

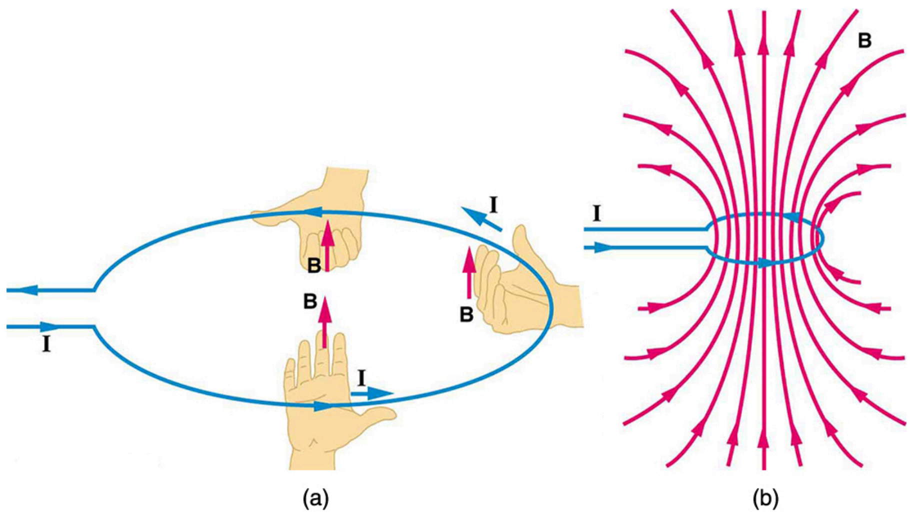


Figure 11.25 (a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. **Figure 11.26** shows how the field looks and how its direction is given by RHR-2.

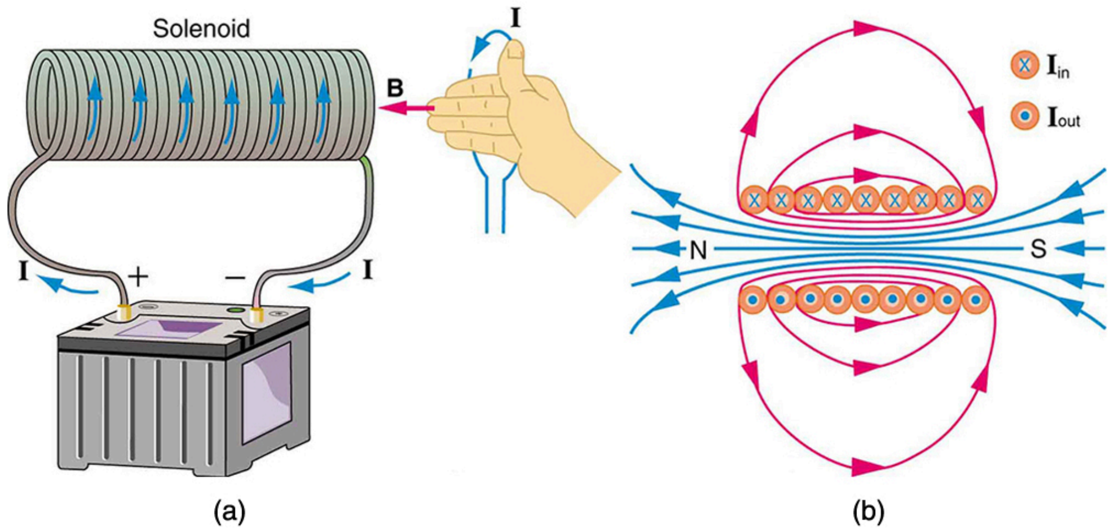


Figure 11.26 (a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the **magnetic field strength inside a solenoid** is simply

$$B = \mu_0 nI \ \ \text{(inside a solenoid)}, \tag{11.10}$$

where n is the number of loops per unit length of the solenoid $(n = N/l$, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as **Example 11.3** implies.

Example 11.3 Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 nI$. First, we note the number of loops per unit length is

$$n^{-1} = \frac{N}{l} = \frac{2000}{2.00 \text{ m}} = 1000 \text{ m}^{-1} = 10 \text{ cm}^{-1}. \tag{11.11}$$

Solution

Substituting known values gives

$$\begin{aligned} B &= \mu_0 nI = \left(4\pi\times10^{-7} \text{ T} \cdot \text{m/A}\right)\left(1000 \text{ m}^{-1}\right)(1600 \text{ A}) \\ &= 2.01 \text{ T}. \end{aligned} \tag{11.12}$$

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter’s length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth’s magnetic field.

11.8 Induced Voltage and Magnetic Flux

The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in **Figure 11.27**. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current. More basic than the current that flows is the voltage that causes it. The current is a result of an *voltage induced by a changing magnetic field*, whether or not there is a path for current to flow.

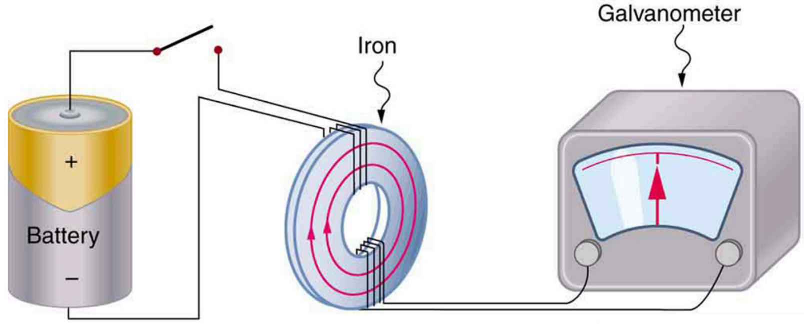


Figure 11.27 Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an voltage and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in **Figure 11.28**. A voltage is induced in the coil when a bar magnet is pushed in and out of it. Voltages of opposite signs are produced by motion in opposite directions, and the voltages are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the voltage, and there is no voltage when the magnet is stationary relative to the coil.

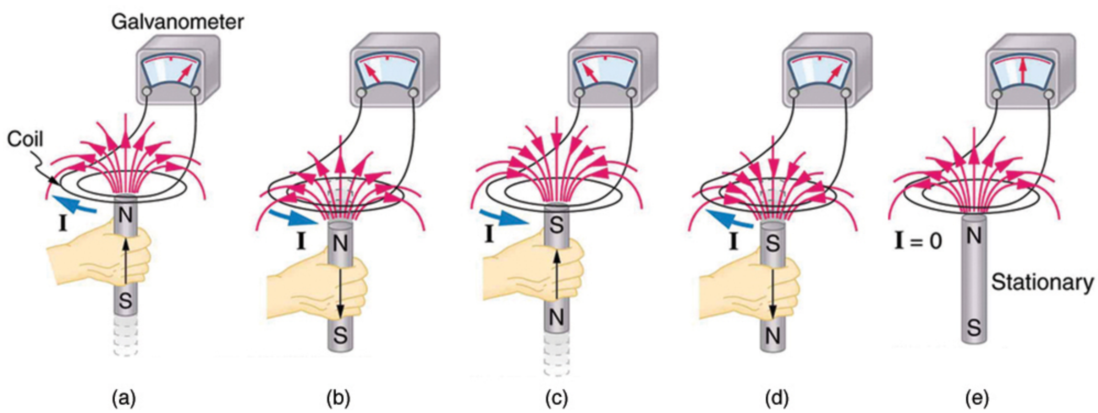


Figure 11.28 Movement of a magnet relative to a coil produces voltage as shown. The same voltages are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the voltage, and the voltage is zero when there is no motion.

The method of inducing a voltage used in most electric generators is shown in **Figure 11.29**. A coil is rotated in a magnetic field, producing an alternating voltage (and current), which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor.

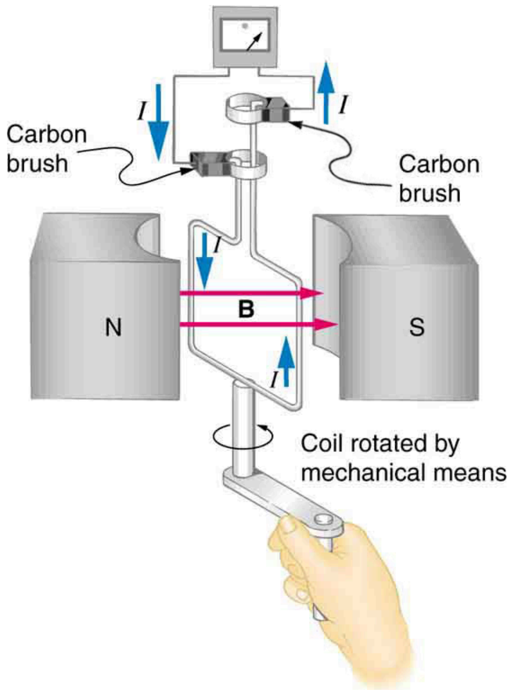


Figure 11.29 Rotation of a coil in a magnetic field produces a voltage. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces a voltage. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ , given by

$$\Phi = B_{\perp} \ A, \tag{11.13}$$

where B is the magnetic field strength over an area A , at an angle θ with the perpendicular to the area as shown in **Figure 11.30**. **Any change in magnetic flux Φ induces a voltage.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$.

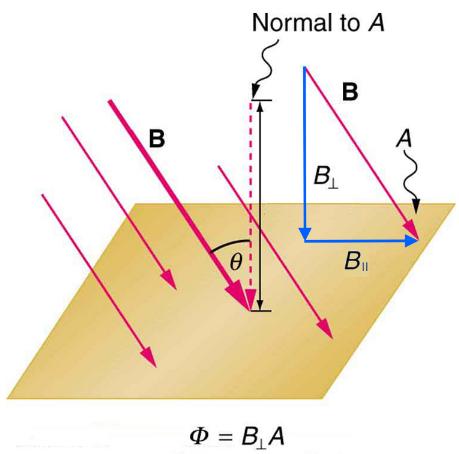


Figure 11.30 Magnetic flux Φ is related to the magnetic field and the area A over which it exists. Only the portion of the magnetic field that is perpendicular to the area ($B_{\perp}$) contributes to the flux. The flux $\Phi = B_{\perp} \ A$ is related to induction; any change in Φ induces a voltage.

All induction, including the examples given so far, arises from some change in magnetic flux Φ . For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in **Figure 11.27**). This is also true for the bar magnet and coil shown in **Figure 11.28**. When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great a voltage and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

11.9 Faraday's Law of Induction: Lenz's Law

Faraday's and Lenz's Law

Faraday's experiments showed that the voltage induced by a change in magnetic flux depends on only a few factors. First, voltage is directly proportional to the change in flux $\Delta\Phi$. Second, voltage is greatest when the change in time Δt is smallest—that is, voltage is inversely proportional to Δt. Finally, if a coil has N turns, a voltage will be produced that is N times greater than for a single coil, so that voltage is directly proportional to N. The equation for the voltage induced by a change in magnetic flux is

$$V = -N\frac{\Delta\Phi}{\Delta t}. \tag{11.14}$$

This relationship is known as **Faraday's law of induction**.

The minus sign in Faraday's law of induction is very important. The minus means that *the induced voltage creates a current I and magnetic field B that oppose the change in flux $\Delta\Phi$ —this is known as Lenz's law*. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See **Figure 11.31**.)

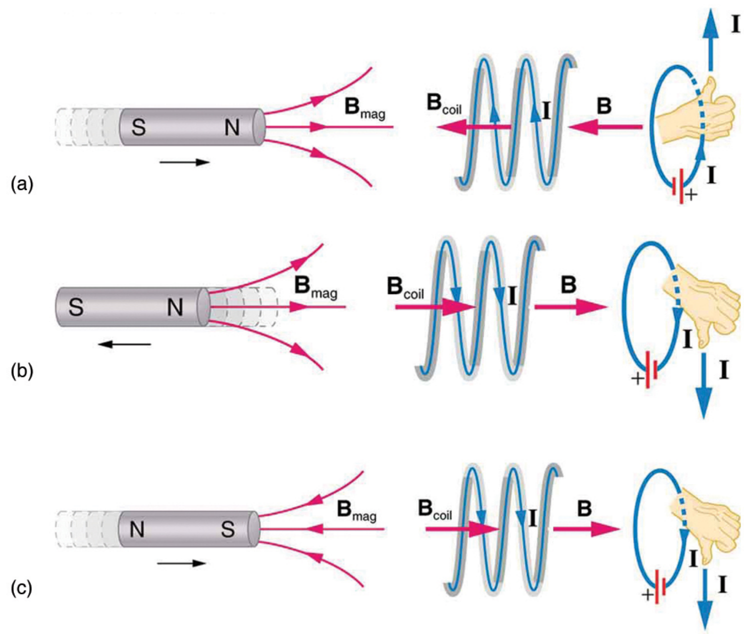


Figure 11.31 (a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced B_{coil} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet (**Figure 11.32**). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.

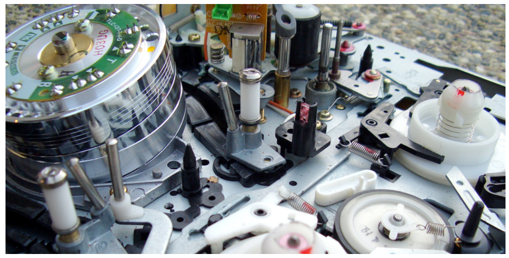


Figure 11.32 Recording and playback heads used with audio and video magnetic tapes. (credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0's or 1's are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.

Figure 11.33 Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic simulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea ("the cessation of breath") affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Making Connections: Conservation of Energy

Lenz's law is a manifestation of the conservation of energy. The induced voltage produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced voltage were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

11.10 Transformers

Transformers do what their name implies—they transform voltages from one value to another. For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in **Figure 11.34**) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in **Figure 11.35**. Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

Figure 11.34 The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range. (credit: Shop Xtreme)

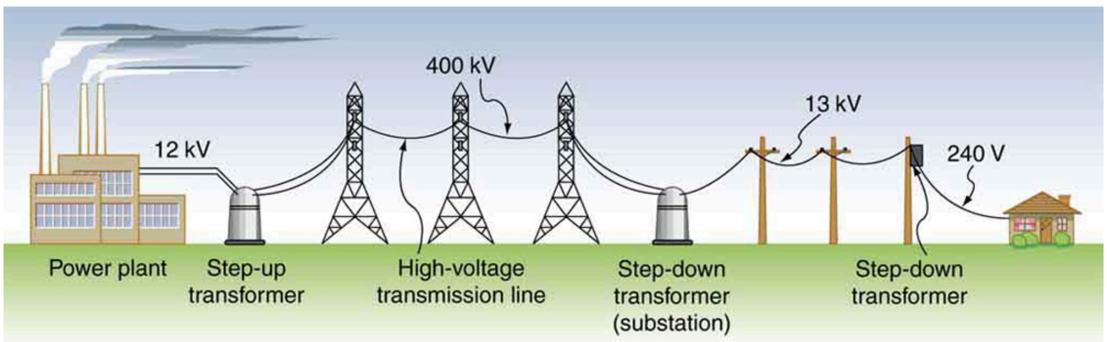


Figure 11.35 Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see **Figure 11.36**—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.

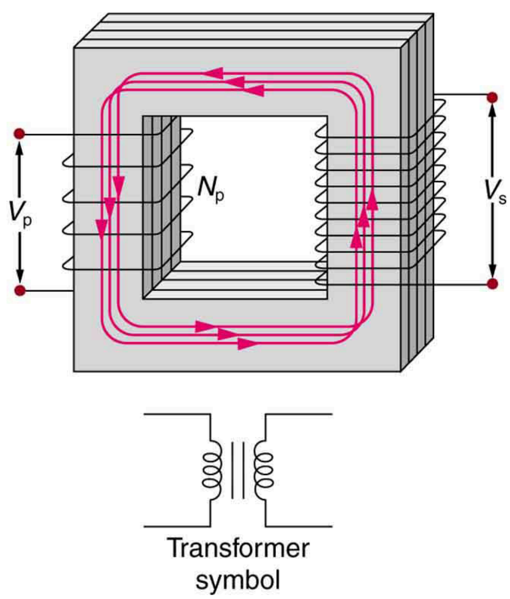


Figure 11.36 A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in **Figure 11.36**, the output voltage V_s depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s to be

$$V_\mathrm{s} = -N_\mathrm{s}\frac{\Delta\Phi}{\Delta t}, \tag{11.15}$$

where N_s is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_p is also related to changing flux by

$$V_p = -N_\mathrm{p}\frac{\Delta\Phi}{\Delta t}. \tag{11.16}$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_p, hence the minus sign. Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced voltage exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

$$\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}}. \tag{11.17}$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up transformer** is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

$$P_\mathrm{p} = I_\mathrm{p}V_\mathrm{p} = I_\mathrm{s}V_\mathrm{s} = P_\mathrm{s}. \tag{11.18}$$

Rearranging terms gives

$$\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{I_\mathrm{p}}{I_\mathrm{s}}. \tag{11.19}$$

Combining this with $\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}}$, we find that

$$\frac{I_\mathrm{s}}{I_\mathrm{p}} = \frac{N_\mathrm{p}}{N_\mathrm{s}} \tag{11.20}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Example 11.4 Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}}$ for N_s, the number of loops in the secondary, and enter the known values. This gives

$$\begin{aligned} N_\mathrm{s} &= N_\mathrm{p}\frac{V_\mathrm{s}}{V_\mathrm{p}} \\ &= (50)\frac{100{,}000\ \mathrm{V}}{120\ \mathrm{V}} = 4.17\times10^{4}. \end{aligned} \tag{11.21}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_s and entering known values. This gives

$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00\text{ A})\frac{50}{4.17 \times 10^4} = 12.0\text{ mA}. \end{aligned} \tag{11.22}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00\text{ A})(120\text{ V}) = 1.20\text{ kW}$. This equals the power output $P_p = I_s V_s = (12.0\text{ mA})(100\text{ kV}) = 1.20\text{ kW}$, as we assumed in the derivation of the equations used.

Example 11.5 Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s and entering known values gives

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (200)\frac{15.0\text{ V}}{120\text{ V}} = 25. \end{aligned} \tag{11.23}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_p and entering known values. This gives

$$\begin{aligned} I_p &= I_s \frac{N_s}{N_p} \\ &= (16.0\text{ A})\frac{25}{200} = 2.00\text{ A}. \end{aligned} \tag{11.24}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_p = P_s$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

11.11 Alternating Current versus Direct Current

Alternating Current

Most of the examples in electric circuits, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Many well-known applications, however, use a time-varying voltage source. **Alternating current** (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. Figure 11.37 shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world. The AC voltages range from 100 V to 240 V; the frequencies range from 50 Hz to 60 Hz.

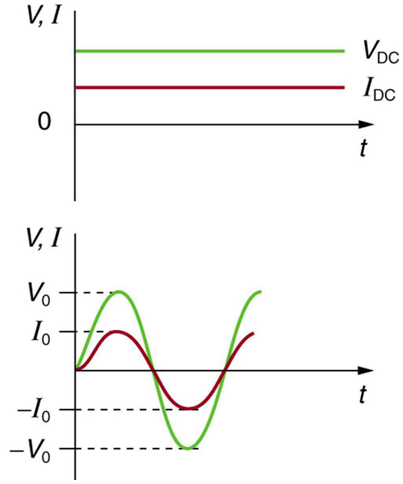


Figure 11.37 (a) DC voltage and current are constant in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.

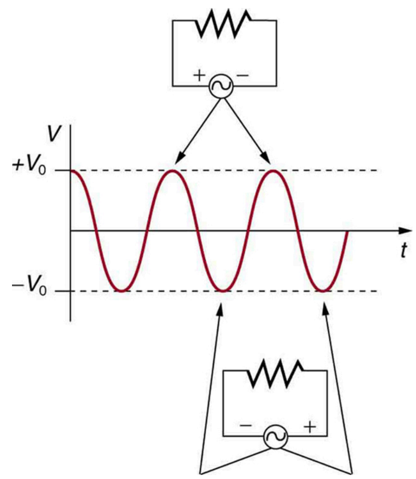


Figure 11.38 The potential difference V between the terminals of an AC voltage source fluctuates as shown.

Figure 11.38 shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown. For this example, the voltage and current are said to be in phase, as seen in **Figure 11.37**(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = IV$.

Making Connections: Take-Home Experiment—AC/DC Lights

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light*.

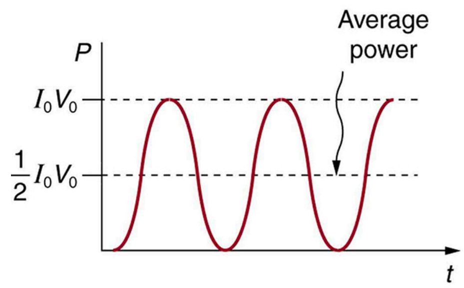


Figure 11.39 AC power as a function of time.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in **Figure 11.39**. One common way to express an average is "root-mean-square," or " **rms**." For example, rms voltage of an AC voltage source is found by first squaring the voltage ("square"), taking an average of this value over one period of oscillation ("mean"), and taking the square root ("root").

It is standard practice to quote I_{rms}, V_{rms}, and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes $P_{\text{ave}} = 1.0\text{ kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}. \tag{11.25}$$

The various expressions for AC power P_{ave} are

$$P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}, \tag{11.26}$$

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}, \tag{11.27}$$

and

$$P_{\text{ave}} = I_{\text{rms}}^2 R. \tag{11.28}$$

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall show below. (See **Figure 11.40**.) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that AC voltages can be increased and decreased efficiently with transformers (which uses electromagnetic induction to produce time-varying voltages), while it is more difficult to change DC voltages without power losses. So AC is used in most large power distribution systems.

Figure 11.40 Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. (Credit: GeorgHH, Wikimedia Commons)

Example 11.6 Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of $1.00\ \Omega$? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\ \text{MW}$, $V_{\text{rms}} = 200\ \text{kV}$, and the resistance of the lines is $R = 1.00\ \ \Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100\times10^6\ \text{W}}{200\times10^3\ \text{V}} = 500\ \text{A}. \tag{11.29}$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2 R$. Substituting the known values gives

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500\ \text{A})^2(1.00\ \ \Omega\ ) = 250\ \text{kW}. \tag{11.30}$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

$$\%\ \text{loss} = \frac{250\ \text{kW}}{100\ \text{MW}}\times 100 = 0.250\ \%. \tag{11.31}$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

Glossary

AC current: current that fluctuates sinusoidally with time.

AC voltage: voltage that fluctuates sinusoidally with time.

alternating current: (AC) the flow of electric charge that periodically reverses direction

Ampere's law: the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

***B*-field:** another term for magnetic field

Biot-Savart law: a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Curie temperature: the temperature above which a ferromagnetic material cannot be magnetized

direct current: (DC) the flow of electric charge in only one direction

direction of magnetic field lines: the direction that the north end of a compass needle points

domains: regions within a material that behave like small bar magnets

electromagnet: an object that is temporarily magnetic when an electrical current is passed through it

electromagnetic induction: the process of inducing a voltage with a change in magnetic flux

electromagnetism: the use of electrical currents to induce magnetism

Faraday's law of induction: the means of calculating the voltage in a coil due to changing magnetic flux, given by

$$V = -N\frac{\Delta\Phi}{\Delta t}$$

ferromagnetic: materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

gauss: G, the unit of the magnetic field strength; $1\ \text{G} = 10^{-4}\ \text{T}$

Lenz's law: the minus sign in Faraday's law, signifying that the voltage induced in a coil opposes the change in magnetic flux

magnetic field: the representation of magnetic forces

magnetic field lines: the pictorial representation of the strength and the direction of a magnetic field

magnetic field strength (magnitude) produced by a long straight current-carrying wire: defined as $B = \frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

magnetic field strength at the center of a circular loop: defined as $B = \frac{\mu_0 I}{2R}$ where R is the radius of the loop

magnetic field strength inside a solenoid: defined as $B = \mu_0 nI$ where n is the number of loops per unit length of the solenoid $(n = N/l$, with N being the number of loops and l the length)

magnetic flux: the amount of magnetic field going through a particular area, calculated with $\Phi = B_\perp\ A$, where $B_\perp$ is the magnetic field strength perpendicular to the area A

magnetic force: the force on a charge produced by its motion through a magnetic field

magnetic monopoles: an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

magnetized: to be turned into a magnet; to be induced to be magnetic

Maxwell's equations: a set of four equations that describe electromagnetic phenomena

meter: common application of magnetic force on a current-carrying loop that is very similar in construction to a motor; by design, the force is proportional to I and not θ, so the needle deflection is proportional to the current

motor: loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts force on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

north magnetic pole: the end or the side of a magnet that is attracted toward Earth's geographic north pole

permeability of free space: the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$

right hand rule 1 (RHR-1): the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity $\mathbf{v}$ and the fingers point in the direction of the magnetic field $\mathbf{B}$, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

right hand rule 2 (RHR-2): a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

rms: a type of average taken for a time-varying quantity by squaring it, taking the mean of the square, and then taking the square-root of the mean.

solenoid: a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

south magnetic pole: the end or the side of a magnet that is attracted toward Earth's geographic south pole

step-down transformer: a transformer that decreases voltage

step-up transformer: a transformer that increases voltage

tesla: T, the SI unit of the magnetic field strength; $1\ \text{T} = \frac{1\ \text{N}}{\text{A} \cdot \text{m}}$

transformer: a device that transforms voltages from one value to another using induction

transformer equation: the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_\text{s}}{V_\text{p}} = \frac{N_\text{s}}{N_\text{p}}$

Section Summary

11.1 Magnets

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth's geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

11.2 Ferromagnets and Electromagnets

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

11.3 Magnetic Fields and Magnetic Field Lines

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:

1. The field is tangent to the magnetic field line.
2. Field strength is proportional to the line density.
3. Field lines cannot cross.
4. Field lines are continuous loops.

11.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- The maximum force a magnetic field can exert on a moving charge is $F = qvB$
- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by

$$1\ \text{T} = \frac{1\ \text{N}}{\text{C} \cdot \text{m/s}} = \frac{1\ \text{N}}{\text{A} \cdot \text{m}}.$$

- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F.
- The force is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. Since the force is zero if $\mathbf{v}$ is parallel to $\mathbf{B}$, charged particles often follow magnetic field lines rather than cross them.

11.5 Magnetic Force on a Current-Carrying Conductor

- The magnetic force on current-carrying conductors (when current direction and magnetic field direction are perpendicular) is given by

$$F = IlB,$$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and $I \perp B$. The force follows RHR-1 with the thumb in the direction of I.

11.7 Magnetic Fields Produced by Currents: Ampere's Law

- The strength of the magnetic field created by current in a long straight wire is given by

$$B = \frac{\mu_0 I}{2\pi r} (\text{long straight wire}),$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$ is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by

$$B = \frac{\mu_0 I}{2R} (\text{at center of loop}),$$

where R is the radius of the loop. This equation becomes $B = \mu_0 nI / (2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is

$$B = \mu_0 nI \quad (\text{inside a solenoid}),$$

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

11.8 Induced Voltage and Magnetic Flux

- The crucial quantity in induction is magnetic flux Φ, defined to be $\Phi = B_\perp \; A$, where $B_\perp$ is the magnetic field strength perpendicular to the area A.
- Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$.
- Any change in magnetic flux Φ induces a voltage—the process is defined to be electromagnetic induction.

11.9 Faraday's Law of Induction: Lenz's Law

- Faraday's law of induction states that the emfinduced by a change in magnetic flux is

$$V = -N\frac{\Delta\Phi}{\Delta t}$$

when flux changes by $\Delta\Phi$ in a time Δt.

- If voltage is induced in a coil, N is its number of turns.
- The minus sign means that the induced voltage creates a current I and magnetic field B that *oppose the change in flux* $\Delta\Phi$ —this opposition is known as Lenz's law.

11.10 Transformers

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by

$$\frac{V_\text{s}}{V_\text{p}} = \frac{N_\text{s}}{N_\text{p}},$$

where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.

- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_\text{s}}{I_\text{p}} = \frac{N_\text{p}}{N_\text{s}}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

11.11 Alternating Current versus Direct Current

- Ohm's law for AC is $I_\text{rms} = \frac{V_\text{rms}}{R}$.
- Expressions for the average power of an AC circuit are $P_\text{ave} = I_\text{rms}V_\text{rms}$, $P_\text{ave} = \frac{V_\text{rms}^2}{R}$, and $P_\text{ave} = I_\text{rms}^2 R$, analogous to the expressions for DC circuits.

Conceptual Questions

11.1 Magnets

1. Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

11.3 Magnetic Fields and Magnetic Field Lines

2. Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

3. List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

4. Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

5. Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

11.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

6. If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

11.5 Magnetic Force on a Current-Carrying Conductor

7. Draw a sketch of the situation in **Figure 11.17** showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

8. Verify that the direction of the force in an MHD drive, such as that in **Figure 11.19**, does not depend on the sign of the charges carrying the current across the fluid.

9. Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

10. Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

11.8 Induced Voltage and Magnetic Flux

11. How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in **Figure 11.27** enhance the observation of induced voltage?

12. When a magnet is thrust into a coil as in **Figure 11.28**(a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

13. Explain how magnetic flux can be zero when the magnetic field is not zero.

14. Is a voltage induced in the coil in **Figure 11.41** when it is stretched? If so, state why and give the direction of the induced current.

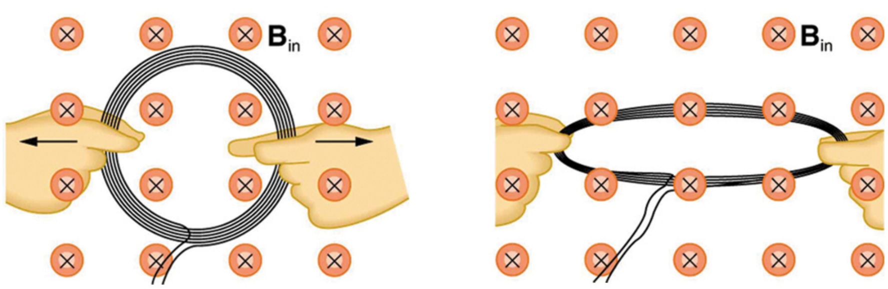


Figure 11.41 A circular coil of wire is stretched in a magnetic field.

11.9 Faraday's Law of Induction: Lenz's Law

15. A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

16. A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

11.11 Alternating Current versus Direct Current

17. Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

18. Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

19. You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problems & Exercises

11.4 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

1. What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in **Figure 11.42**? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."

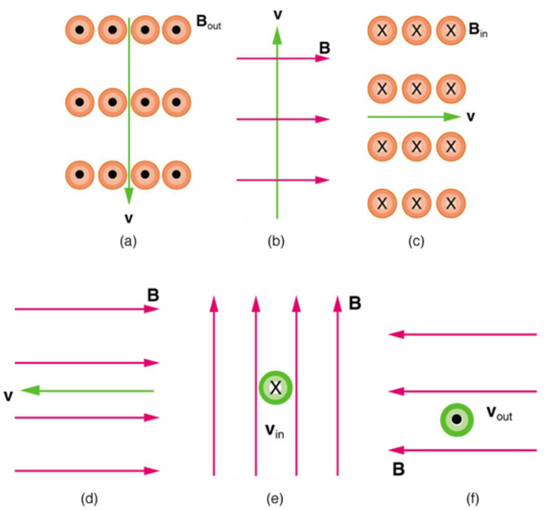


Figure 11.42

2. Repeat **Exercise 11.1** for a negative charge.

3. What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in **Figure 11.43**, assuming it moves perpendicular to $\mathbf{B}$? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."

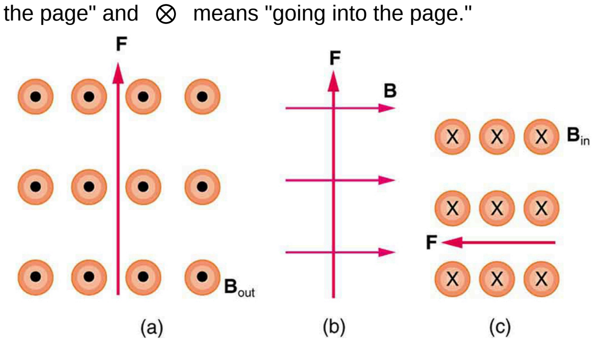


Figure 11.43

4. Repeat **Exercise 11.3** for a positive charge.

5. What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\mathbf{B}$ is perpendicular to $\mathbf{v}$? Note that $\otimes$ means "going into the page."

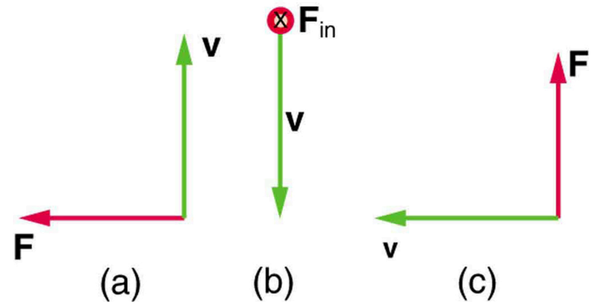


Figure 11.44

6. Repeat **Exercise 11.5** for a negative charge.

7. What is the maximum force on an aluminum rod with a $0.100\text{-}\mu C$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

8. (a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu C$ charge and flies due west at a speed of 660 m/s over the Earth's south magnetic pole, where the $8.00\times10^{-5}\text{-T}$ magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

11.5 Magnetic Force on a Current-Carrying Conductor

9. What is the direction of the magnetic force on the current in each of the six cases in **Figure 11.45**? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."

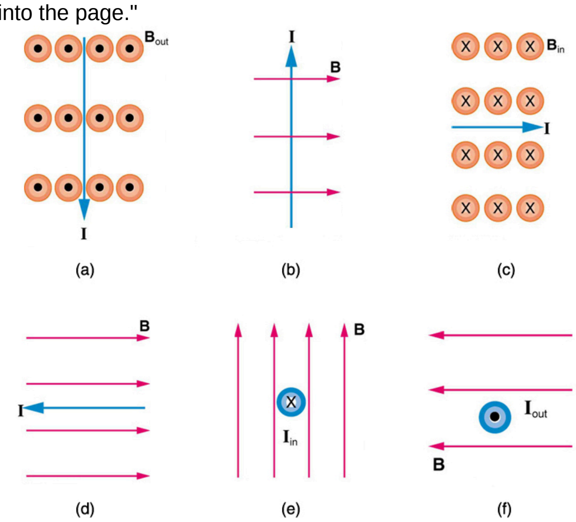


Figure 11.45

10. What is the direction of a current that experiences the magnetic force shown in each of the three cases in **Figure 11.46**, assuming the current runs perpendicular to B ? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."

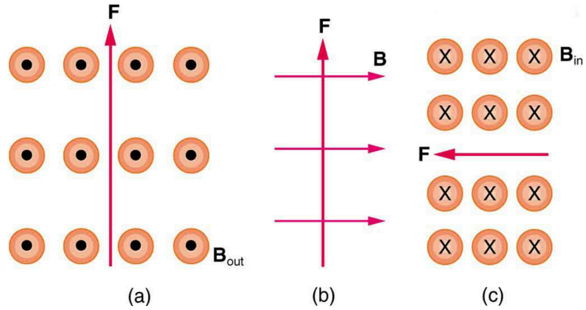


Figure 11.46

11. What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in **Figure 11.47**, assuming $\mathbf{B}$ is perpendicular to $\mathbf{I}$? Note that $\otimes$ means "going into the page."

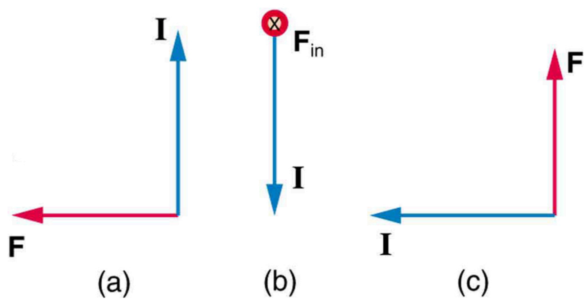


Figure 11.47

12. (a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's $3.00\times10^{-5}\text{-T}$ field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

13. (a) A DC power line for a light-rail system carries 1000 A. If Earth's magnetic field at this location is 5.00×10^{-5} T, what is the maximum possible magnetic force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

14. What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

15. A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

11.9 Faraday's Law of Induction: Lenz's Law

16. Referring to **Figure 11.48**(a), what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant?

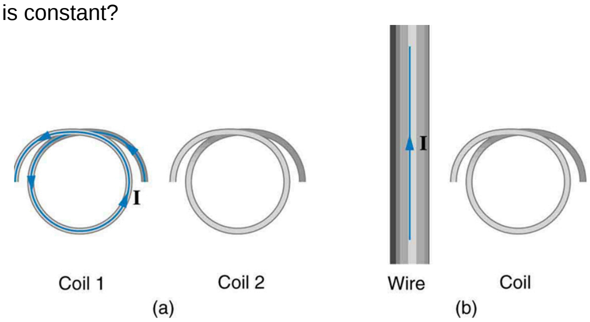


Figure 11.48 (a) The coils lie in the same plane. (b) The wire is in the plane of the coil

17. Referring to **Figure 11.48**(b), what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction?

18. Repeat the previous problem with the battery reversed.

19. Verify that the units of $\Delta\Phi / \Delta t$ are volts. That is, show that $1\ \text{T}\cdot\text{m}^2/\text{s} = 1\ \text{V}$.

20. Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of $0.250\ \text{m}^2$. It is stretched to have no area in 0.100 s. What is the direction and magnitude of the induced voltage if the uniform magnetic field has a strength of 1.50 T?

21. (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average voltage induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.

22. A voltage is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's $5.00\times10^{-5}\ \text{T}$ magnetic field. What average voltage is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

23. A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average voltage of 10,000 V.

11.10 Transformers

24. A plug-in transformer, like that in **???**, supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

25. An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

26. A digital recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

27. (a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

28. (a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

29. A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

30. A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines. (a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

31. If the power output in the previous problem is 1000 MW and line resistance is $2.00\ \Omega$, what were the old and new line losses?

11.11 Alternating Current versus Direct Current

32. Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

33. A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

34. In this problem, you will verify statements made at the end of the power losses for **Example 11.6**. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00 - \ \Omega$ transmission line. (c) What percent loss does this represent?

35. A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\ \text{cents/kW} \cdot \text{h}$?

36. What is the average power consumption of a 120-V AC microwave oven that draws 10.0 A?

37. What is the average current through a 500-W room heater that operates on 120-V AC power?

38. Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

39. Nichrome wire is used in some radiative heaters. Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC.

12 LIGHT

Figure 12.1 Image seen as a result of reflection of light on a plane smooth surface. (credit: NASA Goddard Photo and Video, via Flickr)

Chapter Outline

12.1. Maxwell's Equations: Electromagnetic Waves Predicted and Observed
- Restate Maxwell's equations.

12.2. Production of Electromagnetic Waves
- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.

12.3. The Electromagnetic Spectrum: an Overview
- List three "rules of thumb" that apply to the different frequencies along the electromagnetic spectrum.
- Explain why the higher the frequency, the shorter the wavelength of an electromagnetic wave.
- Draw a simplified electromagnetic spectrum, indicating the relative positions, frequencies, and spacing of the different types of radiation bands.

12.4. The Electromagnetic Spectrum: Application Notes
- List and explain the characteristics and applications of different parts of the electromagnetic spectrum.

12.5. Reflection
- Explain reflection of light from polished and rough surfaces.

12.6. Refraction
- Determine the index of refraction, given the speed of light in a medium.

12.7. Dispersion: The Rainbow and Prisms
- Explain the phenomenon of dispersion and discuss its advantages and disadvantages.

12.8. Image Formation by Lenses
- List the rules for ray tracking for thin lenses.
- Illustrate the formation of images using the technique of ray tracking.
- Determine power of a lens given the focal length.

12.9. Image Formation by Mirrors
- Illustrate image formation in a flat mirror.
- Explain with ray diagrams the formation of an image using spherical mirrors.
- Determine focal length and magnification given radius of curvature, distance of object and image.

12.10. Polarization

- Discuss the meaning of polarization.
- Discuss the property of optical activity of certain materials.

Introduction to Light

Light

Light from this page or screen is formed into an image by the lens of your eye, much as the lens of the camera that made this photograph. Mirrors, like lenses, can also form images that in turn are captured by your eye.

Our lives are filled with light. Through vision, the most valued of our senses, light can evoke spiritual emotions, such as when we view a magnificent sunset or glimpse a rainbow breaking through the clouds. Light can also simply amuse us in a theater, or warn us to stop at an intersection. It has innumerable uses beyond vision. Light can carry telephone signals through glass fibers or cook a meal in a solar oven. Life itself could not exist without light’s energy. From photosynthesis in plants to the sun warming a cold-blooded animal, its supply of energy is vital.

Figure 12.2 Double Rainbow over the bay of Pocitos in Montevideo, Uruguay. (credit: Madrax, Wikimedia Commons)

We will start our discussion of visible light as a type of electromagnetic wave. This knowledge will help us answer questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves.

It is convenient to divide optics into two major parts based on the features we are interested in. The wave characteristics of light, such as frequency and wavelength, relates to the colors we perceive and to how we characterize different types of electromagnetic wave along the electromagnetic spectrum. The wave nature of light is also responsible for phenomena such as diffraction and interference. We call this part of optics "wave optics" or "physical optics." But when light interacts with an object that is several times as large as the light’s wavelength, its observable behavior is like that of a ray; it does not prominently display its wave characteristics, and we can look at primarily how it refracts and reflects. We call this part of optics “geometric optics” or "ray optics."

12.1 Maxwell’s Equations: Electromagnetic Waves Predicted and Observed

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See **Figure 12.3**.) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell’s equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn’s rings.

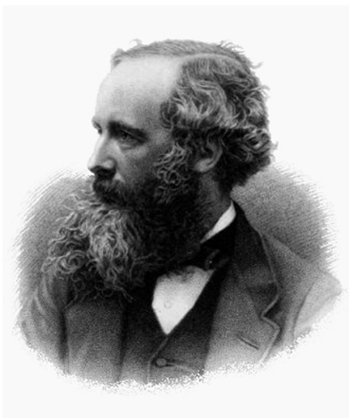


Figure 12.3 James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic waves. (credit: G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. The results of Maxwell’s equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

The Results of Maxwell's Equations

1. Electric field lines originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge.
2. Magnetic field lines are continuous, having no beginning or end. No magnetic monopoles are known to exist.
3. A changing magnetic field induces an electric field.
4. Magnetic fields are generated by moving charges or by changing electric fields.

Maxwell's equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at the speed of light,

$$c = 3.00\times10^{8}\ \text{m/ s}.$$

In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell's theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell's death.

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC circuit that resonates at a known frequency and connected it to a loop of wire as shown in **Figure 12.4**. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.

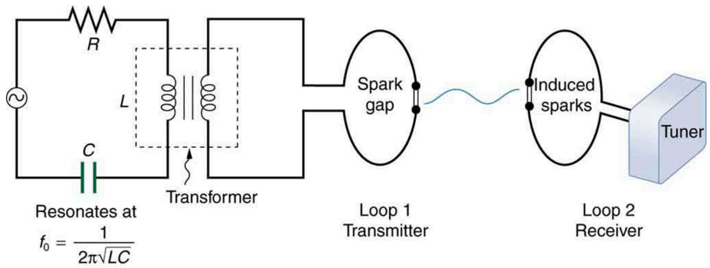


Figure 12.4 The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An AC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v = f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz ($1\ \text{Hz} = 1\ \text{cycle/sec}$), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c. They were predicted by Maxwell, who also showed that
- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

12.2 Production of Electromagnetic Waves

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in **Figure 12.5**.

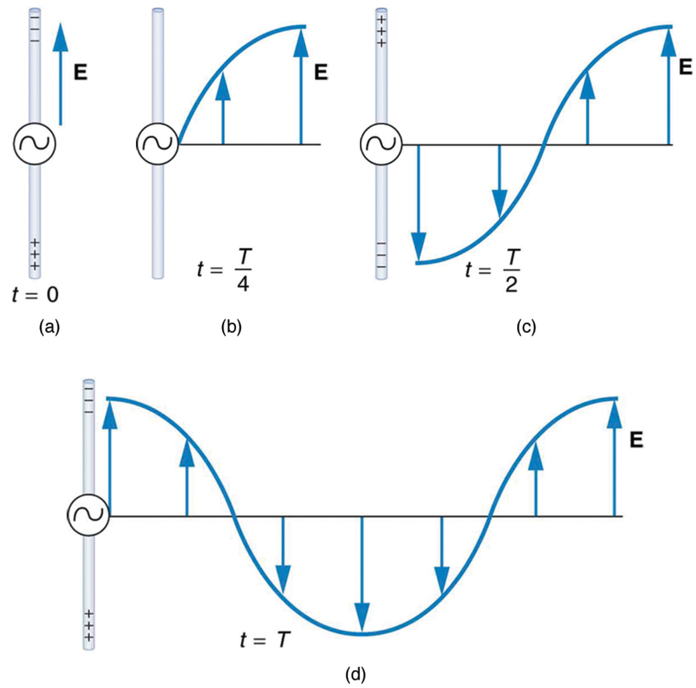


Figure 12.5 This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field ($\mathbf{E}$) propagates away from the antenna at the speed of light, forming part of an electromagnetic wave.

The **electric field** ($\mathbf{E}$) shown surrounding the wire is produced by the charge distribution on the wire. Both the $\mathbf{E}$ and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

There is an associated **magnetic field** ($\mathbf{B}$) which propagates outward as well (see **Figure 12.6**). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

Closer examination of the one complete cycle shown in **Figure 12.5** reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t=0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E-field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E-field has moved away at speed c.

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength** (λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency** (f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

Following Ampere's law, current in the antenna produces a magnetic field, as shown in **Figure 12.6**. The relationship between $\mathbf{E}$ and $\mathbf{B}$ is shown at one instant in **Figure 12.6** (a). As the current varies, the magnetic field varies in magnitude and direction.

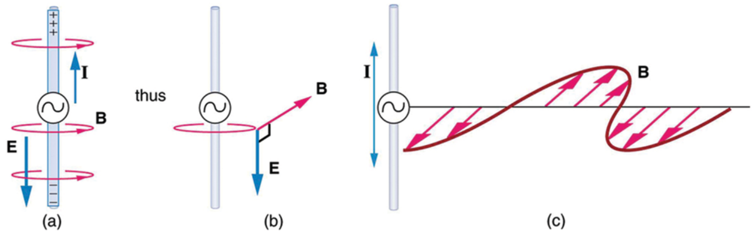


Figure 12.6 (a) The current in the antenna produces the circular magnetic field lines. The current ($\mathbf{I}$) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in Figure 12.6 (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in Figure 12.7. The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.

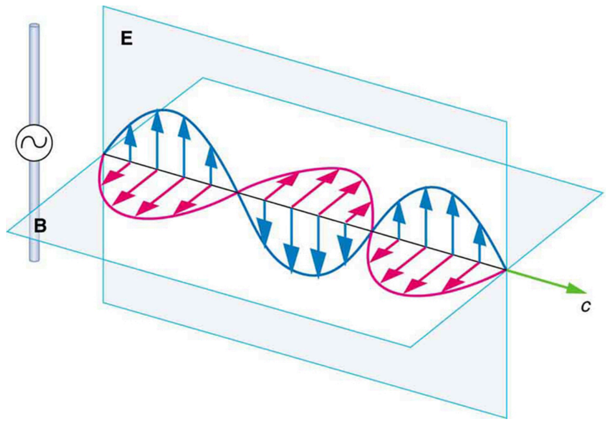


Figure 12.7 A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in Figure 12.7 to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.

12.3 The Electromagnetic Spectrum: an Overview

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in **Table 12.1**. Note that the vast majority of the different types of electromagnetic waves originate from atomic and/or molecular electron transitions—that is, from electrons changing their energy levels within atoms or molecules.

Table 12.1 Electromagnetic Waves

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio and TV	Accelerating charges	Communications, Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges and thermal agitation	Communications, Ovens, Radar	Deep heating	Cell phone use
Infrared	Thermal agitations and atomic/molecular electron transitions	Thermal imaging, Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations and atomic/molecular electron transitions	All pervasive	Photosynthesis, Human vision	
Ultraviolet	Thermal agitations and atomic/molecular electron transitions	Sterilization, Cancer control	Vitamin D production	Ozone depletion, Cancer causing
X-rays	Inner atomic electron transitions and fast collisions	Medical, Security	Medical diagnosis, Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicine, Security	Medical diagnosis, Cancer therapy	Cancer causing, Radiation damage

Connections: Waves

There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression $v_{\text{W}} = f\lambda$. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or c. The relationship among these wave characteristics can be described by $v_{\text{W}} = f\lambda$, where v_{W} is the propagation speed of the wave, f is the frequency, and λ is the wavelength. Here $v_{\text{W}} = c$, so that for all electromagnetic waves,

$$c = f\lambda. \tag{12.1}$$

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

Figure 12.8 shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.

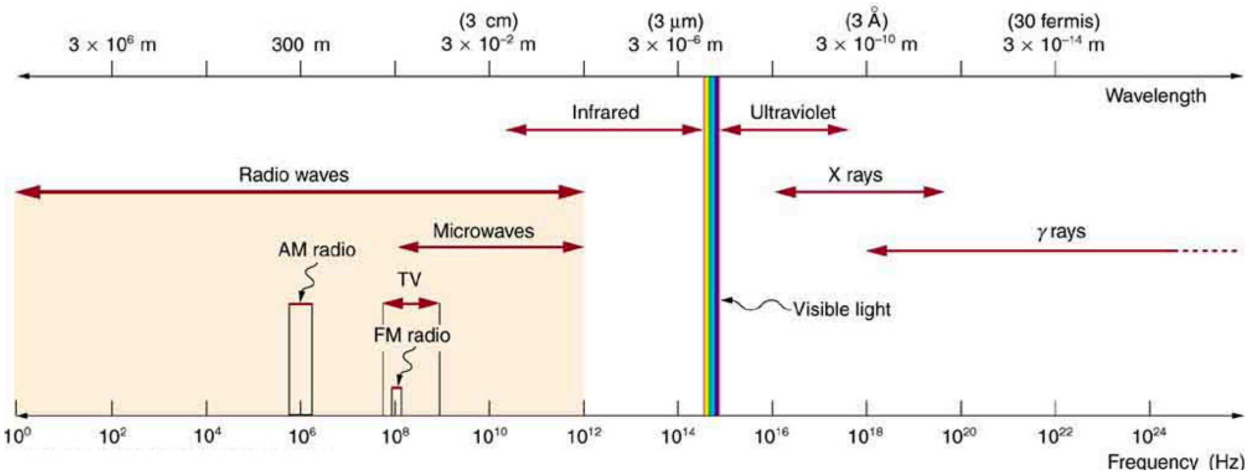


Figure 12.8 The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Electromagnetic Spectrum: Rules of Thumb

Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves. What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Section Summary

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v_{\text{W}} = f\lambda$, so that for electromagnetic waves,

$$c = f\lambda, \tag{12.2}$$

  where f is the frequency, λ is the wavelength, and c is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.

12.4 The Electromagnetic Spectrum: Application Notes

In this module, we look at the properties of different types of electromagnetic waves. Again, **Figure 12.9** shows the electromagnetic spectrum. The characteristics of the various types of electromagnetic waves you will read about below are related to their frequencies and wavelengths.

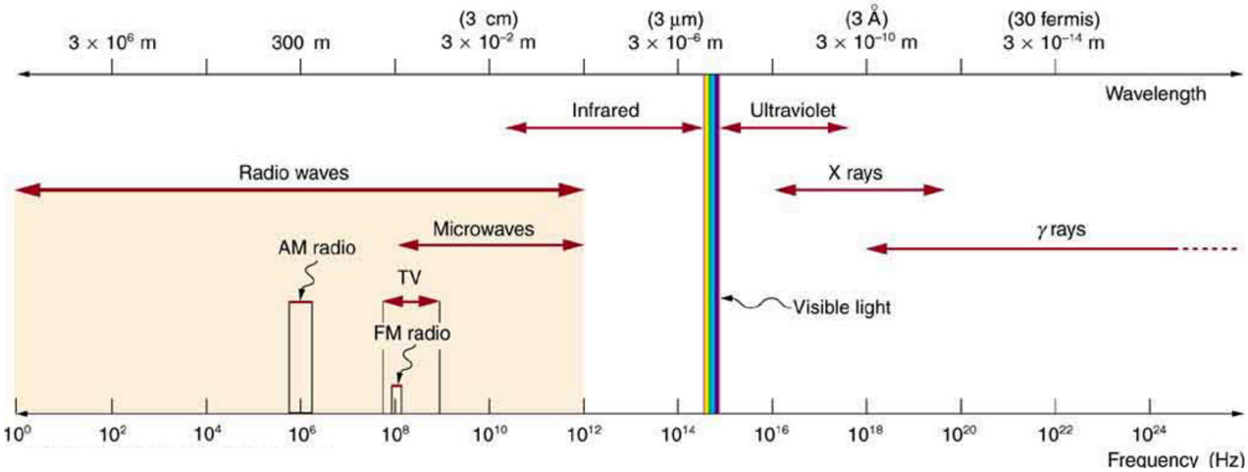


Figure 12.9 The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Radio and TV Waves

The broad category of **radio waves** is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See **Figure 12.10**.) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.

Figure 12.10 This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (E -fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to E -fields.

Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See **Figure 12.11**.)

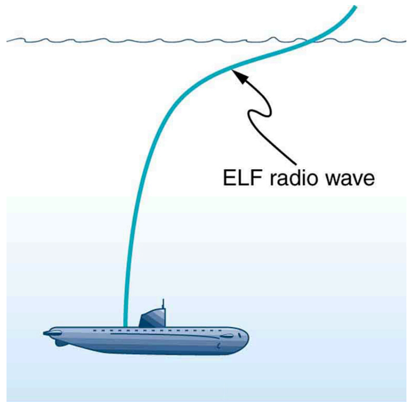


Figure 12.11 Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF). Shorter wavelengths do not penetrate to any significant depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for **amplitude modulation**, which is the method for placing information on these waves. (See **Figure 12.12**.) A **carrier wave** having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver's circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.

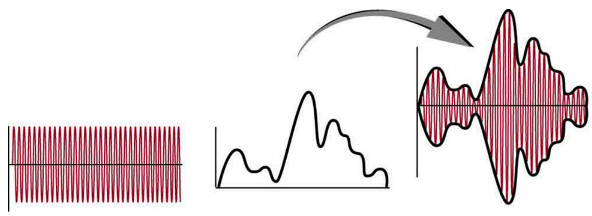


Figure 12.12 Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for **frequency modulation**, another method of carrying information. (See **Figure 12.13**.) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.

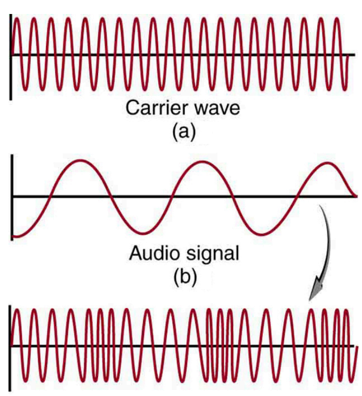


Figure 12.13 Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for **very high frequency**). Other channels called UHF (for **ultra high frequency**) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Example 12.1 Calculating Wavelengths of Radio Waves

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is $c = f\lambda$, where $c = 3.00\times10^8 \text{ m/s}$ is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

$$\lambda = \frac{c}{f}. \tag{12.3}$$

(a) For the $f = 1530 \text{ kHz}$ AM radio signal, then,

$$\begin{aligned} \lambda &= \frac{3.00\times10^8 \text{ m/s}}{1530\times10^3 \text{ cycles/s}} \\ &= 196 \text{ m.} \end{aligned} \tag{12.4}$$

(b) For the $f = 105.1 \text{ MHz}$ FM radio signal,

$$\begin{aligned} \lambda &= \frac{3.00\times10^8 \text{ m/s}}{105.1\times10^6 \text{ cycles/s}} \\ &= 2.85 \text{ m.} \end{aligned} \tag{12.5}$$

(c) And for the $f = 1.90 \text{ GHz}$ cell phone,

$$\begin{aligned} \lambda &= \frac{3.00\times10^8 \text{ m/s}}{1.90\times10^9 \text{ cycles/s}} \\ &= 0.158 \text{ m.} \end{aligned} \tag{12.6}$$

Discussion

These wavelengths are consistent with the spectrum in **Figure 12.9**. The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in "Production of Electromagnetic Waves", is $\lambda / 2$, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See **Figure 12.14**.)

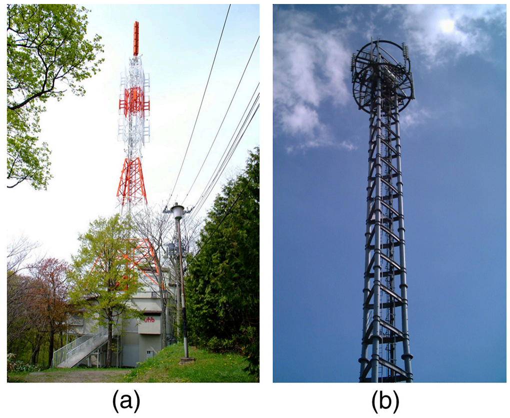


Figure 12.14 (a) A large tower is used to broadcast TV signals. The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons) (b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan. (credit: tokoroten, Wikimedia Commons)

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the "pollution" from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule of thumb (in this case, the rubric that details much smaller than the probe's wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about 10^9 Hz to the highest practical LC resonance at nearly 10^{12} Hz. Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name "microwave."

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by **thermal agitation**. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See **Figure 12.15**.) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.

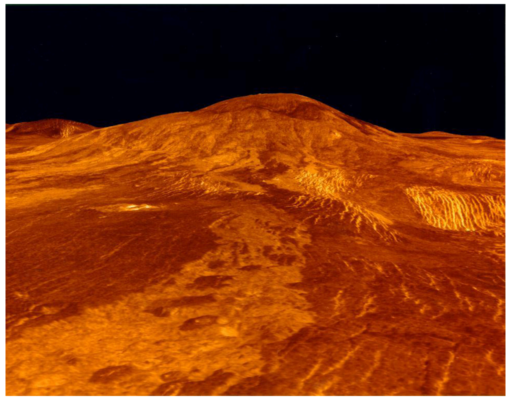


Figure 12.15 An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image. (credit: NSSDC, NASA/JPL)

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish "deep heating" (called microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the $\Delta\mathrm{T}$). Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the $\Delta\mathrm{T}$ for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang's cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see **Figure 12.9**). **Infrared radiation** is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is $e = 0.97$ in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has $e = 1$), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by CO_2 and H_2O in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about $40^{\circ}C$ higher than it would be if there is no absorption. Some scientists think that the increased concentration of CO_2 and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

Figure 12.16 shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.

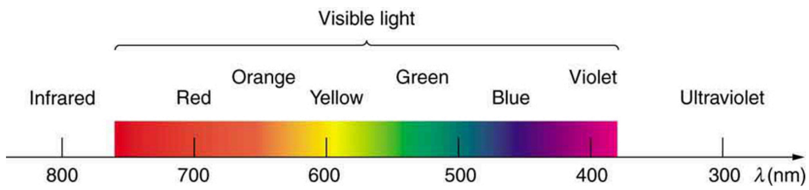


Figure 12.16 A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means "above violet." The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth's surface is UV-A.

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth's surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-

B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth's surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye's lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone (O_3) in our upper atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of $CFCl_3$ with a photon of light (hv) can be written as:

$$CFCl_3 + h\nu \rightarrow CFCl_2 + Cl. \tag{12.7}$$

The Cl atom then catalyzes the breakdown of ozone as follows:

$$Cl + O_3 \rightarrow ClO + O_2 \text{ and } ClO + O_3 \rightarrow Cl + 2O_2. \tag{12.8}$$

A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is India. But the protocol seems to be working, as there are signs of an ozone recovery. (See **Figure 12.17**.)

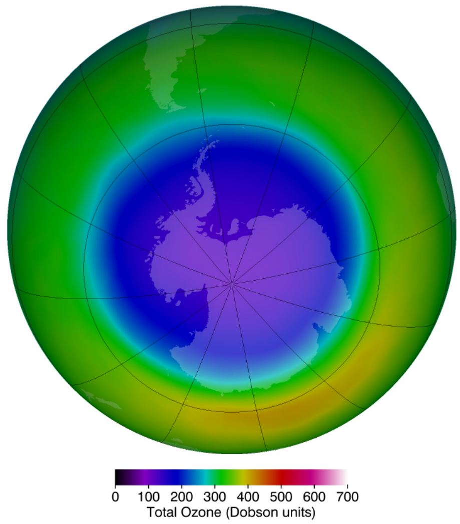


Figure 12.17 This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs. Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37^{o} latitude, most UVB gets blocked by the atmosphere.

UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is also used as an analytical tool to identify substances.

When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

X-Rays

In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an **X-ray**, because its identity and nature were unknown.

Things Great and Small: A Submicroscopic View of X-Ray Production

X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two mechanisms by which the electrons create X-rays.

The first method is illustrated in **Figure 12.18**. An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.

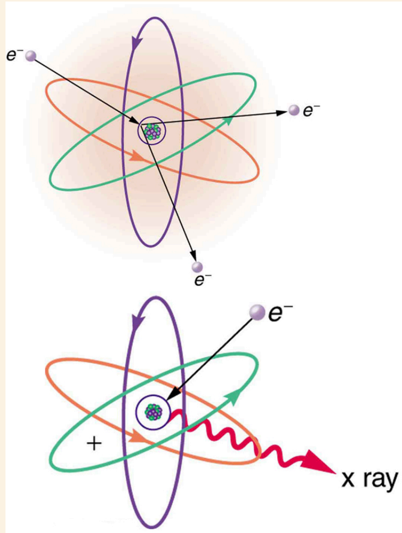


Figure 12.18 Artist's conception of an electron ionizing an atom followed by the recapture of an electron and emission of an X-ray. An energetic electron strikes an atom and knocks an electron out of one of the orbits closest to the nucleus. Later, the atom captures another electron, and the energy released by its fall into a low orbit generates a high-energy EM wave called an X-ray.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray.

The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in **Figure 12.19**. The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.

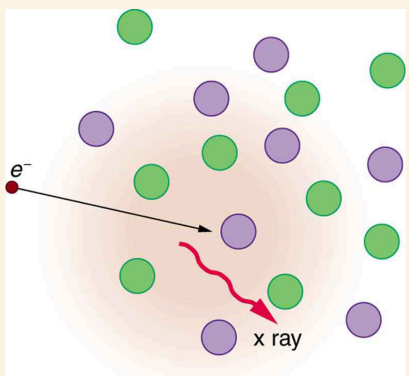


Figure 12.19 Artist's conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An accelerated charge radiates EM waves, a second method by which X-rays are created.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron's velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron, these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called "bremsstrahlung" (German for "braking radiation").

As described above, there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. Figure 12.20 shows an example of the simplest type of X-ray image, an X-ray shadow on film. The amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.

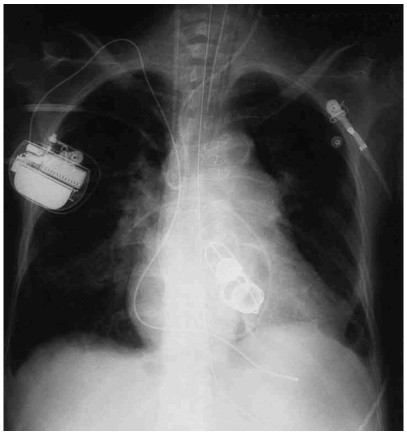


Figure 12.20 This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and the wires used to close the sternum. (credit: P. P. Urone)

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a **gamma ray (γ ray)** (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γ rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the $\gamma\text{-ray}$ frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

Figure 12.21 shows a medical image based on γ rays. Food spoilage can be greatly inhibited by exposing it to large doses of γ radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and $\gamma\text{-ray}$ technologies are also used in scanning luggage at airports.

Figure 12.21 This is an image of the γ rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer is evidenced by nonuniform concentration in similar structures. For example, some ribs are darker than others. (credit: P. P. Urone)

Detecting Electromagnetic Waves from Space

A final note on star gazing. The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γ -ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γ -ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.).

Section Summary

- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared's lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.

- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

Conceptual Questions

Exercise 12.1

If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

Exercise 12.2

Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

Exercise 12.3

How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

Exercise 12.4

Give an example of resonance in the reception of electromagnetic waves.

Exercise 12.5

Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

Exercise 12.6

Why don’t buildings block radio waves as completely as they do visible light?

Exercise 12.7

Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

Exercise 12.8

Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

Exercise 12.9

The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

Exercise 12.10

Give an example of energy carried by an electromagnetic wave.

Exercise 12.11

In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

Exercise 12.12

Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Problems & Exercises

Exercise 12.13

(a) Two microwave frequencies are authorized for use in microwave ovens: 900 and 2560 MHz. Calculate the wavelength of each. (b) Which frequency would produce smaller hot spots in foods due to interference effects?

Solution

(a) 33.3 cm (900 MHz) 11.7 cm (2560 MHz)

(b) The microwave oven with the smaller wavelength would produce smaller hot spots in foods, corresponding to the one with the frequency 2560 MHz.

Exercise 12.14

(a) Calculate the range of wavelengths for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.

Exercise 12.15

A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?

Solution

26.96 MHz

Exercise 12.16

Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

Exercise 12.17

Combing your hair leads to excess electrons on the comb. How fast would you have to move the comb up and down to produce red light?

Solution

5.0×10^{14} Hz

Exercise 12.18

Electromagnetic radiation having a $15.0-\mu\text{m}$ wavelength is classified as infrared radiation. What is its frequency?

Exercise 12.19

Approximately what is the smallest detail observable with a microscope that uses ultraviolet light of frequency

1.20×10^{15} Hz ?

Solution

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{1.20\times10^{15}\ \text{Hz}} = 2.50\times10^{-7}\ \text{m} \tag{12.9}$$

Exercise 12.20

A radar used to detect the presence of aircraft receives a pulse that has reflected off an object 6×10^{-5} s after it was transmitted. What is the distance from the radar station to the reflecting object?

Exercise 12.21

Some radar systems detect the size and shape of objects such as aircraft and geological terrain. Approximately what is the smallest observable detail utilizing 500-MHz radar?

Solution

0.600 m

Exercise 12.22

Determine the amount of time it takes for X-rays of frequency 3×10^{18} Hz to travel (a) 1 mm and (b) 1 cm.

Exercise 12.23

If you wish to detect details of the size of atoms (about 1×10^{-10} m) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?

Solution

(a) $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{1\times10^{-10}\ \text{m}} = 3\times10^{18}\ \text{Hz}$

(b) X-rays

Exercise 12.24

If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.50×10^{11} m away?

Exercise 12.25

Distances in space are often quoted in units of light years, the distance light travels in one year. (a) How many meters is a light year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.00×10^{6} light years away? (c) The most distant galaxy yet discovered is 12.0×10^{9} light years away. How far is this in meters?

Exercise 12.26

A certain 50.0-Hz AC power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very low frequency electromagnetic wave? (b) What is its maximum magnetic field strength?

Solution

(a) 6.00×10^{6} m

(b) 4.33×10^{-5} T

Exercise 12.27

During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

Exercise 12.28

(a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength ($\lambda / 4$) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

Solution

(a) 1.50×10^{6} Hz, AM band

(b) The resonance of currents on an antenna that is 1/4 their wavelength is analogous to the fundamental resonant mode of an air column closed at one end, since the tube also has a length equal to 1/4 the wavelength of the fundamental oscillation.

Exercise 12.29

(a) What is the wavelength of 100-MHz radio waves used in an MRI unit? (b) If the frequencies are swept over a ± 1.00 range centered on 100 MHz, what is the range of wavelengths broadcast?

Exercise 12.30

(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?

Solution

(a) 1.55×10^{15} Hz

(b) The shortest wavelength of visible light is 380 nm, so that

$$\frac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}} = \frac{380 \text{ nm}}{193 \text{ nm}} = 1.97. \tag{12.10}$$

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate!

Exercise 12.31

TV-reception antennas for VHF are constructed with cross wires supported at their centers, as shown in **Figure 12.22**. The ideal length for the cross wires is one-half the wavelength to be received, with the more expensive antennas having one for each channel. Suppose you measure the lengths of the wires for particular channels and find them to be 1.94 and 0.753 m long, respectively. What are the frequencies for these channels?

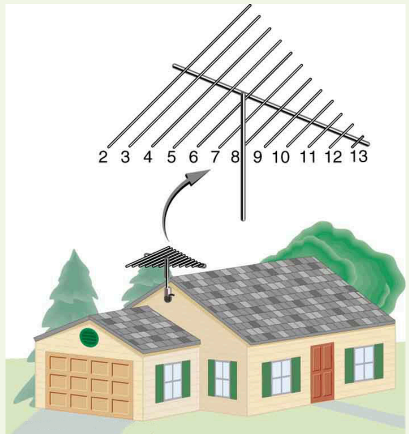


Figure 12.22 A television reception antenna has cross wires of various lengths to most efficiently receive different wavelengths.

Exercise 12.32

Conversations with astronauts on lunar walks had an echo that was used to estimate the distance to the Moon. The sound spoken by the person on Earth was transformed into a radio signal sent to the Moon, and transformed back into sound on a speaker inside the astronaut's space suit. This sound was picked up by the microphone in the space suit (intended for the astronaut's voice) and sent back to Earth as a radio echo of sorts. If the round-trip time was 2.60 s, what was the approximate distance to the Moon, neglecting any delays in the electronics?

Solution

$3.90 \times 10^{8}\ \text{m}$

Exercise 12.33

Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is $3.84 \times 10^{8}\ \text{m}$?

Exercise 12.34

Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a Highway Police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

Solution

(a) $1.50 \times 10^{11}\ \text{m}$

(b) $0.500\ \mu\text{s}$

(c) 66.7 ns

Exercise 12.35

Integrated Concepts

(a) Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (b) Compare this with the ratio of highest to lowest frequencies the ear can hear.

Exercise 12.36

Integrated Concepts

(a) Calculate the rate in watts at which heat transfer through radiation occurs (almost entirely in the infrared) from $1.0\ \text{m}^2$ of the Earth's surface at night. Assume the emissivity is 0.90, the temperature of the Earth is 15°C, and that of outer space is 2.7 K. (b) Compare the intensity of this radiation with that coming to the Earth from the Sun during the day, which averages about $800\ \text{W/m}^2$, only half of which is absorbed. (c) What is the maximum magnetic field strength in the outgoing radiation, assuming it is a continuous wave?

Solution

(a) $-3.5\times10^2\ \text{W/m}^2$

(b) 88%

(c) $1.7\ \mu\text{T}$

12.5 Reflection

Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

The law of reflection is illustrated in **Figure 12.23**, which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but **Figure 12.24** illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in **Figure 12.25**. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in **Figure 12.26**. When the moon reflects from a lake, as shown in **Figure 12.27**, a combination of these effects takes place.

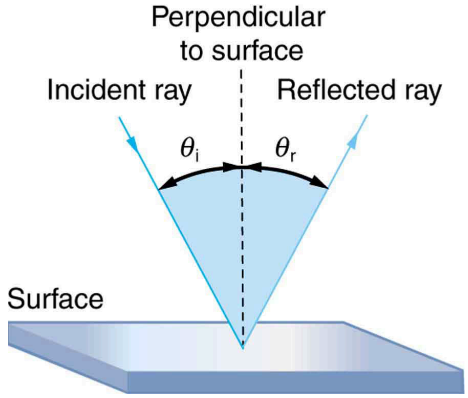


Figure 12.23 The law of reflection states that the angle of reflection equals the angle of incidence— $\theta_r = \theta_i$. The angles are measured relative to the perpendicular to the surface at the point where the ray strikes the surface.

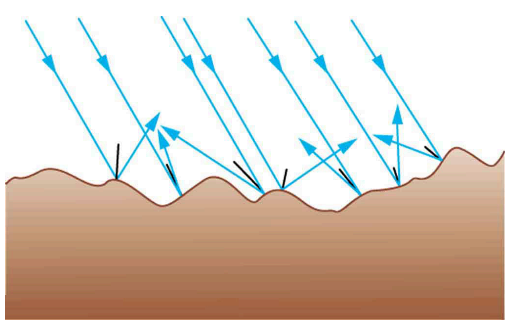


Figure 12.24 Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.

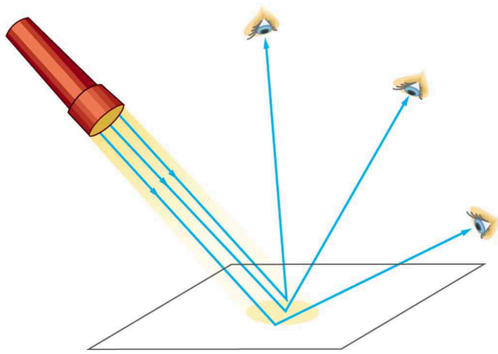


Figure 12.25 When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because its surface is rough and diffuses the light.

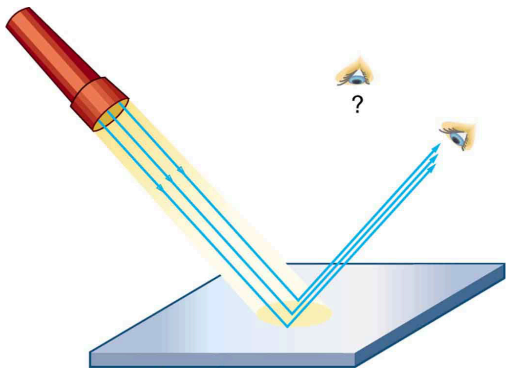


Figure 12.26 A mirror illuminated by many parallel rays reflects them in only one direction, since its surface is very smooth. Only the observer at a particular angle will see the reflected light.

Figure 12.27 Moonlight is spread out when it is reflected by the lake, since the surface is shiny but uneven. (credit: Diego Torres Silvestre, Flickr)

The law of reflection is very simple: The angle of reflection equals the angle of incidence.

The Law of Reflection

The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in **Figure 12.28**. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.

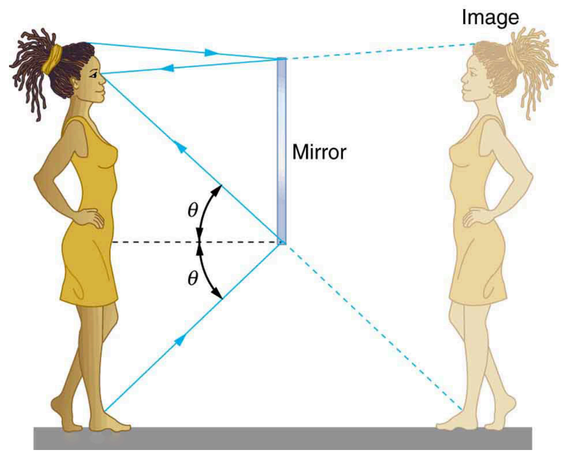


Figure 12.28 Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

Take-Home Experiment: Law of Reflection

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in **Figure 12.25**. Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in **Figure 12.25** and **Figure 12.26**? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

Section Summary

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

Conceptual Questions

Exercise 12.37

Using the law of reflection, explain how powder takes the shine off of a person's nose. What is the name of the optical effect?

12.6 Refraction

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See **Figure 12.29**.) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Refraction

The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

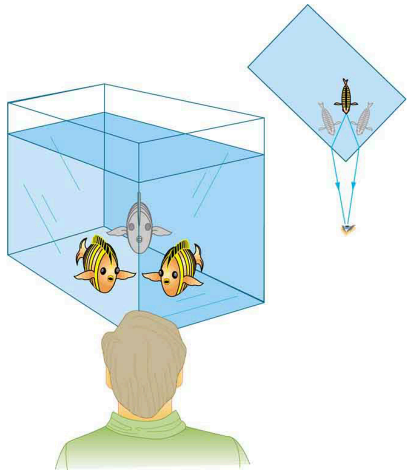


Figure 12.29 Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter's moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be $2.26\times10^{8}\ \text{m/s}$ (only 25% different from today's accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in **Figure 12.30**. Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer's eye.

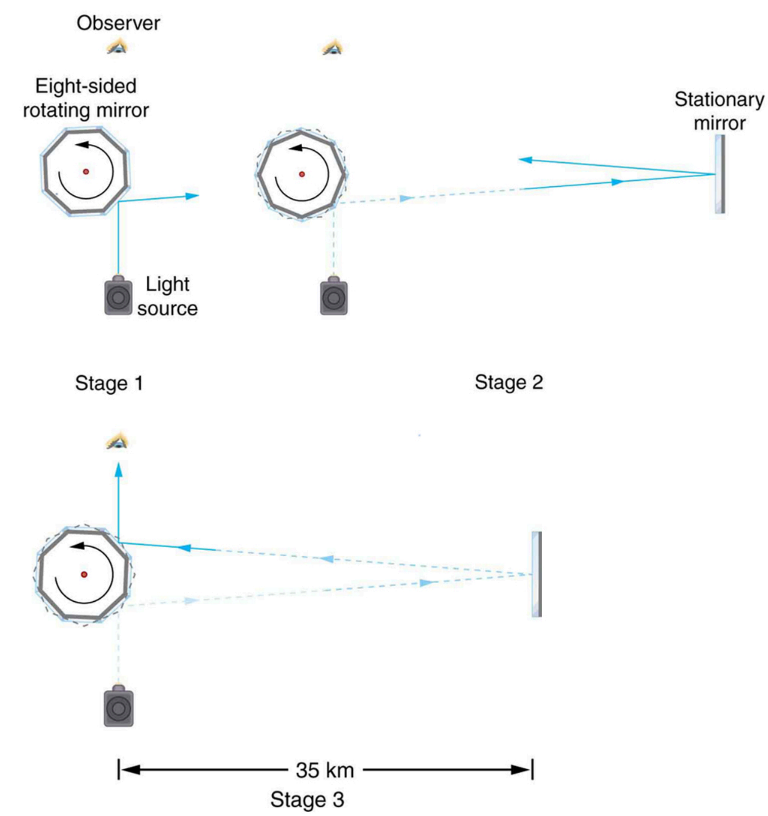


Figure 12.30 A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson's calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

$$c = 2.9972458\times10^{8}\ \text{m/s} \approx 3.00\times10^{8}\ \text{m/s}, \tag{12.11}$$

where the approximate value of $3.00\times10^{8}\ \text{m/s}$ is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

$$n = \frac{c}{v}, \tag{12.12}$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Value of the Speed of Light

$$c = 2.9972458\times10^{8}\ \text{m/s} \approx 3.00\times10^{8}\ \text{m/s} \tag{12.13}$$

Index of Refraction

$$n = \frac{c}{v} \tag{12.14}$$

That is, $n \geq 1$. **Table 12.2** gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n = 1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Table 12.2 Index of Refraction in Various Media

Medium	n
Gases at $0^{\circ}C$ ***, 1 atm***	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at $20^{\circ}C$	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at $20^{\circ}C$	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 20°C	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Example 12.2 Speed of Light in Matter

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Strategy

The speed of light in a material, v, can be calculated from the index of refraction n of the material using the equation $n = c / v$.

Solution

The equation for index of refraction states that $n = c / v$. Rearranging this to determine v gives

$$v = \frac{c}{n}. \tag{12.15}$$

The index of refraction for zircon is given as 1.923 in **Table 12.2**, and c is given in the equation for speed of light. Entering these values in the last expression gives

$$\begin{aligned} v &= \frac{3.00\times10^{8}\ \text{m/s}}{1.923} \\ &= 1.56\times10^{8}\ \text{m/s}. \end{aligned} \tag{12.16}$$

Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in **Table 12.2** that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

Figure 12.31 shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. (Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in **Figure 12.31**, medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in **Figure 12.31**(a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in **Figure 12.31**(b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.

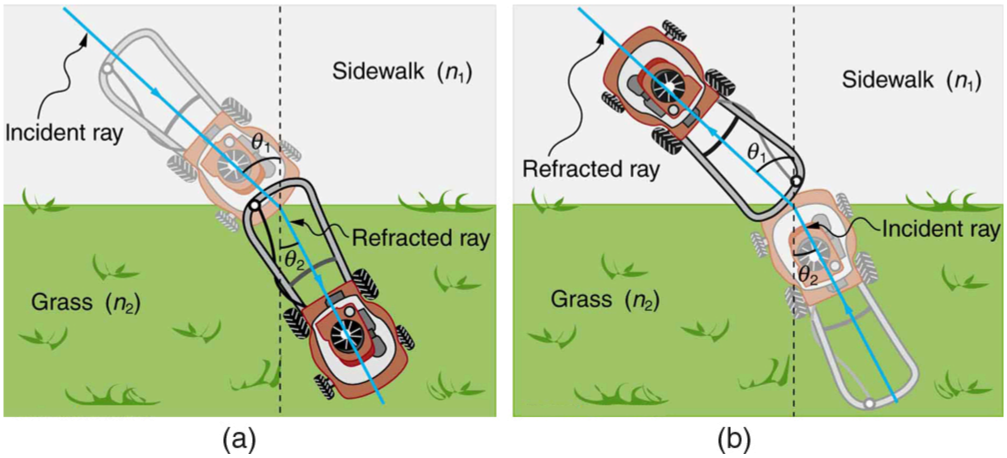


Figure 12.31 The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle.

Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

Section Summary

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum $c = 2.9972458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}.$
- Index of refraction $n = \frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.

Conceptual Questions

Exercise 12.38

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

Exercise 12.39

Why is the index of refraction always greater than or equal to 1?

Exercise 12.40

Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

Exercise 12.41

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

Exercise 12.42

Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

Exercise 12.43

Explain why a person's legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

Exercise 12.44

Why is the front surface of a thermometer curved as shown?

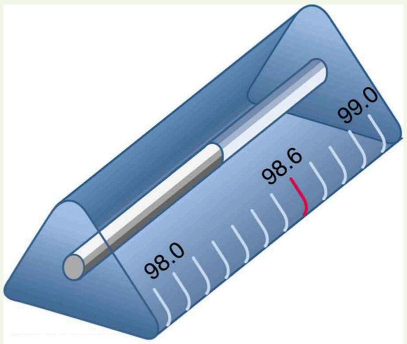


Figure 12.32 The curved surface of the thermometer serves a purpose.

Exercise 12.45

Suppose light were incident from air onto a material that had a negative index of refraction, say –1.3; where does the refracted light ray go?

Problems & Exercises

Exercise 12.46

What is the speed of light in water? In glycerine?

Solution

$2.25\times10^{8}\text{ m/s}$ in water

$2.04\times10^{8}\text{ m/s}$ in glycerine

Exercise 12.47

What is the speed of light in air? In crown glass?

Exercise 12.48

Calculate the index of refraction for a medium in which the speed of light is $2.012\times10^8\ \text{m/s}$, and identify the most likely substance based on Table 12.2.

Solution

1.490 , polystyrene

Exercise 12.49

In what substance in Table 12.2 is the speed of light $2.290\times10^8\ \text{m/s}$?

Exercise 12.50

There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is $3.84\times10^5\ \text{km}$ away, would the light first arrive on Earth?

Solution

$1.28\ \text{s}$

Exercise 12.51

Components of some computers communicate with each other through optical fibers having an index of refraction $n = 1.55$. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

Solution

$1.03\ \text{ns}$

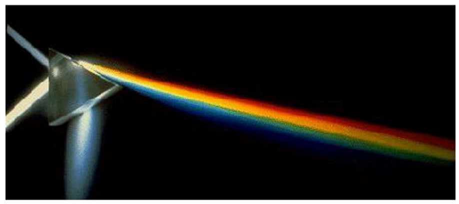
Exercise 12.52

On the Moon's surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth's atmosphere? Assume the distance to the Moon is precisely $3.84\times10^8\ \text{m}$, and Earth's atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction $n = 1.000293$.

12.7 Dispersion: The Rainbow and Prisms

Everyone enjoys the spectacle of a rainbow glimmering against a dark stormy sky. How does sunlight falling on clear drops of rain get broken into the rainbow of colors we see? The same process causes white light to be broken into colors by a clear glass prism or a diamond. (See Figure 12.33.)

(a)



(b)

Figure 12.33 The colors of the rainbow (a) and those produced by a prism (b) are identical. (credit: Alfredo55, Wikimedia Commons; NASA)

We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. Those colors are associated with different wavelengths of light, as shown in **Figure 12.34**. When our eye receives pure-wavelength light, we tend to see only one of the six colors, depending on wavelength. The thousands of other hues we can sense in other situations are our eye's response to various mixtures of wavelengths. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors plotted versus wavelength in **Figure 12.34**. What this implies is that white light is spread out according to wavelength in a rainbow. **Dispersion** is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion, as a general phenomenon, can occur for any type of wave and always involves wavelength-dependent processes.

Dispersion

Dispersion is defined to be the spreading of white light into its full spectrum of wavelengths.

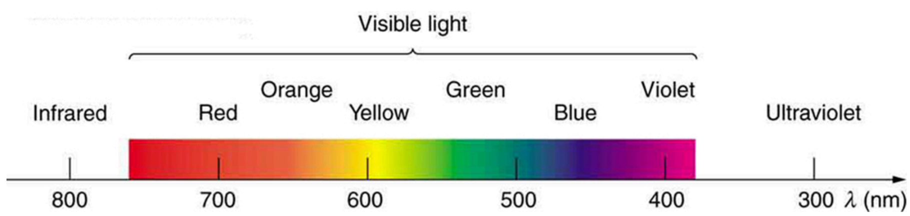


Figure 12.34 Even though rainbows are associated with seven colors, the rainbow is a continuous distribution of colors according to wavelengths.

Refraction is responsible for dispersion in rainbows and many other situations. The angle of refraction depends on the index of refraction. We know that the index of refraction n depends on the medium. But for a given medium, n also depends on wavelength. (See **Table 12.3**. Note that, for a given medium, n increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light, as shown for a prism in **Figure 12.35**(b), and the light is dispersed into the same sequence of wavelengths as seen in **Figure 12.33** and **Figure 12.34**.

Making Connections: Dispersion

Any type of wave can exhibit dispersion. Sound waves, all types of electromagnetic waves, and water waves can be dispersed according to wavelength. Dispersion occurs whenever the speed of propagation depends on wavelength, thus separating and spreading out various wavelengths. Dispersion may require special circumstances and can result in spectacular displays such as in the production of a rainbow. This is also true for sound, since all frequencies ordinarily travel at the same speed. If you listen to sound through a long tube, such as a vacuum cleaner hose, you can easily hear it is dispersed by interaction with the tube. Dispersion, in fact, can reveal a great deal about what the wave has encountered that disperses its wavelengths. The dispersion of electromagnetic radiation from outer space, for example, has revealed much about what exists between the stars—the so-called empty space.

Table 12.3 Index of Refraction *n* in Selected Media at Various Wavelengths

Medium	Red (660 nm)	Orange (610 nm)	Yellow (580 nm)	Green (550 nm)	Blue (470 nm)	Violet (410 nm)
Water	1.331	1.332	1.333	1.335	1.338	1.342
Diamond	2.410	2.415	2.417	2.426	2.444	2.458
Glass, crown	1.512	1.514	1.518	1.519	1.524	1.530
Glass, flint	1.662	1.665	1.667	1.674	1.684	1.698
Polystyrene	1.488	1.490	1.492	1.493	1.499	1.506
Quartz, fused	1.455	1.456	1.458	1.459	1.462	1.468

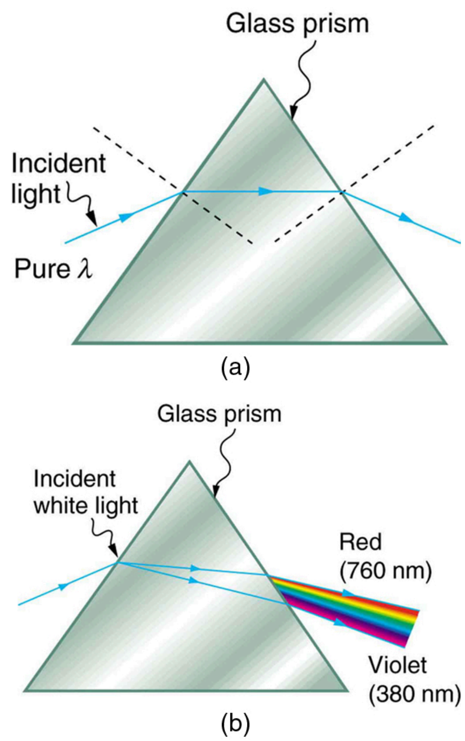


Figure 12.35 (a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (shown exaggerated). Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you look away from the sun. Light enters a drop of water and is reflected from the back of the drop, as shown in **Figure 12.36**. The light is refracted both as it enters and as it leaves the drop. Since the index of refraction of water varies with wavelength, the light is dispersed, and a rainbow is observed, as shown in **Figure 12.37** (a). (There is no dispersion caused by reflection at the back surface, since the law of reflection does not depend on wavelength.) The actual rainbow of colors seen by an observer depends on the myriad of rays being refracted and reflected toward the observer's eyes from numerous drops of water. The effect is most spectacular when the background is dark, as in stormy weather, but can also be observed in waterfalls and lawn sprinklers. The arc of a rainbow comes from the need to be looking at a specific angle relative to the direction of the sun, as illustrated in **Figure 12.37** (b). (If there are two reflections of light within the water drop, another "secondary" rainbow is produced. This rare event produces an arc that lies above the primary rainbow arc—see **Figure 12.37** (c).)

Rainbows

Rainbows are produced by a combination of refraction and reflection.

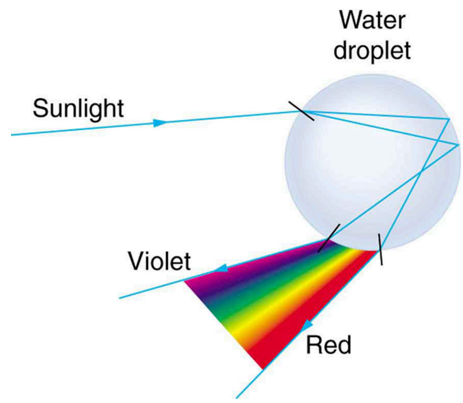


Figure 12.36 Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.

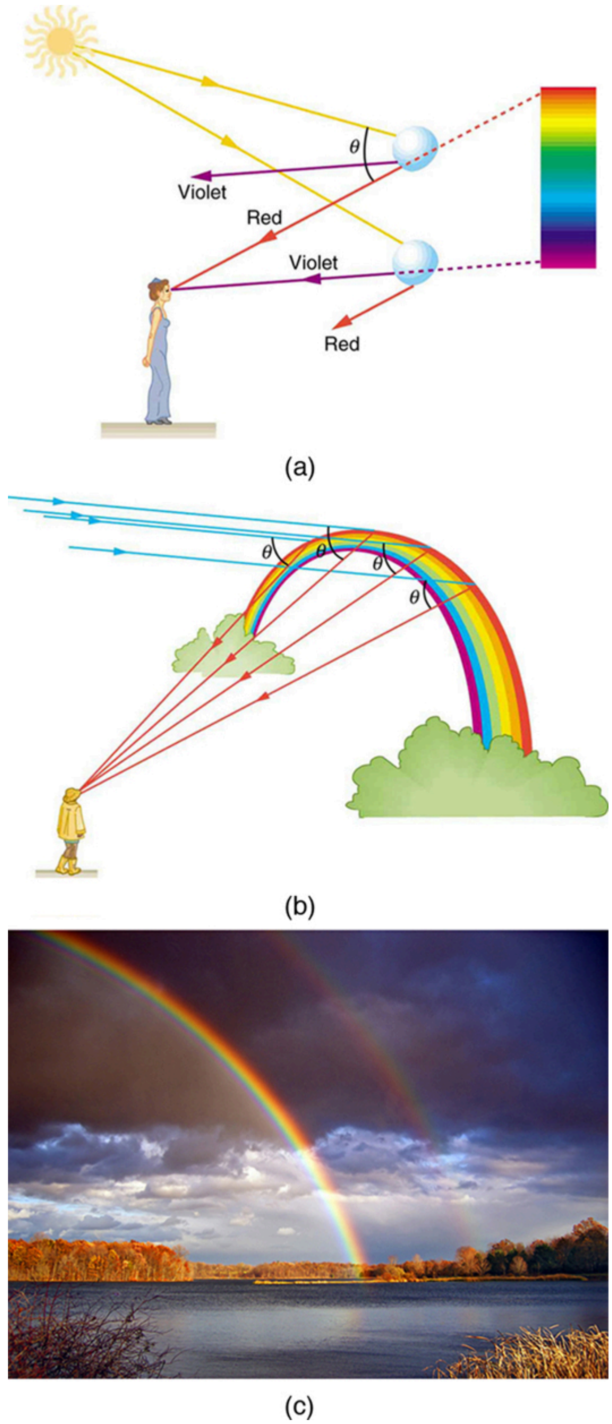


Figure 12.37 (a) Different colors emerge in different directions, and so you must look at different locations to see the various colors of a rainbow. (b) The arc of a rainbow results from the fact that a line between the observer and any point on the arc must make the correct angle with the parallel rays of sunlight to receive the refracted rays. (c) Double rainbow. (credit: Nicholas, Wikimedia Commons)

Dispersion may produce beautiful rainbows, but it can cause problems in optical systems. White light used to transmit messages in a fiber is dispersed, spreading out in time and eventually overlapping with other messages. Since a laser produces a nearly pure wavelength, its light experiences little dispersion, an advantage over white light for transmission of information. In contrast, dispersion of electromagnetic waves coming to us from outer space can be used to determine the amount of matter they pass through. As with many phenomena, dispersion can be useful or a nuisance, depending on the situation and our human goals.

Section Summary

- The spreading of white light into its full spectrum of wavelengths is called dispersion.
- Rainbows are produced by a combination of refraction and reflection and involve the dispersion of sunlight into a continuous distribution of colors.
- Dispersion produces beautiful rainbows but also causes problems in certain optical systems.

Problems & Exercises

Exercise 12.53

(a) What is the ratio of the speed of red light to violet light in diamond, based on **Table 12.3**? (b) What is this ratio in polystyrene? (c) Which is more dispersive?

12.8 Image Formation by Lenses

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we will use the law of refraction to explore the properties of lenses and how they form images.

The word *lens* derives from the Latin word for a lentil bean, the shape of which is similar to the convex lens in **Figure 12.38**. The convex lens shown has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. (The axis is defined to be a line normal to the lens at its center, as shown in **Figure 12.38**.) Such a lens is called a **converging (or convex) lens** for the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown, to illustrate how the ray changes direction both as it enters and as it leaves the lens. Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) Due to the lens's shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined to be the **focal point** F of the lens. The distance from the center of the lens to its focal point is defined to be the **focal length** f of the lens. **Figure 12.39** shows how a converging lens, such as that in a magnifying glass, can converge the nearly parallel light rays from the sun to a small spot.

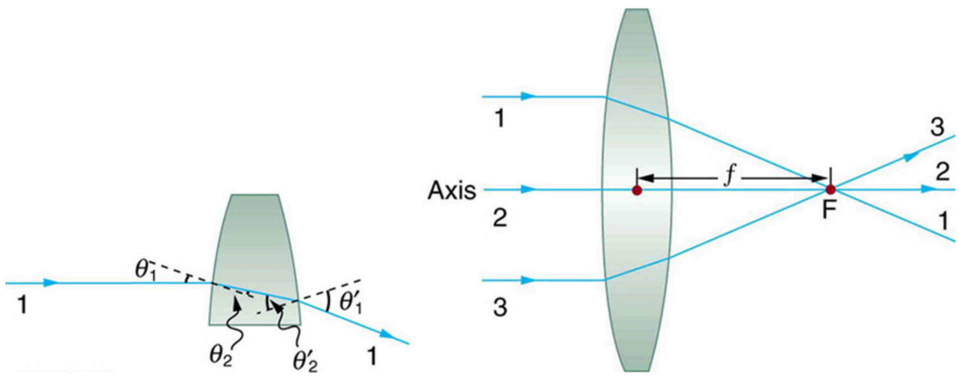


Figure 12.38 Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Converging or Convex Lens

The lens in which light rays that enter it parallel to its axis cross one another at a single point on the opposite side with a converging effect is called converging lens.

Focal Point F

The point at which the light rays cross is called the focal point F of the lens.

Focal Length f

The distance from the center of the lens to its focal point is called focal length f.

Figure 12.39 Sunlight focused by a converging magnifying glass can burn paper. Light rays from the sun are nearly parallel and cross at the focal point of the lens. The more powerful the lens, the closer to the lens the rays will cross.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The light will also focus into a smaller and more intense spot for a more powerful lens. The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

$$P = \frac{1}{f}. \tag{12.17}$$

Power P

The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

$$P = \frac{1}{f}. \tag{12.18}$$

where f is the focal length of the lens, which must be given in meters (and not cm or mm). The power of a lens P has the unit diopters (D), provided that the focal length is given in meters. That is, $1\text{ D} = 1/\text{m}$, or 1 m^{-1}. (Note that this power (optical power, actually) is not the same as power in watts. It is a concept related to the effect of optical devices on light.) Optometrists prescribe common spectacles and contact lenses in units of diopters.

Example 12.3 What is the Power of a Common Magnifying Glass?

Suppose you take a magnifying glass out on a sunny day and you find that it concentrates sunlight to a small spot 8.00 cm away from the lens. What are the focal length and power of the lens?

Strategy

The situation here is the same as those shown in **Figure 12.38** and **Figure 12.39**. The Sun is so far away that the Sun's rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus the focal length of the lens is the distance from the lens to the spot, and its power is the inverse of this distance (in m).

Solution

The focal length of the lens is the distance from the center of the lens to the spot, given to be 8.00 cm. Thus,

$$f = 8.00\text{ cm}. \tag{12.19}$$

To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power. This gives

$$P = \frac{1}{f} = \frac{1}{0.0800\text{ m}} = 12.5\text{ D}. \tag{12.20}$$

Discussion

This is a relatively powerful lens. The power of a lens in diopters should not be confused with the familiar concept of power in watts. It is an unfortunate fact that the word "power" is used for two completely different concepts. If you examine a prescription for eyeglasses, you will note lens powers given in diopters. If you examine the label on a motor, you will note energy consumption rate given as a power in watts.

Figure 12.40 shows a concave lens and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens**, because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so that all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of a diverging lens are defined to be negative. For example, if the distance to F in **Figure 12.40** is 5.00 cm, then the focal length is $f = -5.00\text{ cm}$ and the power of the lens is $P = -20\text{ D}$. An expanded view of the path of one ray through the lens is shown in the figure to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and be diverged.

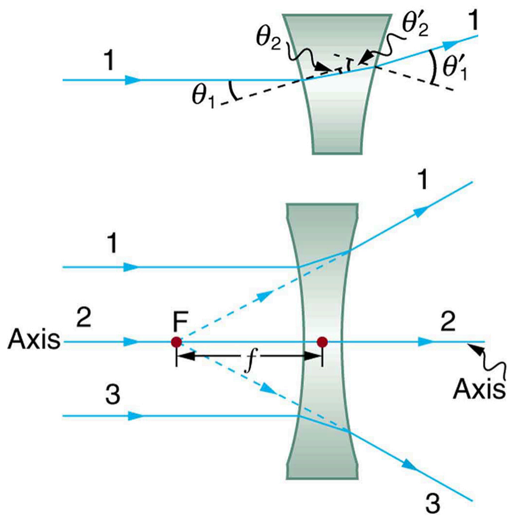


Figure 12.40 Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point F. The dashed lines are not rays—they indicate the directions from which the rays appear to come. The focal length f of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Diverging Lens

A lens that causes the light rays to bend away from its axis is called a diverging lens.

As noted in the initial discussion of the law of refraction, the paths of light rays are exactly reversible. This means that the direction of the arrows could be reversed for all of the rays in **Figure 12.38** and **Figure 12.40**. For example, if a point light source is placed at the focal point of a convex lens, as shown in **Figure 12.41**, parallel light rays emerge from the other side.

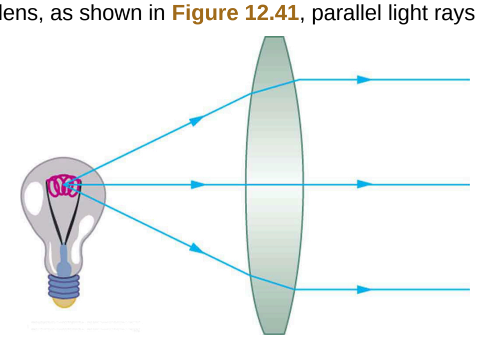


Figure 12.41 A small light source, like a light bulb filament, placed at the focal point of a convex lens, results in parallel rays of light emerging from the other side. The paths are exactly the reverse of those shown in **Figure 12.38**. This technique is used in lighthouses and sometimes in traffic lights to produce a directional beam of light from a source that emits light in all directions.

Ray Tracing and Thin Lenses

Ray tracing is the technique of determining or following (tracing) the paths that light rays take. For rays passing through matter, the law of refraction is used to trace the paths. Here we use ray tracing to help us understand the action of lenses in situations ranging from forming images on film to magnifying small print to correcting nearsightedness. While ray tracing for complicated lenses, such as those found in sophisticated cameras, may require computer techniques, there is a set of simple rules for tracing rays through thin lenses. A **thin lens** is defined to be one whose thickness allows rays to refract, as illustrated in **Figure 12.38**, but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See **Figure 12.42**.) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in **Figure 12.43**.

Thin Lens

A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.

Take-Home Experiment: A Visit to the Optician

Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they act like thin lenses.

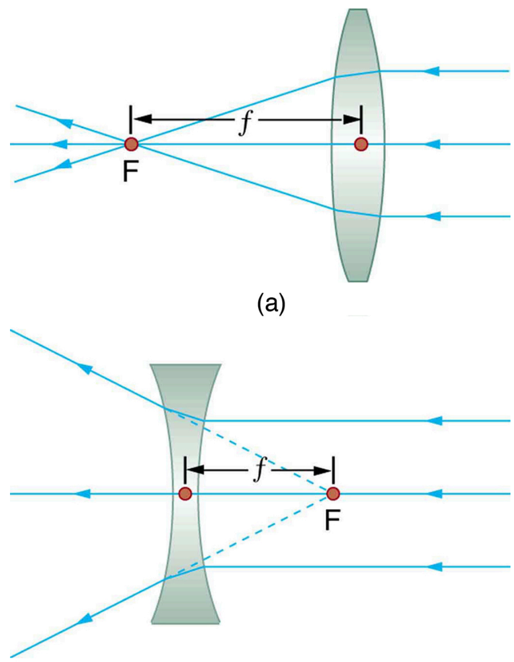


Figure 12.42 Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.

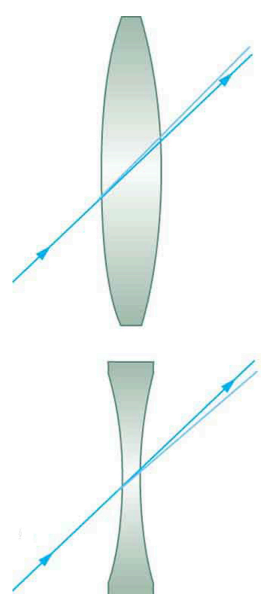


Figure 12.43 The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge parallel to its original path (shown as a shaded line).

Using paper, pencil, and a straight edge, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are based on the illustrations already discussed:

1. **Principal Ray 1**: a ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side. (See rays 1 and 3 in **Figure 12.38**.); a ray entering a diverging lens parallel to its axis seems to come from the focal point F. (See rays 1 and 3 in **Figure 12.40**.)
2. **Principal Ray 2**: a ray passing through the center of either a converging or a diverging lens does not change direction. (See **Figure 12.43**, and see ray 2 in **Figure 12.38** and **Figure 12.40**.)
3. **Principal Ray 3**: a ray entering a converging lens through its focal point exits parallel to its axis. (The reverse of rays 1 and 3 in **Figure 12.38**.); a ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis. (The reverse of rays 1 and 3 in **Figure 12.40**.) *The third principal ray is optional and may be used to verify the accuracy of image location.*

Image Formation by Thin Lenses

In some circumstances, a lens forms an obvious image, such as when a movie projector casts an image onto a screen. In other cases, the image is less obvious. Where, for example, is the image formed by eyeglasses? We will use ray tracing for thin lenses to illustrate how they form images, and we will develop equations to describe the image formation quantitatively.

Consider an object some distance away from a converging lens, as shown in **Figure 12.44**. To find the location and size of the image formed, we trace the paths of selected light rays originating from one point on the object, in this case the top of the person’s head. The figure shows three rays from the top of the object that can be traced using the ray tracing rules given above. (Rays leave this point going in many directions, but we concentrate on only a few with paths that are easy to trace.) The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). The three rays cross at the same point on the other side of the lens. The image of the top of the person’s head is located at this point. All rays that come from the same point on the top of the person’s head are refracted in such a way as to cross at the point shown. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in **Figure 12.44**, only two are necessary to locate the image. It is best to trace rays for which there are simple ray tracing rules. Before applying ray tracing to other situations, let us consider the example shown in **Figure 12.44** in more detail.

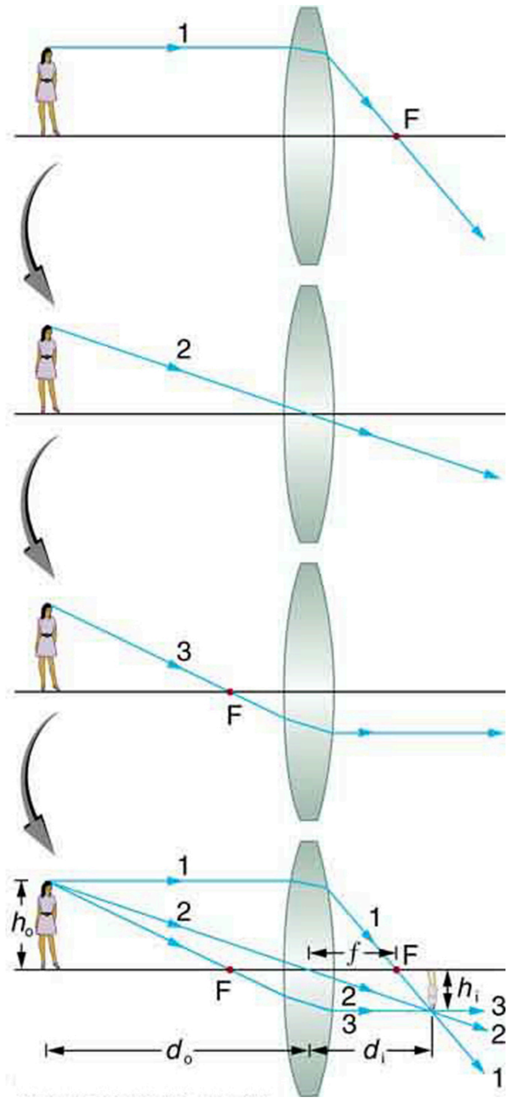
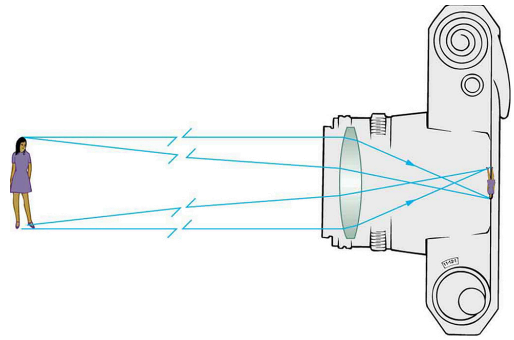


Figure 12.44 Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

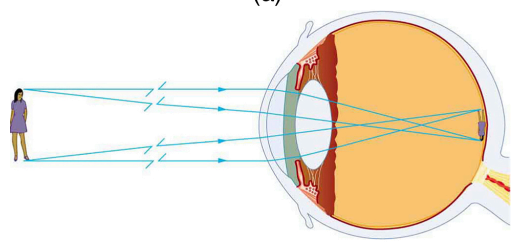
The image formed in **Figure 12.44** is a **real image**, meaning that it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye, for example. **Figure 12.45** shows how such an image would be projected onto film by a camera lens. This figure also shows how a real image is projected onto the retina by the lens of an eye. Note that the image is there whether it is projected onto a screen or not.

Real Image

The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.



(a)



(b)

Figure 12.45 Real images can be projected. (a) A real image of the person is projected onto film. (b) The converging nature of the multiple surfaces that make up the eye result in the projection of a real image on the retina.

Several important distances appear in **Figure 12.44**. We define d_o to be the object distance, the distance of an object from the center of a lens. **Image distance** d_i is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o and h_i, respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in **Figure 12.44**, we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of equations that can be derived from a geometric analysis of ray tracing for thin lenses. The **thin lens equations** are

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{12.21}$$

and

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m. \tag{12.22}$$

We define the ratio of image height to object height (h_i / h_o) to be the **magnification** m . (The minus sign in the equation above will be discussed shortly.) The thin lens equations are broadly applicable to all situations involving thin lenses (and "thin" mirrors, as we will see later). We will explore many features of image formation in the following worked examples.

Image Distance

The distance of the image from the center of the lens is called image distance.

Thin Lens Equations and Magnification

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{12.23}$$

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m \tag{12.24}$$

Example 12.4 Finding the Image of a Light Bulb Filament by Ray Tracing and by the Thin Lens Equations

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in **Figure 12.46**. Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.

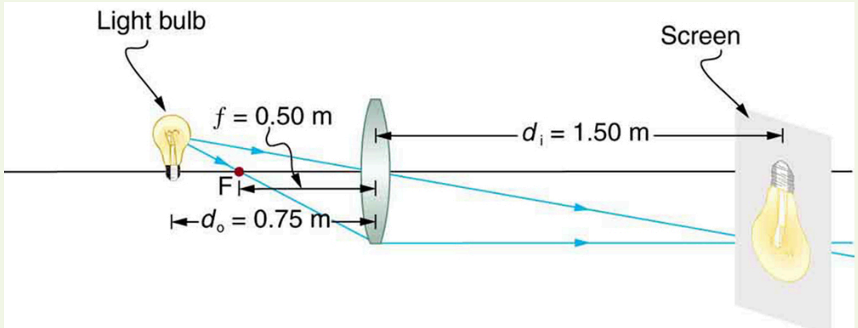


Figure 12.46 A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

Strategy and Concept

Since the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to those illustrated in **Figure 12.44** and **Figure 12.45**. Ray tracing to scale should produce similar results for d_i. Numerical solutions for d_i and m can be obtained using the thin lens equations, noting that $d_\text{o} = 0.750\text{ m and }f = 0.500\text{ m}$.

Solutions (Ray tracing)

The ray tracing to scale in **Figure 12.46** shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus the image distance d_i is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of 2, and the image is inverted. Thus m is about –2. The minus sign indicates that the image is inverted.

The thin lens equations can be used to find d_i from the given information:

$$\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f}. \tag{12.25}$$

Rearranging to isolate d_i gives

$$\frac{1}{d_\text{i}} = \frac{1}{f} - \frac{1}{d_\text{o}}. \tag{12.26}$$

Entering known quantities gives a value for $1 / d_\text{i}$:

$$\frac{1}{d_\text{i}} = \frac{1}{0.500\text{ m}} - \frac{1}{0.750\text{ m}} = \frac{0.667}{\text{m}}. \tag{12.27}$$

This must be inverted to find d_i:

$$d_\text{i} = \frac{\text{m}}{0.667} = 1.50\text{ m}. \tag{12.28}$$

Note that another way to find d_i is to rearrange the equation:

$$\frac{1}{d_\text{i}} = \frac{1}{f} - \frac{1}{d_\text{o}}. \tag{12.29}$$

This yields the equation for the image distance as:

$$d_\text{i} = \frac{fd_\text{o}}{d_\text{o} - f}. \tag{12.30}$$

Note that there is no inverting here.

The thin lens equations can be used to find the magnification m, since both d_i and d_o are known. Entering their values gives

$$m = -\frac{d_\text{i}}{d_\text{o}} = -\frac{1.50\text{ m}}{0.750\text{ m}} = -2.00. \tag{12.31}$$

Discussion

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

Real images, such as the one considered in the previous example, are formed by converging lenses whenever an object is farther from the lens than its focal length. This is true for movie projectors, cameras, and the eye. We shall refer to these as *case 1* images. A case 1 image is formed when $d_o > f$ and f is positive, as in **Figure 12.47**(a). (A summary of the three cases or types of image formation appears at the end of this section.)

A different type of image is formed when an object, such as a person's face, is held close to a convex lens. The image is upright and larger than the object, as seen in **Figure 12.47**(b), and so the lens is called a magnifier. If you slowly pull the magnifier away from the face, you will see that the magnification steadily increases until the image begins to blur. Pulling the magnifier even farther away produces an inverted image as seen in **Figure 12.47**(a). The distance at which the image blurs, and beyond which it inverts, is the focal length of the lens. To use a convex lens as a magnifier, the object must be closer to the converging lens than its focal length. This is called a *case 2* image. A case 2 image is formed when $d_o < f$ and f is positive.

(a)

(b)

Figure 12.47 (a) When a converging lens is held farther away from the face than the lens's focal length, an inverted image is formed. This is a case 1 image. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (credit: DaMongMan, Flickr) (b) A magnified image of a face is produced by placing it closer to the converging lens than its focal length. This is a case 2 image. (credit: Casey Fleser, Flickr)

Figure 12.48 uses ray tracing to show how an image is formed when an object is held closer to a converging lens than its focal length. Rays coming from a common point on the object continue to diverge after passing through the lens, but all appear to originate from a point at the location of the image. The image is on the same side of the lens as the object and is farther away from the lens than the object. This image, like all case 2 images, cannot be projected and, hence, is called a **virtual image**. Light rays only appear to originate at a virtual image; they do not actually pass through that location in space. A screen placed at the location of a virtual image will receive only diffuse light from the object, not focused rays from the lens. Additionally, a screen placed on the opposite side of the lens will receive rays that are still diverging, and so no image will be projected on it. We can see the magnified image with our eyes, because the lens of the eye converges the rays into a real image projected on our retina. Finally, we note that a virtual image is upright and larger than the object, meaning that the magnification is positive and greater than 1.

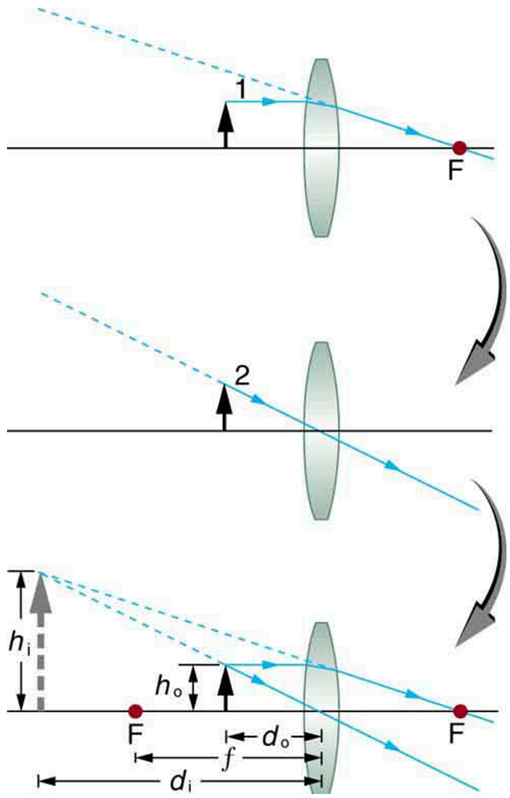


Figure 12.48 Ray tracing predicts the image location and size for an object held closer to a converging lens than its focal length. Ray 1 enters parallel to the axis and exits through the focal point on the opposite side, while ray 2 passes through the center of the lens without changing path. The two rays continue to diverge on the other side of the lens, but both appear to come from a common point, locating the upright, magnified, virtual image. This is a case 2 image.

Virtual Image

An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Example 12.5 Image Produced by a Magnifying Glass

Suppose the book page in **Figure 12.48** (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?

Strategy and Concept

We are given that $d_o = 7.50\text{ cm}$ and $f = 10.0\text{ cm}$, so we have a situation where the object is placed closer to the lens than its focal length. We therefore expect to get a case 2 virtual image with a positive magnification that is greater than 1. Ray tracing produces an image like that shown in **Figure 12.48**, but we will use the thin lens equations to get numerical solutions in this example.

Solution

To find the magnification m, we try to use magnification equation, $m = –d_i / d_o$. We do not have a value for d_i, so that we must first find the location of the image using lens equation. (The procedure is the same as followed in the preceding example, where d_o and f were known.) Rearranging the magnification equation to isolate d_i gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}. \tag{12.32}$$

Entering known values, we obtain a value for $1/d_i$:

$$\frac{1}{d_i} = \frac{1}{10.0\text{ cm}} - \frac{1}{7.50\text{ cm}} = \frac{-0.0333}{\text{cm}}. \tag{12.33}$$

This must be inverted to find d_i:

$$d_i = -\frac{\text{cm}}{0.0333} = -30.0\text{ cm}. \tag{12.34}$$

Now the thin lens equation can be used to find the magnification m, since both d_{i} and d_{o} are known. Entering their values gives

$$m = -\frac{d_{\text{i}}}{d_{\text{o}}} = -\frac{-30.0 \text{ cm}}{10.0 \text{ cm}} = 3.00. \tag{12.35}$$

Discussion

A number of results in this example are true of all case 2 images, as well as being consistent with **Figure 12.48**. Magnification is indeed positive (as predicted), meaning the image is upright. The magnification is also greater than 1, meaning that the image is larger than the object—in this case, by a factor of 3. Note that the image distance is negative. This means the image is on the same side of the lens as the object. Thus the image cannot be projected and is virtual. (Negative values of d_{i} occur for virtual images.) The image is farther from the lens than the object, since the image distance is greater in magnitude than the object distance. The location of the image is not obvious when you look through a magnifier. In fact, since the image is bigger than the object, you may think the image is closer than the object. But the image is farther away, a fact that is useful in correcting farsightedness, as we shall see in a later section.

A third type of image is formed by a diverging or concave lens. Try looking through eyeglasses meant to correct nearsightedness. (See **Figure 12.49**.) You will see an image that is upright but smaller than the object. This means that the magnification is positive but less than 1. The ray diagram in **Figure 12.50** shows that the image is on the same side of the lens as the object and, hence, cannot be projected—it is a virtual image. Note that the image is closer to the lens than the object. This is a *case 3* image, formed for any object by a negative focal length or diverging lens.

Figure 12.49 A car viewed through a concave or diverging lens looks upright. This is a case 3 image. (credit: Daniel Oines, Flickr)

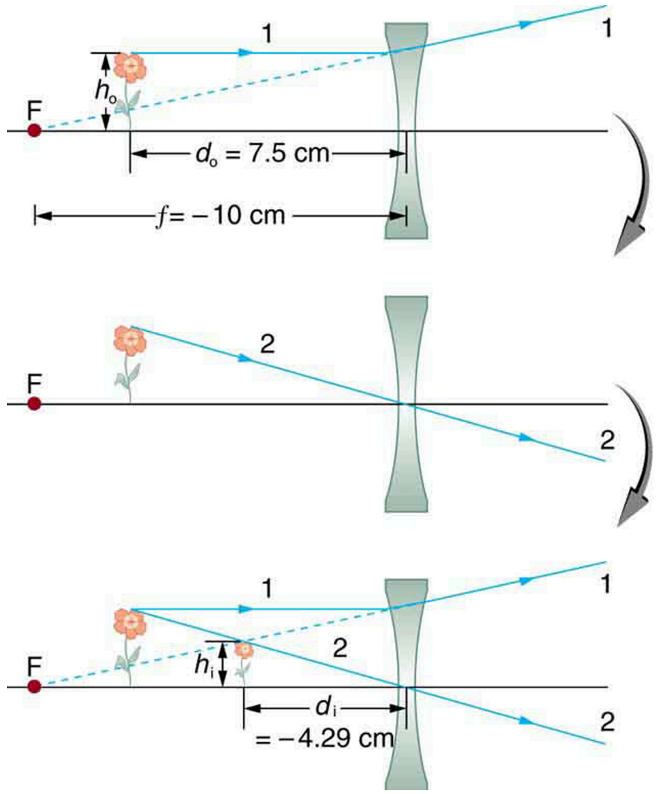


Figure 12.50 Ray tracing predicts the image location and size for a concave or diverging lens. Ray 1 enters parallel to the axis and is bent so that it appears to originate from the focal point. Ray 2 passes through the center of the lens without changing path. The two rays appear to come from a common point, locating the upright image. This is a case 3 image, which is closer to the lens than the object and smaller in height.

Example 12.6 Image Produced by a Concave Lens

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length –10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

Strategy and Concept

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is thus the same, but the results are different in important ways.

Solution

To find the magnification m, we must first find the image distance d_i using thin lens equation

$$\frac{1}{d_\text{i}} = \frac{1}{f} - \frac{1}{d_\text{o}}, \tag{12.36}$$

or its alternative rearrangement

$$d_i = \frac{fd_\text{o}}{d_\text{o} - f}. \tag{12.37}$$

We are given that $f = -10.0\text{ cm}$ and $d_\text{o} = 7.50\text{ cm}$. Entering these yields a value for $1/d_\text{i}$:

$$\frac{1}{d_\text{i}} = \frac{1}{-10.0\text{ cm}} - \frac{1}{7.50\text{ cm}} = \frac{-0.2333}{\text{cm}}. \tag{12.38}$$

This must be inverted to find d_i:

$$d_\text{i} = -\frac{\text{cm}}{0.2333} = -4.29\text{ cm}. \tag{12.39}$$

Or

$$d_\text{i} = \frac{(7.5)(-10)}{(7.5-(-10))} = -75/17.5 = -4.29\text{ cm}. \tag{12.40}$$

Now the magnification equation can be used to find the magnification m, since both d_{i} and d_{o} are known. Entering their values gives

$$m = -\frac{d_{\text{i}}}{d_{\text{o}}} = -\frac{-4.29 \text{ cm}}{7.50 \text{ cm}} = 0.571. \tag{12.41}$$

Discussion

A number of results in this example are true of all case 3 images, as well as being consistent with **Figure 12.50**. Magnification is positive (as predicted), meaning the image is upright. The magnification is also less than 1, meaning the image is smaller than the object—in this case, a little over half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. (The image is virtual.) The image is closer to the lens than the object, since the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, since the image is smaller than the object, you may think it is farther away. But the image is closer than the object, a fact that is useful in correcting nearsightedness, as we shall see in a later section.

Table 12.4 summarizes the three types of images formed by single thin lenses. These are referred to as case 1, 2, and 3 images. Convex (converging) lenses can form either real or virtual images (cases 1 and 2, respectively), whereas concave (diverging) lenses can form only virtual images (always case 3). Real images are always inverted, but they can be either larger or smaller than the object. For example, a slide projector forms an image larger than the slide, whereas a camera makes an image smaller than the object being photographed. Virtual images are always upright and cannot be projected. Virtual images are larger than the object only in case 2, where a convex lens is used. The virtual image produced by a concave lens is always smaller than the object—a case 3 image. We can see and photograph virtual images only by using an additional lens to form a real image.

Table 12.4 Three Types of Images Formed By Thin Lenses

Type	Formed when	Image type	d_{i}	m
Case 1	f positive, $d_o > f$	real	positive	negative
Case 2	f positive, $d_o < f$	virtual	negative	positive, $m > 1$
Case 3	f negative	virtual	negative	positive, $m < 1$

Take-Home Experiment: Concentrating Sunlight

Find several lenses and determine whether they are converging or diverging. In general those that are thicker near the edges are diverging and those that are thicker near the center are converging. On a bright sunny day take the converging lenses outside and try focusing the sunlight onto a piece of paper. Determine the focal lengths of the lenses. Be careful because the paper may start to burn, depending on the type of lens you have selected.

Section Summary

- Light rays entering a converging lens parallel to its axis cross one another at a single point on the opposite side.
- For a converging lens, the focal point is the point at which converging light rays cross; for a diverging lens, the focal point is the point from which diverging light rays appear to originate.
- The distance from the center of the lens to its focal point is called the focal length f.
- Power P of a lens is defined to be the inverse of its focal length, $P = \frac{1}{f}$.
- A lens that causes the light rays to bend away from its axis is called a diverging lens.
- Ray tracing is the technique of graphically determining the paths that light rays take.
- The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.
- Thin lens equations are $\frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i}}} = \frac{1}{f}$ and $\frac{h_{\text{i}}}{h_{\text{o}}} = -\frac{d_{\text{i}}}{d_{\text{o}}} = m$ (magnification).
- The distance of the image from the center of the lens is called image distance.
- An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Conceptual Questions

Exercise 12.54

It can be argued that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise 12.55

You can often see a reflection when looking at a sheet of glass, particularly if it is darker on the other side. Explain why you can often see a double image in such circumstances.

Exercise 12.56

When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?

Exercise 12.57

A thin lens has two focal points, one on either side, at equal distances from its center, and should behave the same for light entering from either side. Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they are thin lenses.

Exercise 12.58

Will the focal length of a lens change when it is submerged in water? Explain.

Problems & Exercises

Exercise 12.59

What is the power in diopters of a camera lens that has a 50.0 mm focal length?

Exercise 12.60

Your camera's zoom lens has an adjustable focal length ranging from 80.0 to 200 mm. What is its range of powers?

Solution

$5.00 \text{ to } 12.5 \text{ D}$

Exercise 12.61

What is the focal length of 1.75 D reading glasses found on the rack in a pharmacy?

Exercise 12.62

You note that your prescription for new eyeglasses is –4.50 D. What will their focal length be?

Solution

-0.222 m

Exercise 12.63

How far from the lens must the film in a camera be, if the lens has a 35.0 mm focal length and is being used to photograph a flower 75.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

Exercise 12.64

A certain slide projector has a 100 mm focal length lens. (a) How far away is the screen, if a slide is placed 103 mm from the lens and produces a sharp image? (b) If the slide is 24.0 by 36.0 mm, what are the dimensions of the image? Explicitly show

how you follow the steps in the Problem-Solving Strategy for lenses.

Solution

(a) 3.43 m

(b) 0.800 by 1.20 m

Exercise 12.65

A doctor examines a mole with a 15.0 cm focal length magnifying glass held 13.5 cm from the mole (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 5.00 mm diameter mole?

Solution

(a) -1.35 m (on the object side of the lens).

(b) $+10.0$

(c) 5.00 cm

Exercise 12.66

How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?

Solution

44.4 cm

Exercise 12.67

A camera with a 50.0 mm focal length lens is being used to photograph a person standing 3.00 m away. (a) How far from the lens must the film be? (b) If the film is 36.0 mm high, what fraction of a 1.75 m tall person will fit on it? (c) Discuss how reasonable this seems, based on your experience in taking or posing for photographs.

Exercise 12.68

A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm. (a) What is the closest object that can be photographed? (b) What is the magnification of this closest object?

Solution

(a) 6.60 cm

(b) –0.333

Exercise 12.69

Suppose your 50.0 mm focal length camera lens is 51.0 mm away from the film in the camera. (a) How far away is an object that is in focus? (b) What is the height of the object if its image is 2.00 cm high?

Exercise 12.70

(a) What is the focal length of a magnifying glass that produces a magnification of 3.00 when held 5.00 cm from an object, such as a rare coin? (b) Calculate the power of the magnifier in diopters. (c) Discuss how this power compares to those for store-bought reading glasses (typically 1.0 to 4.0 D). Is the magnifier's power greater, and should it be?

Solution

(a) $+7.50\text{ cm}$

(b) 13.3 D

(c) Much greater

Exercise 12.71

What magnification will be produced by a lens of power –4.00 D (such as might be used to correct myopia) if an object is held 25.0 cm away?

Exercise 12.72

In Example 12.5, the magnification of a book held 7.50 cm from a 10.0 cm focal length lens was found to be 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Do the same for when it is held 9.50 cm from the magnifier. (c) Comment on the trend in m as the object distance increases as in these two calculations.

Solution

(a) +6.67

(b) +20.0

(c) The magnification increases without limit (to infinity) as the object distance increases to the limit of the focal distance.

Exercise 12.73

Suppose a 200 mm focal length telephoto lens is being used to photograph mountains 10.0 km away. (a) Where is the image? (b) What is the height of the image of a 1000 m high cliff on one of the mountains?

Exercise 12.74

A camera with a 100 mm focal length lens is used to photograph the sun and moon. What is the height of the image of the sun on the film, given the sun is $1.40\times10^{6}\text{ km}$ in diameter and is $1.50\times10^{8}\text{ km}$ away?

Solution

-0.933 mm

Exercise 12.75

Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f - d_\text{o})$.

12.9 Image Formation by Mirrors

We only have to look as far as the nearest bathroom to find an example of an image formed by a mirror. Images in flat mirrors are the same size as the object and are located behind the mirror. Like lenses, mirrors can form a variety of images. For example, dental mirrors may produce a magnified image, just as makeup mirrors do. Security mirrors in shops, on the other hand, form images that are smaller than the object. We will use the law of reflection to understand how mirrors form images, and we will find that mirror images are analogous to those formed by lenses.

Figure 12.51 helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and being reflected into the observer’s eye. The rays can diverge slightly, and both still get into the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, locating the image. (The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror.) Using the law of reflection—the angle of reflection equals the angle of incidence—we can see that the image and object are the same distance from the mirror. This is a virtual image, since it cannot be projected—the rays only appear to originate from a common point behind the mirror. Obviously, if you walk behind the mirror, you cannot see the image, since the rays do not go there. But in front of the mirror, the rays behave exactly as if they had come from behind the mirror, so that is where the image is situated.

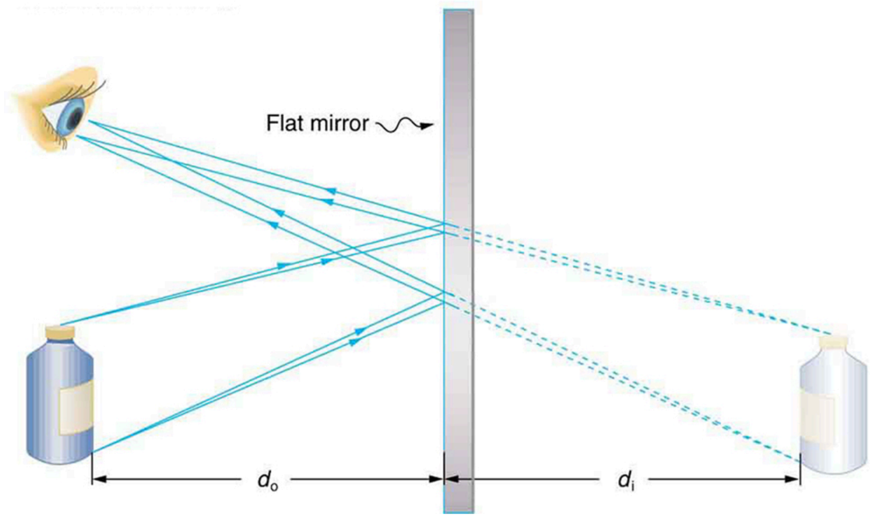


Figure 12.51 Two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer. The reflected rays seem to originate from behind the mirror, locating the virtual image.

Now let us consider the focal length of a mirror—for example, the concave spherical mirrors in **Figure 12.52**. Rays of light that strike the surface follow the law of reflection. For a mirror that is large compared with its radius of curvature, as in **Figure 12.52**(a), we see that the reflected rays do not cross at the same point, and the mirror does not have a well-defined focal point. If the mirror had the shape of a parabola, the rays would all cross at a single point, and the mirror would have a well-defined focal point. But parabolic mirrors are much more expensive to make than spherical mirrors. The solution is to use a mirror that is small compared with its radius of curvature, as shown in **Figure 12.52**(b). (This is the mirror equivalent of the thin lens approximation.) To a very good approximation, this mirror has a well-defined focal point at F that is the focal distance f from the center of the mirror. The focal length f of a concave mirror is positive, since it is a converging mirror.

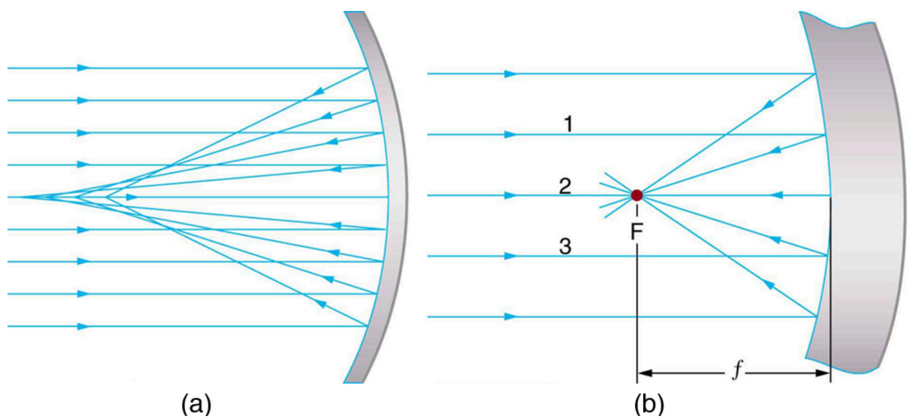


Figure 12.52 (a) Parallel rays reflected from a large spherical mirror do not all cross at a common point. (b) If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point. The distance of the focal point from the center of the mirror is its focal length f. Since this mirror is converging, it has a positive focal length.

Just as for lenses, the shorter the focal length, the more powerful the mirror; thus, $P = 1/f$ for a mirror, too. A more strongly curved mirror has a shorter focal length and a greater power. Using the law of reflection and some simple trigonometry, it can be shown that the focal length is half the radius of curvature, or

$$f = \frac{R}{2}, \tag{12.42}$$

where R is the radius of curvature of a spherical mirror. The smaller the radius of curvature, the smaller the focal length and, thus, the more powerful the mirror.

The convex mirror shown in **Figure 12.53** also has a focal point. Parallel rays of light reflected from the mirror seem to originate from the point F at the focal distance f behind the mirror. The focal length and power of a convex mirror are negative, since it is a diverging mirror.

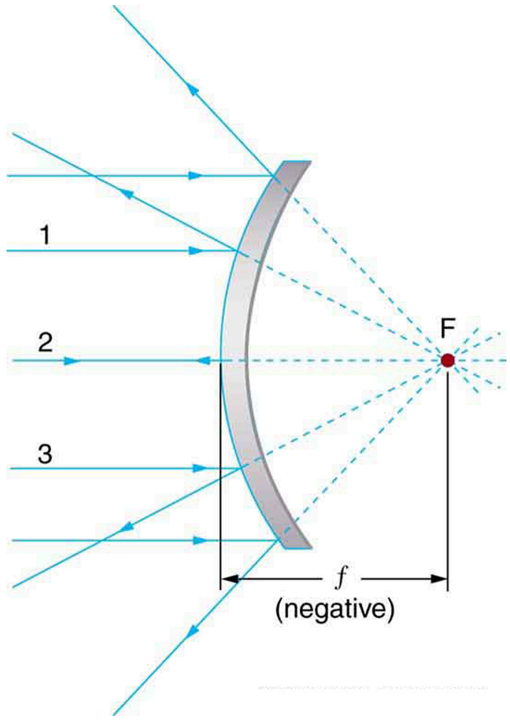


Figure 12.53 Parallel rays of light reflected from a convex spherical mirror (small in size compared with its radius of curvature) seem to originate from a well-defined focal point at the focal distance f behind the mirror. Convex mirrors diverge light rays and, thus, have a negative focal length.

Ray tracing is as useful for mirrors as for lenses. The rules for ray tracing for mirrors are based on the illustrations just discussed:

1. **Principal Ray 1**: a ray approaching a concave converging mirror parallel to its axis is reflected through the focal point F of the mirror on the same side. (See rays 1 and 3 in **Figure 12.52**(b).); a ray approaching a convex diverging mirror parallel to its axis is reflected so that it seems to come from the focal point F behind the mirror. (See rays 1 and 3 in **Figure 12.53**.)
2. **Principal Ray 2**: any ray striking the center of a mirror is followed by applying the law of reflection; it makes the same angle with the axis when leaving as when approaching. (See ray 2 in **Figure 12.54**.)
3. **Principal Ray 3**: a ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (The reverse of rays 1 and 3 in **Figure 12.52**.); a ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis. (The reverse of rays 1 and 3 in **Figure 12.53**.) *The third principal ray is optional and may be used to verify the accuracy of image location.*

We will use ray tracing to illustrate how images are formed by mirrors, and we can use ray tracing quantitatively to obtain numerical information. But since we assume each mirror is small compared with its radius of curvature, we can use the thin lens equations for mirrors just as we did for lenses.

Consider the situation shown in **Figure 12.54**, concave spherical mirror reflection, in which an object is placed farther from a concave (converging) mirror than its focal length. That is, f is positive and $d_{\text{o}} > f$, so that we may expect an image similar to the case 1 real image formed by a converging lens. Ray tracing in **Figure 12.54** shows that the rays from a common point on the object all cross at a point on the same side of the mirror as the object. Thus a real image can be projected onto a screen placed at this location. The image distance is positive, and the image is inverted, so its magnification is negative. This is a *case 1 image for mirrors*. It differs from the case 1 image for lenses only in that the image is on the same side of the mirror as the object. It is otherwise identical.

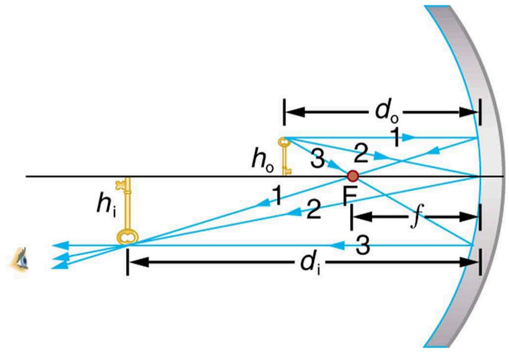


Figure 12.54 A case 1 image for a mirror. An object is farther from the converging mirror than its focal length. Rays from a common point on the object are traced using the rules in the text. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 goes through the focal point on the way toward the mirror. All three rays cross at the same point after being reflected, locating the inverted real image. Although three rays are shown, only two of the three are needed to locate the image and determine its height.

Example 12.7 A Concave Reflector

Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m away from the mirror, where are the coils?

Strategy and Concept

We are given that the concave mirror projects a real image of the coils at an image distance $d_{\text{i}} = 3.00\text{ m}$. The coils are the object, and we are asked to find their location—that is, to find the object distance d_{o}. We are also given the radius of curvature of the mirror, so that its focal length is $f = R/\ 2 = 25.0\text{ cm}$ (positive since the mirror is concave or converging). Assuming the mirror is small compared with its radius of curvature, we can use the thin lens equations, to solve this problem.

Solution

Since d_{i} and f are known, thin lens equation can be used to find d_{o}:

$$\frac{1}{d_{\text{o}}} + \frac{1}{d_{\text{i}}} = \frac{1}{f}. \tag{12.43}$$

Rearranging to isolate d_{o} gives

$$\frac{1}{d_{\text{o}}} = \frac{1}{f} - \frac{1}{d_{\text{i}}}. \tag{12.44}$$

Entering known quantities gives a value for $1/d_{\text{o}}$:

$$\frac{1}{d_{\text{o}}} = \frac{1}{0.250\text{ m}} - \frac{1}{3.00\text{ m}} = \frac{3.667}{\text{m}}. \tag{12.45}$$

This must be inverted to find d_{o}:

$$d_{\text{o}} = \frac{1\text{ m}}{3.667} = 27.3\text{ cm}. \tag{12.46}$$

Discussion

Note that the object (the filament) is farther from the mirror than the mirror's focal length. This is a case 1 image ($d_{\text{o}} > f$ and f positive), consistent with the fact that a real image is formed. You will get the most concentrated thermal energy directly in front of the mirror and 3.00 m away from it. Generally, this is not desirable, since it could cause burns. Usually, you want the rays to emerge parallel, and this is accomplished by having the filament at the focal point of the mirror.

Note that the filament here is not much farther from the mirror than its focal length and that the image produced is considerably farther away. This is exactly analogous to a slide projector. Placing a slide only slightly farther away from the projector lens than its focal length produces an image significantly farther away. As the object gets closer to the focal distance, the image gets farther away. In fact, as the object distance approaches the focal length, the image distance approaches infinity and the rays are sent out parallel to one another.

Example 12.8 Solar Electric Generating System

One of the solar technologies used today for generating electricity is a device (called a parabolic trough or concentrating collector) that concentrates the sunlight onto a blackened pipe that contains a fluid. This heated fluid is pumped to a heat exchanger, where its heat energy is transferred to another system that is used to generate steam—and so generate electricity through a conventional steam cycle. Figure 12.55 shows such a working system in southern California. Concave mirrors are used to concentrate the sunlight onto the pipe. The mirror has the approximate shape of a section of a cylinder. For the problem, assume that the mirror is exactly one-quarter of a full cylinder.

a. If we wish to place the fluid-carrying pipe 40.0 cm from the concave mirror at the mirror's focal point, what will be the radius of curvature of the mirror?
b. Per meter of pipe, what will be the amount of sunlight concentrated onto the pipe, assuming the insolation (incident solar radiation) is $0.900\ \text{kW/m}^2$?
c. If the fluid-carrying pipe has a 2.00-cm diameter, what will be the temperature increase of the fluid per meter of pipe over a period of one minute? Assume all the solar radiation incident on the reflector is absorbed by the pipe, and that the fluid is mineral oil.

Strategy

To solve an *Integrated Concept Problem* we must first identify the physical principles involved. Part (a) is related to the current topic. Part (b) involves a little math, primarily geometry. Part (c) requires an understanding of heat and density.

Solution to (a)

To a good approximation for a concave or semi-spherical surface, the point where the parallel rays from the sun converge will be at the focal point, so $R = 2f = 80.0\ \text{cm}$.

Solution to (b)

The insolation is $900\ \text{W/m}^2$. We must find the cross-sectional area A of the concave mirror, since the power delivered is $900\ \text{W/m}^2 \times \text{A}$. The mirror in this case is a quarter-section of a cylinder, so the area for a length L of the mirror is $\text{A} = \frac{1}{4}(2\pi R)\text{L}$. The area for a length of 1.00 m is then

$$\text{A} = \frac{\pi}{2}R(1.00\ \text{m}) = \frac{(3.14)}{2}(0.800\ \text{m})(1.00\ \text{m}) = 1.26\ \text{m}^2. \tag{12.47}$$

The insolation on the 1.00-m length of pipe is then

$$\left(9.00\times10^2\ \frac{\text{W}}{\text{m}^2}\right)\left(1.26\ \text{m}^2\right) = 1130\ \text{W}. \tag{12.48}$$

Solution to (c)

The increase in temperature is given by $Q = mc\Delta T$. The mass m of the mineral oil in the one-meter section of pipe is

$$\begin{aligned} m &= \rho \text{V} = \rho\pi\left(\frac{d}{2}\right)^2(1.00\ \text{m}) \\ &= \left(8.00\times10^2\ \text{kg/m}^3\right)(3.14)(0.0100\ \text{m})^2(1.00\ \text{m}) \\ &= 0.251\ \text{kg}. \end{aligned} \tag{12.49}$$

Therefore, the increase in temperature in one minute is

$$\begin{aligned} \Delta T &= Q/mc \\ &= \frac{(1130\ \text{W})(60.0\ \text{s})}{(0.251\ \text{kg})(1670\ \text{J}\cdot\text{kg/ºC})} \\ &= 162\text{ºC}. \end{aligned} \tag{12.50}$$

Discussion for (c)

An array of such pipes in the California desert can provide a thermal output of 250 MW on a sunny day, with fluids reaching temperatures as high as 400ºC. We are considering only one meter of pipe here, and ignoring heat losses along the pipe.

Figure 12.55 Parabolic trough collectors are used to generate electricity in southern California. (credit: kjkolb, Wikimedia Commons)

What happens if an object is closer to a concave mirror than its focal length? This is analogous to a case 2 image for lenses ($d_o < f$ and f positive), which is a magnifier. In fact, this is how makeup mirrors act as magnifiers. **Figure 12.56**(a) uses ray tracing to locate the image of an object placed close to a concave mirror. Rays from a common point on the object are reflected in such a manner that they appear to be coming from behind the mirror, meaning that the image is virtual and cannot be projected. As with a magnifying glass, the image is upright and larger than the object. This is a *case 2 image for mirrors* and is exactly analogous to that for lenses.

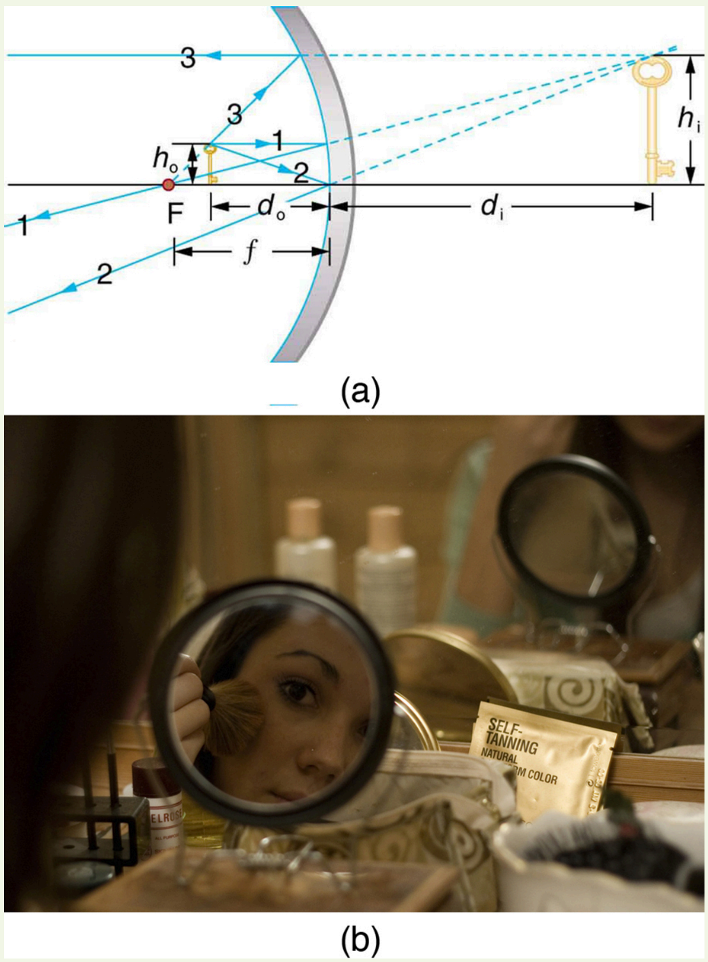


Figure 12.56 (a) Case 2 images for mirrors are formed when a converging mirror has an object closer to it than its focal length. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches the mirror as if it came from the focal point. (b) A magnifying mirror showing the reflection. (credit: Mike Melrose, Flickr)

All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be larger than the object. (b) Makeup mirrors are perhaps the most common use of a concave mirror to produce a larger, upright image.

A convex mirror is a diverging mirror (f is negative) and forms only one type of image. It is a *case 3* image—one that is upright and smaller than the object, just as for diverging lenses. **Figure 12.57**(a) uses ray tracing to illustrate the location and size of the case 3 image for mirrors. Since the image is behind the mirror, it cannot be projected and is thus a virtual image. It is also seen to be smaller than the object.

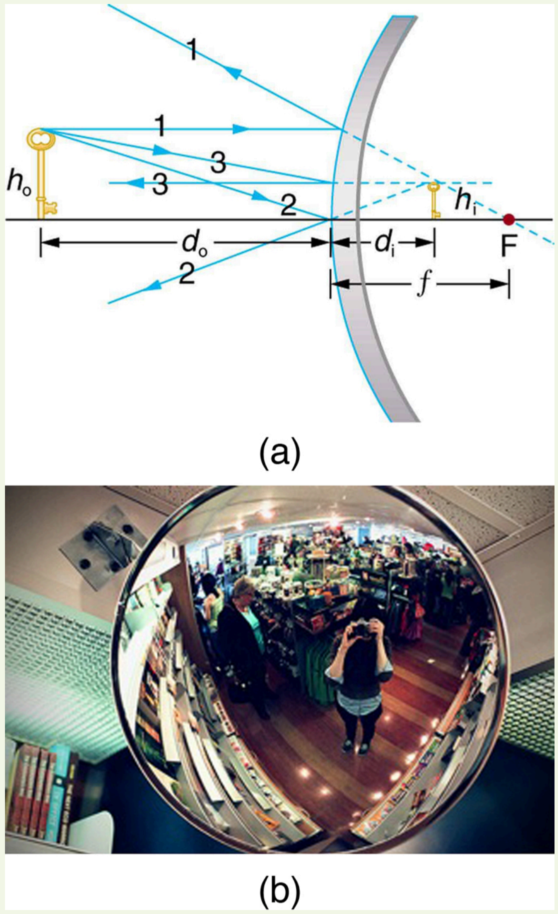


Figure 12.57 Case 3 images for mirrors are formed by any convex mirror. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches toward the focal point. All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be smaller than the object. (b) Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared to what would be observed for a flat mirror (and hence security is improved). (credit: Laura D'Alessandro, Flickr)

Example 12.9 Image in a Convex Mirror

A keratometer is a device used to measure the curvature of the cornea, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea. If the light source is 12.0 cm from the cornea and the image's magnification is 0.0320, what is the cornea's radius of curvature?

Strategy

If we can find the focal length of the convex mirror formed by the cornea, we can find its radius of curvature (the radius of curvature is twice the focal length of a spherical mirror). We are given that the object distance is $d_\text{o} = 12.0 \text{ cm}$ and that $m = 0.0320$. We first solve for the image distance d_i, and then for f.

Solution

$m = –d_\text{i} / d_\text{o}$. Solving this expression for d_i gives

$$d_\text{i} = -md_\text{o}. \tag{12.51}$$

Entering known values yields

$$d_\text{i} = -(0.0320)(12.0 \text{ cm}) = –0.384 \text{ cm}. \tag{12.52}$$

$$\frac{1}{f} = \frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} \tag{12.53}$$

Substituting known values,

$$\frac{1}{f} = \frac{1}{12.0\text{ cm}} + \frac{1}{-0.384\text{ cm}} = \frac{-2.52}{\text{cm}}. \tag{12.54}$$

This must be inverted to find f:

$$f = \frac{\text{cm}}{-2.52} = -0.400\text{ cm}. \tag{12.55}$$

The radius of curvature is twice the focal length, so that

$$R = 2\mid f \mid = 0.800\text{ cm}. \tag{12.56}$$

Discussion

Although the focal length f of a convex mirror is defined to be negative, we take the absolute value to give us a positive value for R. The radius of curvature found here is reasonable for a cornea. The distance from cornea to retina in an adult eye is about 2.0 cm. In practice, many corneas are not spherical, complicating the job of fitting contact lenses. Note that the image distance here is negative, consistent with the fact that the image is behind the mirror, where it cannot be projected. In this section's Problems and Exercises, you will show that for a fixed object distance, the smaller the radius of curvature, the smaller the magnification.

The three types of images formed by mirrors (cases 1, 2, and 3) are exactly analogous to those formed by lenses, as summarized in the table at the end of "Image Formation by Lenses". It is easiest to concentrate on only three types of images—then remember that concave mirrors act like convex lenses, whereas convex mirrors act like concave lenses.

Take-Home Experiment: Concave Mirrors Close to Home

Find a flashlight and identify the curved mirror used in it. Find another flashlight and shine the first flashlight onto the second one, which is turned off. Estimate the focal length of the mirror. You might try shining a flashlight on the curved mirror behind the headlight of a car, keeping the headlight switched off, and determine its focal length.

Section Summary

- The characteristics of an image formed by a flat mirror are: (a) The image and object are the same distance from the mirror, (b) The image is a virtual image, and (c) The image is situated behind the mirror.
- Image length is half the radius of curvature.

$$f = \frac{R}{2} \tag{12.57}$$

- A convex mirror is a diverging mirror and forms only one type of image, namely a virtual image.

Conceptual Questions

Exercise 12.76

What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?

Exercise 12.77

Can you see a virtual image? Can you photograph one? Can one be projected onto a screen with additional lenses or mirrors? Explain your responses.

Exercise 12.78

Is it necessary to project a real image onto a screen for it to exist?

Exercise 12.79

At what distance is an image *always* located—at d_o, d_i, or f?

Exercise 12.80

Under what circumstances will an image be located at the focal point of a lens or mirror?

Exercise 12.81

What is meant by a negative magnification? What is meant by a magnification that is less than 1 in magnitude?

Exercise 12.82

Can a case 1 image be larger than the object even though its magnification is always negative? Explain.

Exercise 12.83

Figure 12.58 shows a light bulb between two mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam. Where is the filament of the light in relation to the focal point or radius of curvature of each mirror?

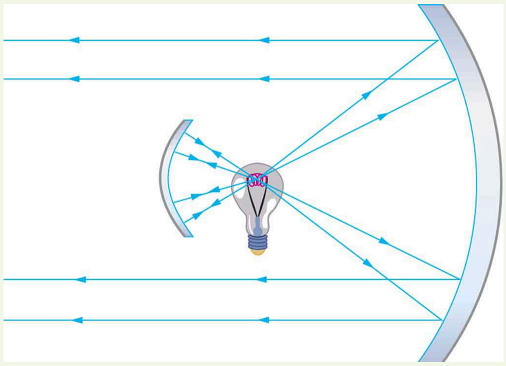


Figure 12.58 The two mirrors trap most of the bulb's light and form a directional beam as in a headlight.

Exercise 12.84

Devise an arrangement of mirrors allowing you to see the back of your head. What is the minimum number of mirrors needed for this task?

Exercise 12.85

If you wish to see your entire body in a flat mirror (from head to toe), how tall should the mirror be? Does its size depend upon your distance away from the mirror? Provide a sketch.

Exercise 12.86

It can be argued that a flat mirror has an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise 12.87

Why are diverging mirrors often used for rear-view mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?

Problems & Exercises

Exercise 12.88

What is the focal length of a makeup mirror that has a power of 1.50 D?

Solution
+0.667 m

Exercise 12.89

Some telephoto cameras use a mirror rather than a lens. What radius of curvature mirror is needed to replace a 800 mm focal length telephoto lens?

Exercise 12.90

(a) Calculate the focal length of the mirror formed by the shiny back of a spoon that has a 3.00 cm radius of curvature. (b) What is its power in diopters?

Solution

(a) $-1.5\times10^{-2}\text{ m}$

(b) -66.7 D

Exercise 12.91

Find the magnification of the heater element in **Example 12.7**. Note that its large magnitude helps spread out the reflected energy.

Exercise 12.92

What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away?

Solution
+0.360 m (concave)

Exercise 12.93

A shopper standing 3.00 m from a convex security mirror sees his image with a magnification of 0.250. (a) Where is his image? (b) What is the focal length of the mirror? (c) What is its radius of curvature?

Exercise 12.94

An object 1.50 cm high is held 3.00 cm from a person's cornea, and its reflected image is measured to be 0.167 cm high. (a) What is the magnification? (b) Where is the image? (c) Find the radius of curvature of the convex mirror formed by the cornea. (Note that this technique is used by optometrists to measure the curvature of the cornea for contact lens fitting. The instrument used is called a keratometer, or curve measurer.)

Solution

(a) +0.111

(b) -0.334 cm (behind "mirror")

(c) 0.752cm

Exercise 12.95

Ray tracing for a flat mirror shows that the image is located a distance behind the mirror equal to the distance of the object from the mirror. This is stated $d_{\text{i}} = -d_{\text{o}}$, since this is a negative image distance (it is a virtual image). (a) What is the focal length of a flat mirror? (b) What is its power?

Exercise 12.96

Show that for a flat mirror $h_{\text{i}} = h_{\text{o}}$, knowing that the image is a distance behind the mirror equal in magnitude to the distance of the object from the mirror.

Solution

$$m = \frac{h_{\text{i}}}{h_{\text{o}}} = -\frac{d_{\text{i}}}{d_{\text{o}}} = -\frac{-d_{\text{o}}}{d_{\text{o}}} = \frac{d_{\text{o}}}{d_{\text{o}}} = 1 \Rightarrow h_{\text{i}} = h_{\text{o}} \tag{12.58}$$

Exercise 12.97

Use the law of reflection to prove that the focal length of a mirror is half its radius of curvature. That is, prove that $f = R/2$. Note this is true for a spherical mirror only if its diameter is small compared with its radius of curvature.

Exercise 12.98

Referring to the electric room heater considered in the first example in this section, calculate the intensity of IR radiation in W/m^2 projected by the concave mirror on a person 3.00 m away. Assume that the heating element radiates 1500 W and has an area of $100\ \text{cm}^2$, and that half of the radiated power is reflected and focused by the mirror.

Solution

$6.82\ \text{kW/m}^2$

Exercise 12.99

Consider a 250-W heat lamp fixed to the ceiling in a bathroom. If the filament in one light burns out then the remaining three still work. Construct a problem in which you determine the resistance of each filament in order to obtain a certain intensity projected on the bathroom floor. The ceiling is 3.0 m high. The problem will need to involve concave mirrors behind the filaments. Your instructor may wish to guide you on the level of complexity to consider in the electrical components.

12.10 Polarization

Polaroid sunglasses are familiar to most of us. They have a special ability to cut the glare of light reflected from water or glass (see **Figure 12.59**). Polaroids have this ability because of a wave characteristic of light called polarization. What is polarization? How is it produced? What are some of its uses? The answers to these questions are related to the wave character of light.

(a) (b)

Figure 12.59 These two photographs of a river show the effect of a polarizing filter in reducing glare in light reflected from the surface of water. Part (b) of this figure was taken with a polarizing filter and part (a) was not. As a result, the reflection of clouds and sky observed in part (a) is not observed in part (b). Polarizing sunglasses are particularly useful on snow and water. (credit: Amithshs, Wikimedia Commons)

Light is one type of electromagnetic (EM) wave. As noted earlier, EM waves are *transverse waves* consisting of varying electric and magnetic fields that oscillate perpendicular to the direction of propagation (see **Figure 12.60**). There are specific directions for the oscillations of the electric and magnetic fields. **Polarization** is the attribute that a wave's oscillations have a definite direction relative to the direction of propagation of the wave. (This is not the same type of polarization as that discussed for the separation of charges.) Waves having such a direction are said to be **polarized**. For an EM wave, we define the **direction of polarization** to be the direction parallel to the electric field. Thus we can think of the electric field arrows as showing the direction of polarization, as in **Figure 12.60**.

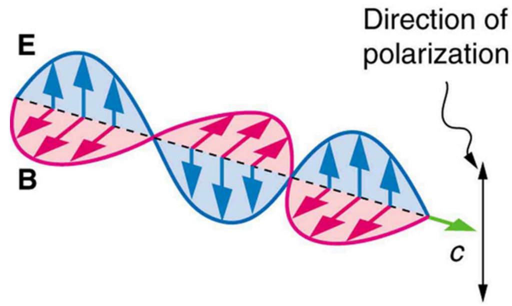


Figure 12.60 An EM wave, such as light, is a transverse wave. The electric and magnetic fields are perpendicular to the direction of propagation.

To examine this further, consider the transverse waves in the ropes shown in Figure 12.61. The oscillations in one rope are in a vertical plane and are said to be **vertically polarized**. Those in the other rope are in a horizontal plane and are **horizontally polarized**. If a vertical slit is placed on the first rope, the waves pass through. However, a vertical slit blocks the horizontally polarized waves. For EM waves, the direction of the electric field is analogous to the disturbances on the ropes.

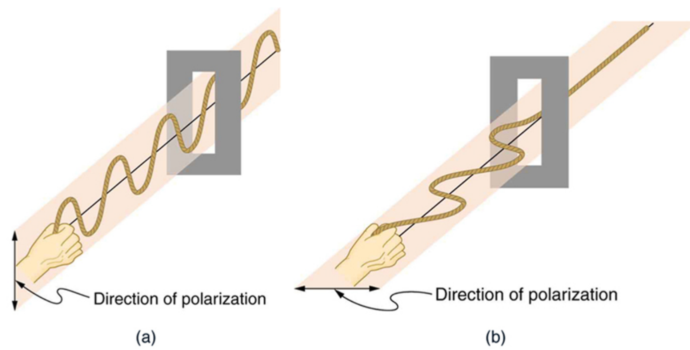


Figure 12.61 The transverse oscillations in one rope are in a vertical plane, and those in the other rope are in a horizontal plane. The first is said to be vertically polarized, and the other is said to be horizontally polarized. Vertical slits pass vertically polarized waves and block horizontally polarized waves.

The Sun and many other light sources produce waves that are randomly polarized (see Figure 12.62). Such light is said to be **unpolarized** because it is composed of many waves with all possible directions of polarization. Polaroid materials, invented by the founder of Polaroid Corporation, Edwin Land, act as a *polarizing* slit for light, allowing only polarization in one direction to pass through. Polarizing filters are composed of long molecules aligned in one direction. Thinking of the molecules as many slits, analogous to those for the oscillating ropes, we can understand why only light with a specific polarization can get through. The **axis of a polarizing filter** is the direction along which the filter passes the electric field of an EM wave (see Figure 12.63).

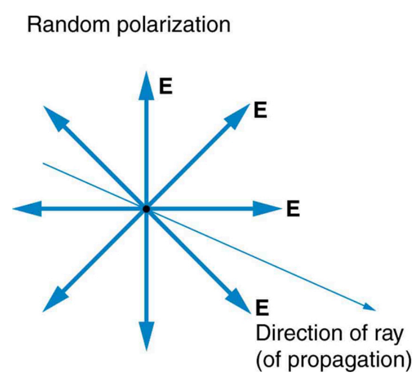


Figure 12.62 The slender arrow represents a ray of unpolarized light. The bold arrows represent the direction of polarization of the individual waves composing the ray. Since the light is unpolarized, the arrows point in all directions.

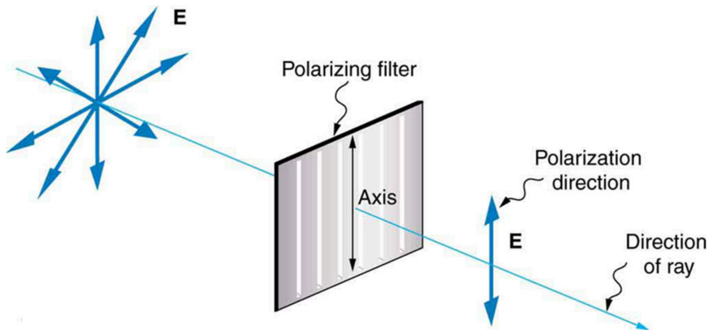


Figure 12.63 A polarizing filter has a polarization axis that acts as a slit passing through electric fields parallel to its direction. The direction of polarization of an EM wave is defined to be the direction of its electric field.

Figure 12.64 shows the effect of two polarizing filters on originally unpolarized light. The first filter polarizes the light along its axis. When the axes of the first and second filters are aligned (parallel), then all of the polarized light passed by the first filter is also passed by the second. If the second polarizing filter is rotated, only the component of the light parallel to the second filter's axis is passed. When the axes are perpendicular, no light is passed by the second.

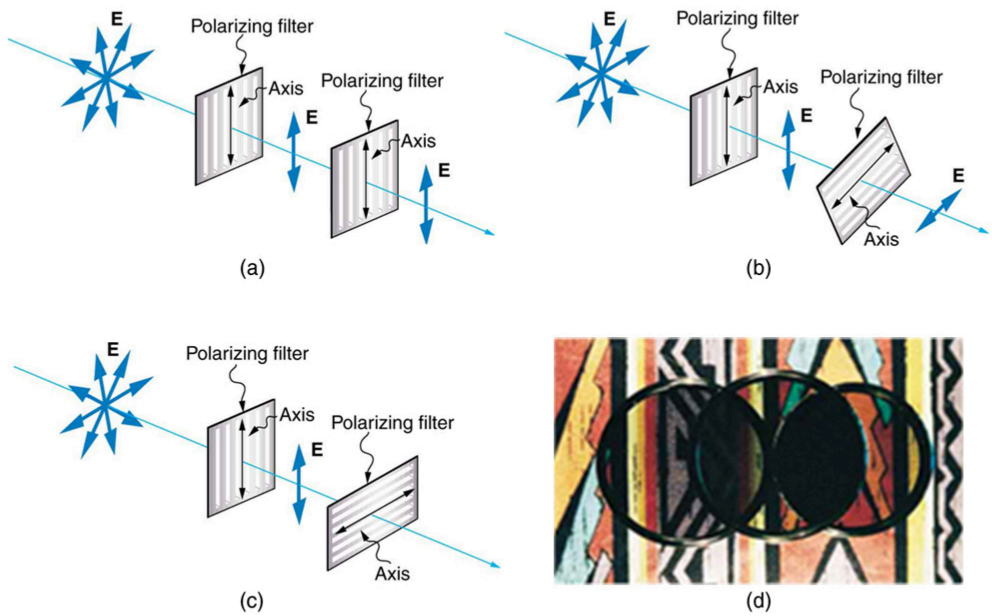


Figure 12.64 The effect of rotating two polarizing filters, where the first polarizes the light. (a) All of the polarized light is passed by the second polarizing filter, because its axis is parallel to the first. (b) As the second is rotated, only part of the light is passed. (c) When the second is perpendicular to the first, no light is passed. (d) In this photograph, a polarizing filter is placed above two others. Its axis is perpendicular to the filter on the right (dark area) and parallel to the filter on the left (lighter area). (credit: P.P. Urone)

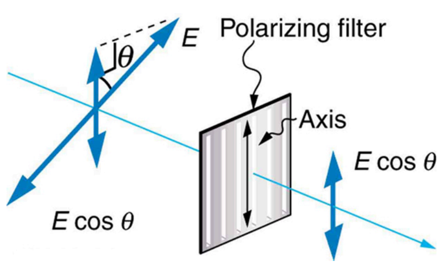


Figure 12.65 A polarizing filter transmits only the component of the wave parallel to its axis, $E\cos\theta$, reducing the intensity of any light not polarized parallel to its axis.

Polarization by Reflection

By now you can probably guess that Polaroid sunglasses cut the glare in reflected light because that light is polarized. You can check this for yourself by holding Polaroid sunglasses in front of you and rotating them while looking at light reflected from water or glass. As you rotate the sunglasses, you will notice the light gets bright and dim, but not completely black. This implies the reflected light is partially polarized and cannot be completely blocked by a polarizing filter.

Figure 12.66 illustrates what happens when unpolarized light is reflected from a surface. Vertically polarized light is preferentially refracted at the surface, so that *the reflected light is left more horizontally polarized*. The reasons for this phenomenon are beyond the scope of this text, but a convenient mnemonic for remembering this is to imagine the polarization direction to be like an arrow. Vertical polarization would be like an arrow perpendicular to the surface and would be more likely to stick and not be reflected. Horizontal polarization is like an arrow bouncing on its side and would be more likely to be reflected. Sunglasses with vertical axes would then block more reflected light than unpolarized light from other sources.

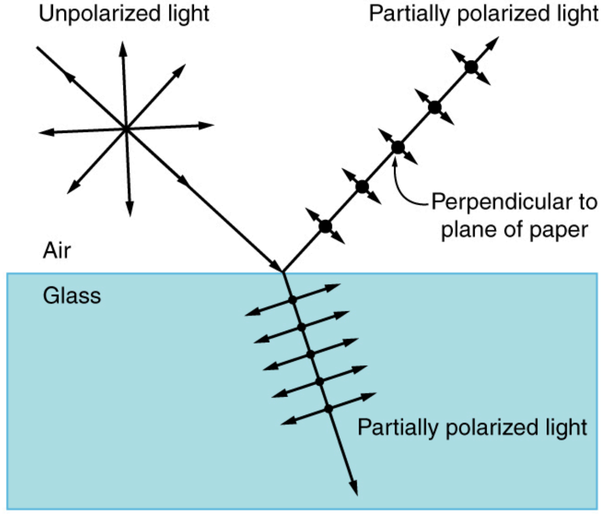


Figure 12.66 Polarization by reflection. Unpolarized light has equal amounts of vertical and horizontal polarization. After interaction with a surface, the vertical components are preferentially absorbed or refracted, leaving the reflected light more horizontally polarized. This is akin to arrows striking on their sides bouncing off, whereas arrows striking on their tips go into the surface.

Things Great and Small: Atomic Explanation of Polarizing Filters

Polarizing filters have a polarization axis that acts as a slit. This slit passes electromagnetic waves (often visible light) that have an electric field parallel to the axis. This is accomplished with long molecules aligned perpendicular to the axis as shown in **Figure 12.67**.

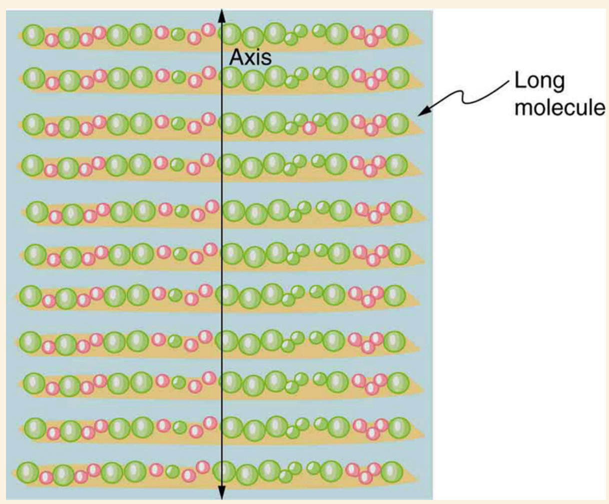


Figure 12.67 Long molecules are aligned perpendicular to the axis of a polarizing filter. The component of the electric field in an EM wave perpendicular to these molecules passes through the filter, while the component parallel to the molecules is absorbed.

Figure 12.68 illustrates how the component of the electric field parallel to the long molecules is absorbed. An electromagnetic wave is composed of oscillating electric and magnetic fields. The electric field is strong compared with the magnetic field and is more effective in exerting force on charges in the molecules. The most affected charged particles are the electrons in the molecules, since electron masses are small. If the electron is forced to oscillate, it can absorb energy from the EM wave. This reduces the fields in the wave and, hence, reduces its intensity. In long molecules, electrons can more easily oscillate parallel to the molecule than in the perpendicular direction. The electrons are bound to the molecule and are more restricted in their movement perpendicular to the molecule. Thus, the electrons can absorb EM waves that have a component of their electric field parallel to the molecule. The electrons are much less responsive to electric fields perpendicular to the molecule and will allow those fields to pass. Thus the axis of the polarizing filter is perpendicular to the length of the molecule.

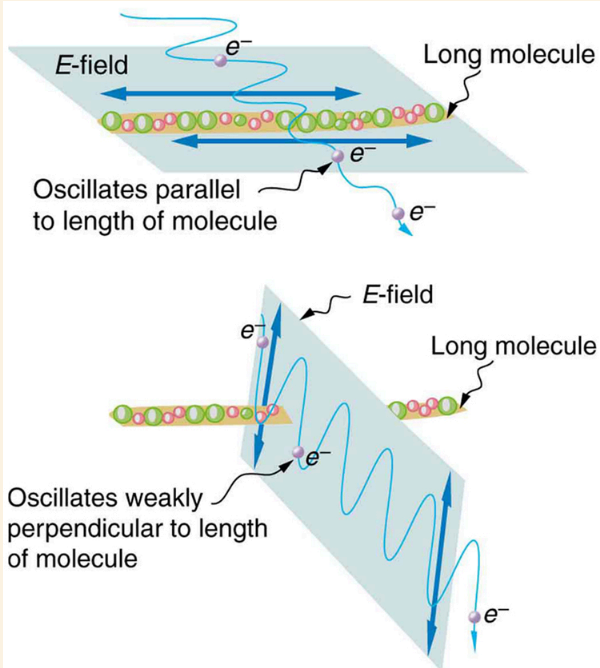


Figure 12.68 Artist's conception of an electron in a long molecule oscillating parallel to the molecule. The oscillation of the electron absorbs energy and reduces the intensity of the component of the EM wave that is parallel to the molecule.

Polarization by Scattering

If you hold your Polaroid sunglasses in front of you and rotate them while looking at blue sky, you will see the sky get bright and dim. This is a clear indication that light scattered by air is partially polarized. **Figure 12.69** helps illustrate how this happens. Since light is a transverse EM wave, it vibrates the electrons of air molecules perpendicular to the direction it is traveling. The electrons then radiate like small antennae. Since they are oscillating perpendicular to the direction of the light ray, they produce EM radiation that is polarized perpendicular to the direction of the ray. When viewing the light along a line perpendicular to the original ray, as in **Figure 12.69**, there can be no polarization in the scattered light parallel to the original ray, because that would require the original ray to be a longitudinal wave. Along other directions, a component of the other polarization can be projected along the line of sight, and the scattered light will only be partially polarized. Furthermore, multiple scattering can bring light to your eyes from other directions and can contain different polarizations.

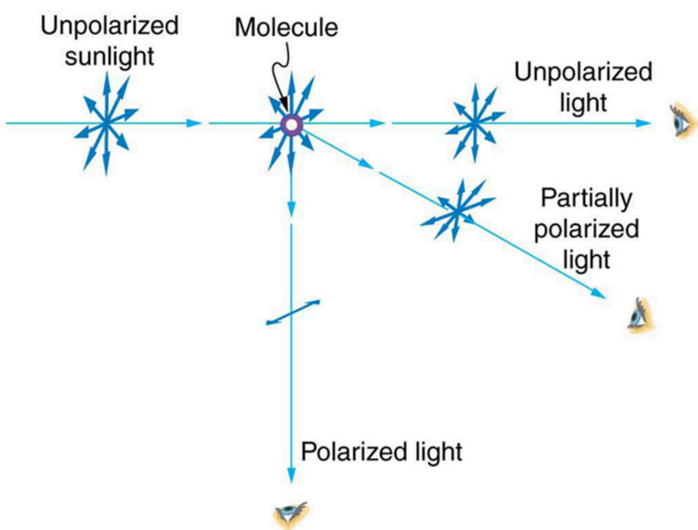


Figure 12.69 Polarization by scattering. Unpolarized light scattering from air molecules shakes their electrons perpendicular to the direction of the original ray. The scattered light therefore has a polarization perpendicular to the original direction and none parallel to the original direction.

Photographs of the sky can be darkened by polarizing filters, a trick used by many photographers to make clouds brighter by contrast. Scattering from other particles, such as smoke or dust, can also polarize light. Detecting polarization in scattered EM waves can be a useful analytical tool in determining the scattering source.

There is a range of optical effects used in sunglasses. Besides being Polaroid, other sunglasses have colored pigments embedded in them, while others use non-reflective or even reflective coatings. A recent development is photochromic lenses, which darken in the sunlight and become clear indoors. Photochromic lenses are embedded with organic microcrystalline molecules that change their properties when exposed to UV in sunlight, but become clear in artificial lighting with no UV.

Take-Home Experiment: Polarization

Find Polaroid sunglasses and rotate one while holding the other still and look at different surfaces and objects. Explain your observations. What is the difference in angle from when you see a maximum intensity to when you see a minimum intensity? Find a reflective glass surface and do the same. At what angle does the glass need to be oriented to give minimum glare?

Liquid Crystals and Other Polarization Effects in Materials

While you are undoubtedly aware of liquid crystal displays (LCDs) found in watches, calculators, computer screens, cellphones, flat screen televisions, and other myriad places, you may not be aware that they are based on polarization. Liquid crystals are so named because their molecules can be aligned even though they are in a liquid. Liquid crystals have the property that they can rotate the polarization of light passing through them by 90°. Furthermore, this property can be turned off by the application of a voltage, as illustrated in **Figure 12.70**. It is possible to manipulate this characteristic quickly and in small well-defined regions to create the contrast patterns we see in so many LCD devices.

In flat screen LCD televisions, there is a large light at the back of the TV. The light travels to the front screen through millions of tiny units called pixels (picture elements). One of these is shown in **Figure 12.70** (a) and (b). Each unit has three cells, with red, blue, or green filters, each controlled independently. When the voltage across a liquid crystal is switched off, the liquid crystal passes the light through the particular filter. One can vary the picture contrast by varying the strength of the voltage applied to the liquid crystal.

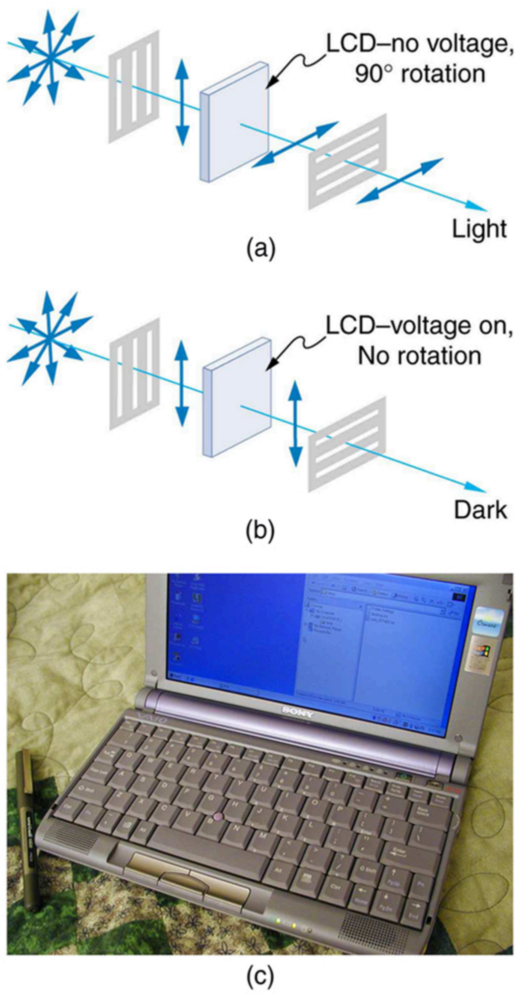


Figure 12.70 (a) Polarized light is rotated 90° by a liquid crystal and then passed by a polarizing filter that has its axis perpendicular to the original polarization direction. (b) When a voltage is applied to the liquid crystal, the polarized light is not rotated and is blocked by the filter, making the region dark in comparison with its surroundings. (c) LCDs can be made color specific, small, and fast enough to use in laptop computers and TVs. (credit: Jon Sullivan)

Many crystals and solutions rotate the plane of polarization of light passing through them. Such substances are said to be

optically active. Examples include sugar water, insulin, and collagen (see **Figure 12.71**). In addition to depending on the type of substance, the amount and direction of rotation depends on a number of factors. Among these is the concentration of the substance, the distance the light travels through it, and the wavelength of light. Optical activity is due to the asymmetric shape of molecules in the substance, such as being helical. Measurements of the rotation of polarized light passing through substances can thus be used to measure concentrations, a standard technique for sugars. It can also give information on the shapes of molecules, such as proteins, and factors that affect their shapes, such as temperature and pH.

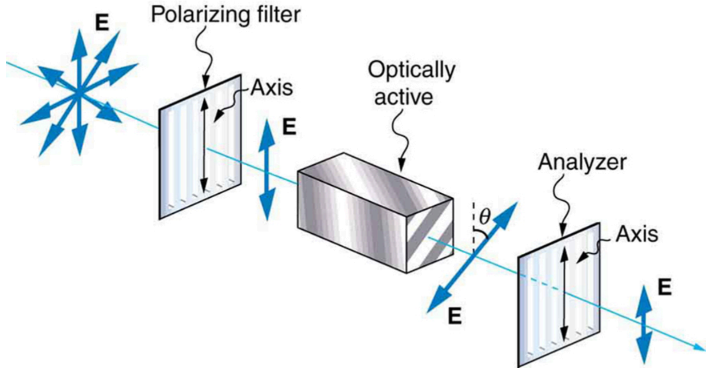


Figure 12.71 Optical activity is the ability of some substances to rotate the plane of polarization of light passing through them. The rotation is detected with a polarizing filter or analyzer.

Glass and plastic become optically active when stressed; the greater the stress, the greater the effect. Optical stress analysis on complicated shapes can be performed by making plastic models of them and observing them through crossed filters, as seen in **Figure 12.72**. It is apparent that the effect depends on wavelength as well as stress. The wavelength dependence is sometimes also used for artistic purposes.

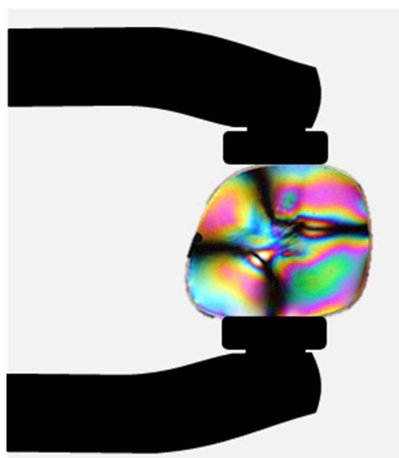


Figure 12.72 Optical stress analysis of a plastic lens placed between crossed polarizers. (credit: Infopro, Wikimedia Commons)

Another interesting phenomenon associated with polarized light is the ability of some crystals to split an unpolarized beam of light into two. Such crystals are said to be **birefringent** (see **Figure 12.73**). Each of the separated rays has a specific polarization. One behaves normally and is called the ordinary ray, whereas the other does not obey Snell's law and is called the extraordinary ray. Birefringent crystals can be used to produce polarized beams from unpolarized light. Some birefringent materials preferentially absorb one of the polarizations. These materials are called dichroic and can produce polarization by this preferential absorption. This is fundamentally how polarizing filters and other polarizers work. The interested reader is invited to further pursue the numerous properties of materials related to polarization.

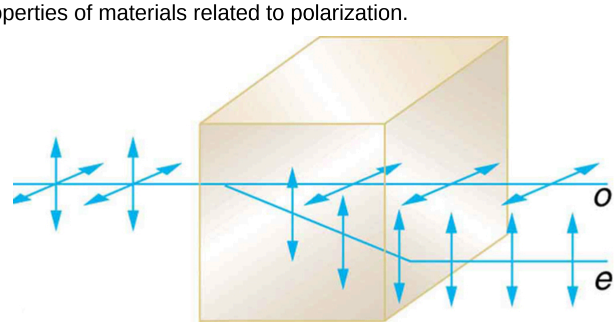


Figure 12.73 Birefringent materials, such as the common mineral calcite, split unpolarized beams of light into two. The ordinary ray behaves as expected, but the extraordinary ray does not obey Snell's law.

Section Summary

- Polarization is the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave.
- EM waves are transverse waves that may be polarized.
- The direction of polarization is defined to be the direction parallel to the electric field of the EM wave.
- Unpolarized light is composed of many rays having random polarization directions.
- Light can be polarized by passing it through a polarizing filter or other polarizing material. The intensity of polarized light after passing through a polarizing filter depends on the direction of polarization and the axis of the filter.
- Polarization is also produced by reflection.
- Polarization can also be produced by scattering.
- There are a number of types of optically active substances that rotate the direction of polarization of light passing through them.

Conceptual Questions

Exercise 12.100

Under what circumstances is the phase of light changed by reflection? Is the phase related to polarization?

Exercise 12.101

Can a sound wave in air be polarized? Explain.

Exercise 12.102

No light passes through two perfect polarizing filters with perpendicular axes. However, if a third polarizing filter is placed between the original two, some light can pass. Why is this? Under what circumstances does most of the light pass?

Exercise 12.103

Explain what happens to the energy carried by light that it is dimmed by passing it through two crossed polarizing filters.

Exercise 12.104

When particles scattering light are much smaller than its wavelength, the amount of scattering is proportional to $1 / \lambda^4$. Does this mean there is more scattering for small λ than large λ? How does this relate to the fact that the sky is blue?

Exercise 12.105

Using the information given in the preceding question, explain why sunsets are red.

Glossary

amplitude: the height, or magnitude, of an electromagnetic wave

amplitude modulation (AM): a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

axis of a polarizing filter: the direction along which the filter passes the electric field of an EM wave

birefringent: crystals that split an unpolarized beam of light into two beams

carrier wave: an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

converging lens: a convex lens in which light rays that enter it parallel to its axis converge at a single point on the opposite side

converging mirror: a concave mirror in which light rays that strike it parallel to its axis converge at one or more points along the axis

direction of polarization: the direction parallel to the electric field for EM waves

dispersion: spreading of white light into its full spectrum of wavelengths

diverging lens: a concave lens in which light rays that enter it parallel to its axis bend away (diverge) from its axis

diverging mirror: a convex mirror in which light rays that strike it parallel to its axis bend away (diverge) from its axis

electric field: a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electric field lines: a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

electric field strength: the magnitude of the electric field, denoted *E*-field

electromagnetic spectrum: the full range of wavelengths or frequencies of electromagnetic radiation

electromagnetic spectrum: the full range of wavelengths or frequencies of electromagnetic radiation

electromagnetic waves: radiation in the form of waves of electric and magnetic energy

extremely low frequency (ELF): electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

focal length: distance from the center of a lens or curved mirror to its focal point

focal point: for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate

frequency: the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

frequency modulation (FM): a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

gamma ray: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

gamma ray: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

hertz: an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

horizontally polarized: the oscillations are in a horizontal plane

index of refraction: for a material, the ratio of the speed of light in vacuum to that in the material

infrared radiation (IR): a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from 0.74 μm to 300 μm

infrared radiation (IR): a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from 0.74 μm to 300 μm

law of reflection: angle of reflection equals the angle of incidence

law of reflection: angle of reflection equals the angle of incidence

magnetic field: a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

magnetic field lines: a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

magnetic field strength: the magnitude of the magnetic field, denoted *B*-field

magnification: ratio of image height to object height

Maxwell's equations: a set of four equations that comprise a complete, overarching theory of electromagnetism

microwaves: electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

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mirror: smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

optically active: substances that rotate the plane of polarization of light passing through them

oscillate: to fluctuate back and forth in a steady beat

polarization: the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave

polarized: waves having the electric and magnetic field oscillations in a definite direction

power: inverse of focal length

radar: a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

radio waves: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

radio waves: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

rainbow: dispersion of sunlight into a continuous distribution of colors according to wavelength, produced by the refraction and reflection of sunlight by water droplets in the sky

real image: image that can be projected

refraction: changing of a light ray’s direction when it passes through variations in matter

resonant: a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

speed of light: in a vacuum, such as space, the speed of light is a constant 3×10^8 m/s

standing wave: a wave that oscillates in place, with nodes where no motion happens

thermal agitation: the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

transverse wave: a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

TV: video and audio signals broadcast on electromagnetic waves

ultra-high frequency (UHF): TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

ultraviolet radiation (UV): electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

ultraviolet radiation (UV): electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

unpolarized: waves that are randomly polarized

vertically polarized: the oscillations are in a vertical plane

very high frequency (VHF): TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

virtual image: image that cannot be projected

visible light: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

visible light: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

wavelength: the distance from one peak to the next in a wave

X-ray: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ -ray range

X-ray: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ -ray range

APPENDIX A | ATOMIC MASSES

Table A1 Atomic Masses

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38\times10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	1.28×10^9 y
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	4.2×10^6y
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	4.4×10^{14}y
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	2.5×10^{21}y
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	4.6×10^{10}y
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	$1.60 \times 10^3 y$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	$1.41 \times 10^{10} y$
91	Protactinium	231	^{231}Pa	231.035 880	α	$3.28 \times 10^4 y$
92	Uranium	233	^{233}U	233.039 628	α	$1.59 \times 10^3 y$
		235	^{235}U	235.043 924	0.720%, α	$7.04 \times 10^8 y$
		236	^{236}U	236.045 562	α	$2.34 \times 10^7 y$
		238	^{238}U	238.050 784	99.2745%, α	$4.47 \times 10^9 y$
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	$2.41 \times 10^4 y$
95	Americium	243	^{243}Am	243.061 375	α, fission	$7.37 \times 10^3 y$
96	Curium	245	^{245}Cm	245.065 483	α	$8.50 \times 10^3 y$
97	Berkelium	247	^{247}Bk	247.070 300	α	$1.38 \times 10^3 y$

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

APPENDIX B | SELECTED RADIOACTIVE ISOTOPES

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Table B1 Selected Radioactive Isotopes

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
^3H	12.33 y	β^-	0.0186	100%			
^{14}C	5730 y	β^-	0.156	100%			
^{13}N	9.96 min	β^+	1.20	100%			
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27	100%
^{32}P	14.28 d	β^-	1.71	100%			
^{35}S	87.4 d	β^-	0.167	100%			
^{36}Cl	3.00×10^5y	β^-	0.710	100%			
^{40}K	1.28×10^9y	β^-	1.31	89%			
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373	87%
						0.618	87%
^{45}Ca	165 d	β^-	0.257	100%			
^{51}Cr	27.70 d	EC			γ	0.320	10%
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33	28%
						1.43	28%
^{52}Fe	8.27 h	β^+	1.80	43%		0.169	43%
						0.378	43%
^{59}Fe	44.6 d	β^-s	0.273	45%	γs	1.10	57%
			0.466	55%		1.29	43%
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17	100%
						1.33	100%
^{65}Zn	244.1 d	EC			γ	1.12	51%
^{67}Ga	78.3 h	EC			γs	0.0933	70%
						0.185	35%
						0.300	19%
						others	
^{75}Se	118.5 d	EC			γs	0.121	20%
						0.136	65%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
						0.265	68%
						0.280	20%
						others	
^{86}Rb	18.8 d	β^-s	0.69	9%	γ	1.08	9%
			1.77	91%			
^{85}Sr	64.8 d	EC			γ	0.514	100%
^{90}Sr	28.8 y	β^-	0.546	100%			
^{90}Y	64.1 h	β^-	2.28	100%			
^{99m}Tc	6.02 h	IT			γ	0.142	100%
^{113m}In	99.5 min	IT			γ	0.392	100%
^{123}I	13.0 h	EC			γ	0.159	$\approx 100\%$
^{131}I	8.040 d	β^-s	0.248	7%	γs	0.364	85%
			0.607	93%		others	
			others				
^{129}Cs	32.3 h	EC			γs	0.0400	35%
						0.372	32%
						0.411	25%
						others	
^{137}Cs	30.17 y	β^-s	0.511	95%	γ	0.662	95%
			1.17	5%			
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γs	0.030	25%
						0.044	65%
						0.537	24%
						others	
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412	$\approx 100\%$
^{197}Hg	64.1 h	EC			γ	0.0733	100%
^{210}Po	138.38 d	α	5.41	100%			
^{226}Ra	1.60×10^3y	αs	4.68	5%	γ	0.186	100%
			4.87	95%			
^{235}U	7.038×10^8y	α	4.68	$\approx 100\%$	γs	numerous	<0.400%
^{238}U	4.468×10^9y	αs	4.22	23%	γ	0.050	23%
			4.27	77%			
^{237}Np	2.14×10^6y	αs	numerous		γs	numerous	<0.250%
			4.96 (max.)				
^{239}Pu	2.41×10^4y	αs	5.19	11%	γs	7.5×10^{-5}	73%
			5.23	15%		0.013	15%

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)	Percent
			5.24	73%		0.052	10%
						others	
^{243}Am	$7.37\times10^{3}y$	αs	Max. 5.44		γs	0.075	
			5.37	88%		others	
			5.32	11%			
			others				

APPENDIX C | USEFUL INFORMATION

This appendix is broken into several tables.

- **Table C1**, Important Constants
- **Table C2**, Submicroscopic Masses
- **Table C3**, Solar System Data
- **Table C4**, Metric Prefixes for Powers of Ten and Their Symbols
- **Table C5**, The Greek Alphabet
- **Table C6**, SI units
- **Table C7**, Selected British Units
- **Table C8**, Other Units
- **Table C9**, Useful Formulae

Table C1 Important Constants[1]

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458\times10^{8}\ \text{m/s}$	$3.00\times10^{8}\ \text{m/s}$
G	Gravitational constant	$6.67384(80)\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$	$6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
N_A	Avogadro's number	$6.02214129(27)\times10^{23}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488(13)\times10^{-23}\ \text{J/K}$	$1.38\times10^{-23}\ \text{J/K}$
R	Gas constant	$8.3144621(75)\ \text{J/mol}\cdot\text{K}$	$8.31\ \text{J/mol}\cdot\text{K} = 1.99\ \text{cal/mol}\cdot\text{K} = 0.0821\text{atm}\cdot\text{L/mol}\cdot\text{K}$
σ	Stefan-Boltzmann constant	$5.670373(21)\times10^{-8}\ \text{W/m}^2\cdot\text{K}$	$5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}$
k	Coulomb force constant	$8.987551788...\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$	$8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35)\times10^{-19}\ \text{C}$	$-1.60\times10^{-19}\ \text{C}$
ε_0	Permittivity of free space	$8.854187817...\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$	$8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$
μ_0	Permeability of free space	$4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$	$1.26\times10^{-6}\ \text{T}\cdot\text{m/A}$
h	Planck's constant	$6.62606957(29)\times10^{-34}\ \text{J}\cdot\text{s}$	$6.63\times10^{-34}\ \text{J}\cdot\text{s}$

Table C2 Submicroscopic Masses[2]

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40)\times10^{-31}\text{kg}$	$9.11\times10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74)\times10^{-27}\text{kg}$	$1.6726\times10^{-27}\text{kg}$
m_n	Neutron mass	$1.674927351(74)\times10^{-27}\text{kg}$	$1.6749\times10^{-27}\text{kg}$

1. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.
2. Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (http://www.physics.nist.gov/cuu) (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
u	Atomic mass unit	$1.660538921(73)\times10^{-27}\text{kg}$	$1.6605\times10^{-27}\text{kg}$

Table C3 Solar System Data

Sun	mass	$1.99\times10^{30}\text{kg}$
	average radius	$6.96\times10^{8}\text{m}$
	Earth-sun distance (average)	$1.496\times10^{11}\text{m}$
Earth	mass	$5.9736\times10^{24}\text{kg}$
	average radius	$6.376\times10^{6}\text{m}$
	orbital period	$3.16\times10^{7}\text{s}$
Moon	mass	$7.35\times10^{22}\text{kg}$
	average radius	$1.74\times10^{6}\text{m}$
	orbital period (average)	$2.36\times10^{6}\text{s}$
	Earth-moon distance (average)	$3.84\times10^{8}\text{m}$

Table C4 Metric Prefixes for Powers of Ten and Their Symbols

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0}(=1)$	femto	f	10^{-15}

Table C5 The Greek Alphabet

Alpha	A	α	Eta	H	η	Nu	N	ν	Tau	T	τ
Beta	B	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	I	ι	Omicron	O	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	K	κ	Pi	Π	π	Chi	X	χ
Epsilon	E	ε	Lambda	Λ	λ	Rho	P	ρ	Psi	Ψ	ψ
Zeta	Z	ζ	Mu	M	μ	Sigma	Σ	σ	Omega	Ω	ω

Table C6 SI Units

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram

	Entity	Abbreviation	Name
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$\mathrm{N = kg \cdot m / s^2}$	newton
	Energy	$\mathrm{J = kg \cdot m^2 / s^2}$	joule
	Power	$\mathrm{W = J / s}$	watt
	Pressure	$\mathrm{Pa = N / m^2}$	pascal
	Frequency	$\mathrm{Hz = 1 / s}$	hertz
	Electronic potential	$\mathrm{V = J / C}$	volt
	Capacitance	$\mathrm{F = C / V}$	farad
	Charge	$\mathrm{C = s \cdot A}$	coulomb
	Resistance	$\Omega = \mathrm{V / A}$	ohm
	Magnetic field	$\mathrm{T = N / (A \cdot m)}$	tesla
	Nuclear decay rate	$\mathrm{Bq = 1 / s}$	becquerel

Table C7 Selected British Units

Length	$1 \text{ inch (in.)} = 2.54 \text{ cm (exactly)}$
	$1 \text{ foot (ft)} = 0.3048 \text{ m}$
	$1 \text{ mile (mi)} = 1.609 \text{ km}$
Force	$1 \text{ pound (lb)} = 4.448 \text{ N}$
Energy	$1 \text{ British thermal unit (Btu)} = 1.055 \times 10^3 \text{ J}$
Power	$1 \text{ horsepower (hp)} = 746 \text{ W}$
Pressure	$1 \text{ lb / in}^2 = 6.895 \times 10^3 \text{ Pa}$

Table C8 Other Units

Length	$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$
	$1 \text{ astronomical unit (au)} = 1.50 \times 10^{11} \text{ m}$
	$1 \text{ nautical mile} = 1.852 \text{ km}$
	$1 \text{ angstrom(Å)} = 10^{-10} \text{ m}$
Area	$1 \text{ acre (ac)} = 4.05 \times 10^3 \text{ m}^2$
	$1 \text{ square foot (ft}^2) = 9.29 \times 10^{-2} \text{ m}^2$
	$1 \text{ barn } (b) = 10^{-28} \text{ m}^2$
Volume	$1 \text{ liter } (L) = 10^{-3} \text{ m}^3$
	$1 \text{ U.S. gallon (gal)} = 3.785 \times 10^{-3} \text{ m}^3$

Mass	$1 \text{ solar mass } = 1.99 \times 10^{30} \text{ kg}$
	$1 \text{ metric ton} = 10^3 \text{ kg}$
	$1 \text{ atomic mass unit } (u) = 1.6605 \times 10^{-27} \text{ kg}$
Time	$1 \text{ year } (y) = 3.16 \times 10^7 \text{ s}$
	$1 \text{ day } (d) = 86{,}400 \text{ s}$
Speed	$1 \text{ mile per hour (mph)} = 1.609 \text{ km/h}$
	$1 \text{ nautical mile per hour (naut)} = 1.852 \text{ km/h}$
Angle	$1 \text{ degree } (°) = 1.745 \times 10^{-2} \text{ rad}$
	$1 \text{ minute of arc } (') = 1/60 \text{ degree}$
	$1 \text{ second of arc } ('') = 1/60 \text{ minute of arc}$
	$1 \text{ grad} = 1.571 \times 10^{-2} \text{ rad}$
Energy	$1 \text{ kiloton TNT (kT)} = 4.2 \times 10^{12} \text{ J}$
	$1 \text{ kilowatt hour } (\text{kW} \cdot h) = 3.60 \times 10^6 \text{ J}$
	$1 \text{ food calorie (kcal)} = 4186 \text{ J}$
	$1 \text{ calorie (cal)} = 4.186 \text{ J}$
	$1 \text{ electron volt (eV)} = 1.60 \times 10^{-19} \text{ J}$
Pressure	$1 \text{ atmosphere (atm)} = 1.013 \times 10^5 \text{ Pa}$
	$1 \text{ millimeter of mercury (mm Hg)} = 133.3 \text{ Pa}$
	$1 \text{ torricelli (torr)} = 1 \text{ mm Hg} = 133.3 \text{ Pa}$
Nuclear decay rate	$1 \text{ curie (Ci)} = 3.70 \times 10^{10} \text{ Bq}$

Table C9 Useful Formulae

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$
Volume of a sphere with radius r	$V = (4/3)\left(\pi r^3\right)$

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URL: **https://legacy.cnx.org/content/m67034/1.1/**
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Based on: 2.6 Motion Equations for Constant Acceleration in One Dimension <**http://legacy.cnx.org/content/m52319/1.2**> by Bobby Bailey.

Module: **2.8 Falling Objects**
Used here as: Falling Objects
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52321/1.1/**
Copyright: Bobby Bailey
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Based on: Falling Objects <**http://legacy.cnx.org/content/m42102/1.5**> by OpenStax.

Module: **Projectile Motion (edited for Introduction to Physics)**
Used here as: Projectile Motion
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67038/1.1/**
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Based on: Projectile Motion <**http://legacy.cnx.org/content/m42042/1.14**> by OpenStax.

Module: **Centripetal Acceleration (edited for Introduction to Physics)**
Used here as: Centripetal Acceleration
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67035/1.1/**
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Based on: Centripetal Acceleration <**http://legacy.cnx.org/content/m42084/1.9**> by OpenStax.

Module: **4.1 Introduction to Dynamics: Newton’s Laws of Motion**
Used here as: Introduction to Dynamics: Newton’s Laws of Motion
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52322/1.1/**

Based on: Introduction to Dynamics: Newton’s Laws of Motion <**http://legacy.cnx.org/content/m42129/1.4**> by OpenStax.

Module: **4.2 Development of Force Concept**
Used here as: Development of Force Concept
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52324/1.1/**

Based on: Development of Force Concept <**http://legacy.cnx.org/content/m42069/1.3**> by OpenStax.

Module: **4.3 Newton’s First Law of Motion: Inertia**
Used here as: Newton’s First Law of Motion: Inertia
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52325/1.1/**

Based on: Newton’s First Law of Motion: Inertia <**http://legacy.cnx.org/content/m42130/1.2**> by OpenStax.

Module: **4.4 Newton’s Second Law of Motion: Concept of a System**
Used here as: Newton’s Second Law of Motion: Concept of a System
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52327/1.1/**

Based on: Newton’s Second Law of Motion: Concept of a System <**http://legacy.cnx.org/content/m42073/1.8**> by OpenStax.

Module: **4.5 Newton’s Third Law of Motion: Symmetry in Forces**
Used here as: Newton’s Third Law of Motion: Symmetry in Forces
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52328/1.1/**

Based on: Newton’s Third Law of Motion: Symmetry in Forces <**http://legacy.cnx.org/content/m42074/1.5**> by OpenStax.

Module: **4.6 Normal, Tension, and Other Examples of Forces**
Used here as: Normal Force and Tension
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52329/1.1/**

Based on: Normal, Tension, and Other Examples of Forces <**http://legacy.cnx.org/content/m42075/1.7**> by OpenStax.

Module: **Spring Force: Hooke’s Law**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67044/1.1/**

Based on: Hooke’s Law: Stress and Strain Revisited <**http://legacy.cnx.org/content/m42240/1.5**> by OpenStax.

Module: **5.2 Friction**
Used here as: Friction
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52332/1.1/**
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Based on: Friction <**http://legacy.cnx.org/content/m42139/1.6**> by OpenStax.

Module: **6.6 Newton's Universal Law of Gravitation**
Used here as: Newton's Universal Law of Gravitation
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52333/1.1/**
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Based on: Newton's Universal Law of Gravitation <**http://legacy.cnx.org/content/m42143/1.5**> by OpenStax.

Module: **Centripetal Force (edited for Introduction to Physics)**
Used here as: Centripetal Force
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67036/1.1/**
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Based on: Centripetal Force <**http://legacy.cnx.org/content/m42086/1.11**> by OpenStax.

Module: **7.1 Introduction to Work and Energy**
Used here as: Introduction to Work and Energy
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52337/1.1/**
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Based on: Introduction to Work, Energy, and Energy Resources <**http://legacy.cnx.org/content/m42145/1.2**> by OpenStax.

Module: **BB -- NSCI 100 -- 7.2 Work: The Scientific Definition**
Used here as: Work: The Scientific Definition
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m51295/1.5/**
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Based on: Work: The Scientific Definition <**http://legacy.cnx.org/content/m42146/1.5**> by OpenStax.

Module: **7.3 Kinetic Energy and the Work-Energy Theorem**
Used here as: Kinetic Energy and the Work-Energy Theorem
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52338/1.1/**
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Based on: Kinetic Energy and the Work-Energy Theorem <**http://legacy.cnx.org/content/m42147/1.5**> by OpenStax.

Module: **7.4 Gravitational Potential Energy**
Used here as: Gravitational Potential Energy
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52340/1.2/**
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Based on: Gravitational Potential Energy <**http://legacy.cnx.org/content/m42148/1.7**> by OpenStax.

Module: **Conservative Forces, Potential Energy, and Conservation of Energy (edited for Introduction to Physics)**
Used here as: Conservative Forces, Potential Energy, and Conservation of Energy
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67037/1.2/**
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Based on: 7.5 Conservative Forces and Potential Energy <**http://legacy.cnx.org/content/m52346/1.1**> by Bobby Bailey.

Module: **Spring Potential Energy**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67045/1.1/**
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Based on: Hooke's Law: Stress and Strain Revisited <**http://legacy.cnx.org/content/m42240/1.5**> by OpenStax.

Module: **7.8 Power**
Used here as: Power
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52349/1.3/**
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Based on: Power <**http://legacy.cnx.org/content/m42152/1.4**> by OpenStax.

Module: **Introduction to Linear Momentum and Collisions**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42155/1.3/**
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Module: **Linear Momentum and Force**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42156/1.3/**
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Module: **Derived copy of Impulse**
Used here as: Impulse
By: James Rittenbach
URL: **https://legacy.cnx.org/content/m59691/1.2/**
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Based on: Impulse <**http://legacy.cnx.org/content/m42159/1.6**> by OpenStax.

Module: **Conservation of Momentum (edited for Introduction to Physics)**
Used here as: Conservation of Momentum
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67039/1.1/**
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Based on: Derived copy of Conservation of Momentum <**http://legacy.cnx.org/content/m59689/1.3**> by James Rittenbach.

Module: **Elastic Collisions in One Dimension (edited for Introduction to Physics)**
Used here as: Elastic Collisions in One Dimension
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67042/1.1/**

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Based on: Elastic Collisions in One Dimension <**http://legacy.cnx.org/content/m42163/1.3**> by OpenStax.

Module: **Inelastic Collisions in One Dimension (edited for Introduction to Physics)**
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By: Andrew Park
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Based on: Inelastic Collisions in One Dimension <**http://legacy.cnx.org/content/m42164/1.10**> by OpenStax.

Module: **16.1 Introduction to Oscillatory Motion and Waves**
Used here as: Introduction to Oscillatory Motion and Waves
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52431/1.1/**
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Based on: Introduction to Oscillatory Motion and Waves <**http://legacy.cnx.org/content/m42239/1.3**> by OpenStax.

Module: **16.3 Period and Frequency in Oscillations**
Used here as: Period and Frequency in Oscillations
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52428/1.1/**
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Based on: Period and Frequency in Oscillations <**http://legacy.cnx.org/content/m42241/1.3**> by OpenStax.

Module: **Simple Harmonic Motion: A Special Periodic Motion**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42242/1.7/**
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Module: **Forced Oscillations and Resonance**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42247/1.5/**
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Module: **16.10 Waves**
Used here as: Waves
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52430/1.1/**
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Based on: Waves <**http://legacy.cnx.org/content/m42248/1.5**> by OpenStax.

Module: **16.11 Superposition and Interference**
Used here as: Superposition and Interference
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52433/1.1/**
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Based on: Superposition and Interference <**http://legacy.cnx.org/content/m42249/1.5**> by OpenStax.

Module: **17.2 Sound**
Used here as: Sound
By: Bobby Bailey

URL: **https://legacy.cnx.org/content/m52438/1.1/**
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Based on: Sound <**http://legacy.cnx.org/content/m42255/1.3**> by OpenStax.

Module: **17.3 Speed of Sound, Frequency, and Wavelength**
Used here as: Speed of Sound, Frequency, and Wavelength
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52435/1.1/**
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Based on: Speed of Sound, Frequency, and Wavelength <**http://legacy.cnx.org/content/m42256/1.4**> by OpenStax.

Module: **17.5 Doppler Effect and Sonic Booms**
Used here as: Doppler Effect and Sonic Booms
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52445/1.1/**
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Based on: Doppler Effect and Sonic Booms <**http://legacy.cnx.org/content/m42712/1.2**> by OpenStax.

Module: **Introduction to Rotational Motion and Angular Momentum**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42176/1.4/**
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Module: **Dynamics of Rotational Motion: Rotational Inertia**
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Module: **Rotational Kinetic Energy (edited for Introduction to Physics)**
Used here as: Rotational Kinetic Energy
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67046/1.2/**
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Based on: Rotational Kinetic Energy: Work and Energy Revisited <**http://legacy.cnx.org/content/m42180/1.8**> by OpenStax.

Module: **Angular Momentum and Its Conservation**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42182/1.5/**
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Module: **Gyroscopic Effects: Vector Aspects of Angular Momentum**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42184/1.2/**
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Module: **Introduction to Fluids**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67047/1.2/**
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Based on: 11.1 Introduction to Fluid Statics <**http://legacy.cnx.org/content/m52357/1.1**> by Bobby Bailey.

Module: **11.2 What Is a Fluid?**
Used here as: What Is a Fluid?
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52358/1.1/**
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Based on: What Is a Fluid? <**http://legacy.cnx.org/content/m42186/1.4**> by OpenStax.

Module: **11.3 Density**
Used here as: Density
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52355/1.1/**
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Based on: Density <**http://legacy.cnx.org/content/m42187/1.3**> by OpenStax.

Module: **11.4 Pressure**
Used here as: Pressure
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52360/1.1/**
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Based on: Pressure <**http://legacy.cnx.org/content/m42189/1.4**> by OpenStax.

Module: **11.8 Archimedes' Principle**
Used here as: Archimedes' Principle
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52361/1.2/**
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Based on: Archimedes' Principle <**http://legacy.cnx.org/content/m42196/1.8**> by OpenStax.

Module: **Flow Rate and Its Relation to Velocity**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42205/1.6/**
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Module: **Bernoulli's Equation**
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Module: **Introduction to Thermal Physics**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67121/1.1/**
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Based on: 13.1 Introduction to Temperature, Heat, and the Gas Laws <**http://legacy.cnx.org/content/m52362/1.1**> by Bobby Bailey.

Module: **13.2 Temperature**
Used here as: Temperature
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52364/1.2/**

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Based on: Temperature <**http://legacy.cnx.org/content/m42214/1.3**> by OpenStax.

Module: **The Ideal Gas Law (edited for Introduction to Physics)**
Used here as: The Ideal Gas Law
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67122/1.2/**
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Based on: 13.4 The Ideal Gas Law <**http://legacy.cnx.org/content/m52366/1.1**> by Bobby Bailey.

Module: **14.2 Heat**
Used here as: Heat
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52368/1.1/**
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Based on: Heat <**http://legacy.cnx.org/content/m42223/1.4**> by OpenStax.

Module: **Heat Transfer Methods (edited for Introduction to Physics)**
Used here as: Heat Transfer Methods
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URL: **https://legacy.cnx.org/content/m67123/1.1/**
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Based on: 14.5 Heat Transfer Methods <**http://legacy.cnx.org/content/m52371/1.1**> by Bobby Bailey.

Module: **14.3 Temperature Change and Heat Capacity**
Used here as: Temperature Change and Heat Capacity
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52369/1.1/**
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Based on: Temperature Change and Heat Capacity <**http://legacy.cnx.org/content/m42224/1.5**> by OpenStax.

Module: **Phase Change and Latent Heat (edited for Introduction to Physics)**
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URL: **https://legacy.cnx.org/content/m67124/1.1/**
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Based on: 14.4 Phase Change and Latent Heat <**http://legacy.cnx.org/content/m52372/1.2**> by Bobby Bailey.

Module: **The First Law of Thermodynamics (edited for Introduction to Physics)**
Used here as: The First Law of Thermodynamics
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67125/1.1/**
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Based on: 15.2 The First Law of Thermodynamics <**http://legacy.cnx.org/content/m52377/1.2**> by Bobby Bailey.

Module: **The First Law of Thermodynamics and Heat Engine Processes (edited for Introduction to Physics)**
Used here as: The First Law of Thermodynamics and Heat Engine Processes
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67126/1.1/**
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Based on: The First Law of Thermodynamics and Some Simple Processes <**http://legacy.cnx.org/content/m42233/1.7**> by OpenStax.

Module: **Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency (edited for Introduction to Physics)**
Used here as: Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67127/1.1/**
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Based on: Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency <**http://legacy.cnx.org/content/m42234/1.4**> by OpenStax.

Module: **Carnot's Perfect Heat Engine: The Second Law of Thermodynamics Restated**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42235/1.4/**
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Module: **Applications of Thermodynamics: Heat Pumps and Refrigerators**
By: OpenStax
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Module: **Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy (edited for Introduction to Physics)**
Used here as: Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67128/1.1/**
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Based on: Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy <**http://legacy.cnx.org/content/m42237/1.9**> by OpenStax.

Module: **Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42238/1.4/**
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Module: **Introduction to Electricity**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67129/1.1/**
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Based on: Introduction to Electric Charge and Electric Field <**http://legacy.cnx.org/content/m42299/1.3**> by OpenStax.

Module: **18.2 Static Electricity and Charge: Conservation of Charge**
Used here as: Static Electricity and Charge: Conservation of Charge
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52381/1.1/**
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Based on: Static Electricity and Charge: Conservation of Charge <**http://legacy.cnx.org/content/m42300/1.5**> by OpenStax.

Module: **18.4 Coulomb's Law**
Used here as: Coulomb's Law
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52384/1.1/**
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Based on: Coulomb's Law <**http://legacy.cnx.org/content/m42308/1.6**> by OpenStax.

Module: **18.5 Electric Field: Concept of a Field Revisited**
Used here as: Electric Field: Concept of a Field Revisited
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52385/1.1/**
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Based on: Electric Field: Concept of a Field Revisited <**http://legacy.cnx.org/content/m42310/1.6**> by OpenStax.

Module: **18.6 Electric Field Lines**
Used here as: Electric Field Lines
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52387/1.1/**
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Based on: Electric Field Lines: Multiple Charges <**http://legacy.cnx.org/content/m42312/1.7**> by OpenStax.

Module: **19.2 Electric Potential Energy: Potential Difference**
Used here as: Electric Potential Energy: Potential Difference
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52392/1.1/**
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Based on: Electric Potential Energy: Potential Difference <**http://legacy.cnx.org/content/m42324/1.4**> by OpenStax.

Module: **18.9 Applications of Electrostatics**
Used here as: Applications of Electrostatics
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52388/1.1/**
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Based on: Applications of Electrostatics <**http://legacy.cnx.org/content/m42329/1.6**> by OpenStax.

Module: **Current (edited for Introduction to Physics)**
Used here as: Current
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67130/1.1/**
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Based on: 20.2 Current <**http://legacy.cnx.org/content/m52400/1.1**> by Bobby Bailey.

Module: **20.3 Ohm's Law: Resistance and Simple Circuits**
Used here as: Ohm's Law: Resistance and Simple Circuits
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52401/1.1/**
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Based on: Ohm's Law: Resistance and Simple Circuits <**http://legacy.cnx.org/content/m42344/1.4**> by OpenStax.

Module: **20.5 Electric Power and Energy**

Used here as: Electric Power and Energy
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52402/1.1/**
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Based on: Electric Power and Energy <**http://legacy.cnx.org/content/m42714/1.5**> by OpenStax.

Module: **Resistors in Series and Parallel**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42356/1.5/**
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Module: **20.7 Electric Hazards and the Human Body**
Used here as: Electric Hazards and the Human Body
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52406/1.1/**
Copyright: Bobby Bailey
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Based on: Electric Hazards and the Human Body <**http://legacy.cnx.org/content/m42350/1.4**> by OpenStax.

Module: **22.1 Introduction to Magnetism**
Used here as: Introduction to Magnetism
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52408/1.1/**
Copyright: Bobby Bailey
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Based on: Introduction to Magnetism <**http://legacy.cnx.org/content/m42365/1.2**> by OpenStax.

Module: **22.2 Magnets**
Used here as: Magnets
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52412/1.1/**
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Based on: Magnets <**http://legacy.cnx.org/content/m42366/1.3**> by OpenStax.

Module: **22.3 Ferromagnets and Electromagnets**
Used here as: Ferromagnets and Electromagnets
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52410/1.1/**
Copyright: Bobby Bailey
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Based on: Ferromagnets and Electromagnets <**http://legacy.cnx.org/content/m42368/1.4**> by OpenStax.

Module: **22.4 Magnetic Fields and Magnetic Field Lines**
Used here as: Magnetic Fields and Magnetic Field Lines
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52413/1.1/**
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Based on: Magnetic Fields and Magnetic Field Lines <**http://legacy.cnx.org/content/m42370/1.2**> by OpenStax.

Module: **22.5 Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field**
Used here as: Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52414/1.1/**
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Based on: Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field <**http://legacy.cnx.org/content/m42372/1.4**> by OpenStax.

Module: **22.7 Magnetic Force on a Current-Carrying Conductor**
Used here as: Magnetic Force on a Current-Carrying Conductor
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52416/1.1/**
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Based on: Magnetic Force on a Current-Carrying Conductor <**http://legacy.cnx.org/content/m42398/1.4**> by OpenStax.

Module: **22.8 Motors and Meters**
Used here as: Motors and Meters
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52417/1.1/**
Copyright: Bobby Bailey
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Based on: Torque on a Current Loop: Motors and Meters <**http://legacy.cnx.org/content/m42380/1.2**> by OpenStax.

Module: **22.9 Magnetic Fields Produced by Currents: Ampere's Law**
Used here as: Magnetic Fields Produced by Currents: Ampere's Law
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52420/1.2/**
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Based on: Magnetic Fields Produced by Currents: Ampere's Law <**http://legacy.cnx.org/content/m42382/1.2**> by OpenStax.

Module: **23.2 Induced Voltage and Magnetic Flux**
Used here as: Induced Voltage and Magnetic Flux
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52421/1.1/**
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Based on: Induced Emf and Magnetic Flux <**http://legacy.cnx.org/content/m42390/1.3**> by OpenStax.

Module: **23.3 Faraday's Law of Induction: Lenz's Law**
Used here as: Faraday's Law of Induction: Lenz's Law
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52425/1.1/**
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Based on: Faraday's Law of Induction: Lenz's Law <**http://legacy.cnx.org/content/m42392/1.4**> by OpenStax.

Module: **23.8 Transformers**
Used here as: Transformers
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52427/1.1/**
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Based on: Transformers <**http://legacy.cnx.org/content/m42414/1.5**> by OpenStax.

Module: **Alternating Current versus Direct Current (edited for Introduction to Physics)**
Used here as: Alternating Current versus Direct Current
By: Andrew Park

URL: **https://legacy.cnx.org/content/m67131/1.1/**
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Based on: 20.6 Alternating Current versus Direct Current <**http://legacy.cnx.org/content/m52407/1.2**> by Bobby Bailey.

Module: **Introduction to Light**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67031/1.3/**
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Based on: Introduction to Geometric Optics <**http://legacy.cnx.org/content/m42449/1.2**> by OpenStax.

Module: **24.2 Maxwell's Equations: Electromagnetic Waves Predicted and Observed**
Used here as: Maxwell's Equations: Electromagnetic Waves Predicted and Observed
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52443/1.1/**
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Based on: Maxwell's Equations: Electromagnetic Waves Predicted and Observed <**http://legacy.cnx.org/content/m42437/1.2**> by OpenStax.

Module: **24.3 Production of Electromagnetic Waves**
Used here as: Production of Electromagnetic Waves
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52452/1.1/**
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Based on: Production of Electromagnetic Waves <**http://legacy.cnx.org/content/m42440/1.5**> by OpenStax.

Module: **The Electromagnetic Spectrum: an Overview**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67132/1.2/**
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Based on: 24.4 The Electromagnetic Spectrum <**http://legacy.cnx.org/content/m52447/1.1**> by Bobby Bailey.

Module: **The Electromagnetic Spectrum: Application Notes**
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67133/1.1/**
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Based on: 24.4 The Electromagnetic Spectrum <**http://legacy.cnx.org/content/m52447/1.1**> by Bobby Bailey.

Module: **25.3 Reflection**
Used here as: Reflection
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52450/1.1/**
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Based on: The Law of Reflection <**http://legacy.cnx.org/content/m42456/1.4**> by OpenStax.

Module: **25.4 Refraction**
Used here as: Refraction
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52454/1.1/**
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Based on: The Law of Refraction <**http://legacy.cnx.org/content/m42459/1.7**> by OpenStax.

Module: **25.6 Dispersion: The Rainbow and Prisms**
Used here as: Dispersion: The Rainbow and Prisms
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52455/1.1/**
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Based on: Dispersion: The Rainbow and Prisms <**http://legacy.cnx.org/content/m42466/1.5**> by OpenStax.

Module: **Image Formation by Lenses (edited for Introduction to Physics)**
Used here as: Image Formation by Lenses
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67134/1.2/**
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Based on: 25.7 Image Formation by Lenses <**http://legacy.cnx.org/content/m52457/1.2**> by Bobby Bailey.

Module: **Image Formation by Mirrors (edited for Introduction to Physics)**
Used here as: Image Formation by Mirrors
By: Andrew Park
URL: **https://legacy.cnx.org/content/m67135/1.1/**
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Based on: 25.8 Image Formation by Mirrors <**http://legacy.cnx.org/content/m52459/1.2**> by Bobby Bailey.

Module: **27.9 Polarization**
Used here as: Polarization
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52456/1.1/**
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Based on: Polarization <**http://legacy.cnx.org/content/m42522/1.4**> by OpenStax.

Module: **30.1 Introduction to Atomic Physics**
Used here as: Introduction to Atomic Physics
By: Bobby Bailey
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Based on: Introduction to Atomic Physics <**http://legacy.cnx.org/content/m42585/1.2**> by OpenStax.

Module: **30.2 Discovery of the Atom**
Used here as: Discovery of the Atom
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52462/1.1/**
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Based on: Discovery of the Atom <**http://legacy.cnx.org/content/m42589/1.2**> by OpenStax.

Module: **30.3 Discovery of the Parts of the Atom: Electrons and Nuclei**
Used here as: Discovery of the Parts of the Atom: Electrons and Nuclei
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52463/1.1/**
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Based on: Discovery of the Parts of the Atom: Electrons and Nuclei <**http://legacy.cnx.org/content/m42592/1.6**> by OpenStax.

Module: **30.4 Bohr’s Theory of the Hydrogen Atom**
Used here as: Bohr’s Theory of the Hydrogen Atom
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52469/1.1/**
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Based on: Bohr’s Theory of the Hydrogen Atom <**http://legacy.cnx.org/content/m42596/1.6**> by OpenStax.

Module: **Introduction to Special Relativity**
By: OpenStax
URL: **https://legacy.cnx.org/content/m42525/1.5/**
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Module: **31.2 Nuclear Radioactivity**
Used here as: Nuclear Radioactivity
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52468/1.1/**
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Based on: Nuclear Radioactivity <**http://legacy.cnx.org/content/m42623/1.7**> by OpenStax.

Module: **31.4 Substructure of the Nucleus**
Used here as: Substructure of the Nucleus
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52466/1.1/**
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Based on: Substructure of the Nucleus <**http://legacy.cnx.org/content/m42631/1.3**> by OpenStax.

Module: **31.5 Nuclear Decay and Conservation Laws**
Used here as: Nuclear Decay and Conservation Laws
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52477/1.3/**
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Based on: Nuclear Decay and Conservation Laws <**http://legacy.cnx.org/content/m42633/1.6**> by OpenStax.

Module: **31.6 Half-Life and Activity**
Used here as: Half-Life and Activity
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52471/1.4/**
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Based on: Half-Life and Activity <**http://legacy.cnx.org/content/m42636/1.6**> by OpenStax.

Module: **31.7 Binding Energy**
Used here as: Binding Energy
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52475/1.1/**
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Based on: Binding Energy <**http://legacy.cnx.org/content/m42640/1.5**> by OpenStax.

Module: **32.6 Fusion**
Used here as: Fusion
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52480/1.1/**

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Based on: Fusion <**http://legacy.cnx.org/content/m42659/1.6**> by OpenStax.

Module: **32.7 Fission**
Used here as: Fission
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52479/1.2/**
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Based on: Fission <**http://legacy.cnx.org/content/m42662/1.8**> by OpenStax.

Module: **Appendix A: Atomic Masses**
Used here as: Atomic Masses
By: Bobby Bailey
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Based on: Atomic Masses <**http://legacy.cnx.org/content/m42699/1.3**> by OpenStax.

Module: **Appendix B: Selected Radioactive Isotopes**
Used here as: Selected Radioactive Isotopes
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52485/1.1/**
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Based on: Selected Radioactive Isotopes <**http://legacy.cnx.org/content/m42702/1.2**> by OpenStax.

Module: **Appendix C: Useful Information**
Used here as: Useful Information
By: Bobby Bailey
URL: **https://legacy.cnx.org/content/m52487/1.1/**
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